Rational Functions
Asymptotes and Holes in the graph

A RATIONAL FUNCTION is the quotient of two polynomials.

$f(x) = \frac{g(x)}{h(x)}$, where $h(x) \neq 0$

A HOLE IN THE GRAPH appears whenever the numerator and the denominator contain a common factor.

The graph of $f(x) = \frac{(x^2-9)}{(x-3)} = \frac{(x-3)(x+3)}{(x-3)}$

has a hole in the graph when , x = 3.

The graph of $f(x)$ looks like the graph of $f(x) = x+3$, but it has a hole at (3 , 6).

The calculator does not show the hole very clearly, but if you tried to trace for y at x = -3, you will not get a value for y.

VERTICAL ASYMPTOTES is a vertical line that the graph approaches but never intercepts.

You can find all vertical asymptotes by setting the factors of the denominator not common in the numerator equal to zero and solving for x. Setting the common factor(s) equal to zero ends up with both the numerator and the denominator equal to zero, which creates a hole.

NOTE: Vertical asymptotes are lines and must be in the form X = K, where K is a real number (a constant).

The graph $f(x) = \frac{(x+3)}{(x-3)}$

has a vertical asymptote at x = 3.

HORIZONTAL ASYMPTOTES are horizontal lines that the graph of $f(x)$ approaches as X approaches infinity (∞). The graph might intersect the horizontal asymptote for relatively small values of X.. Horizontal asymptotes only describe the end behavior.

CASE I
A horizontal asymptote exists if the highest degree in the numerator is the same as in the denominator. In this case the horizontal asymptote is the ratio of the leading coefficients. You'll learn why in calculus.
The function

$$f(x)=\frac{3x^2-5x-2}{4x^2+7x+1}$$

has a horizontal asymptote at $y=\frac{3}{4}$.

CASE II
If the degree of the numerator is less than the degree of the denominator, then there is a horizontal asymptote at y = 0.
The function $f(x)=\frac{x-1}{x^3-1}$
has a horizontal asymptote at y = 0, it also has a hole at x = 1.

SLANT ASYMPTOTES are lines with slopes not equal to zero that the graph does not cross. Slant asymptotes occur when the highest degree in the numerator is one more than the highest degree in the denominator.
The slant asymptotes are found by using long division. The slant asymptote is the equation formed by setting y equal to the quotient of the long division.
The function $f(x) = \frac{3X^2+4X+3}{X+2}$, has a slant asymptote at y = 3X-2

$$\begin{array}{rl} & 3X - 2 \\ X+2 & 3X^2 + 4X + 3 \\ & \underline{-(3X^2 + 6X)} \\ & -2X + 3 \\ & \underline{-(-2X - 4)} \\ & 7 \end{array}$$

⇦ The polynomials are arranged in descending degree. The first term ($3X^2$) is divided by (X), and your result is put as the first term in the quotient. Then you multiply that term in the quotient by the divisor and subtract it from the dividend and bring down your result (-2X+3).
Then you do the same procedure again. The remainder is insignificant as x approaches infinity and therefore ignored.

NOTE: Obviously, horizontal and slant asymptotes do not occur together. Either of them may occur with hole(s) and/or vertical asymptote(s).