

Basic Trigonometric Identities - sec. 7.1

A statement of equality between two expressions that is true for *all* values of the variable(s) for which the expressions are defined is called an **identity**.

An identity involving trigonometric expressions is a **trigonometric identity**.

Some of the following identities were covered in a previous chapter. They are listed here as review.

Reciprocal Identities:

$$\begin{array}{lll} \sin A = \frac{1}{\csc A} & \cos A = \frac{1}{\sec A} & \tan A = \frac{1}{\cot A} \\ \csc A = \frac{1}{\sin A} & \sec A = \frac{1}{\cos A} & \cot A = \frac{1}{\tan A} \end{array}$$

Example 1: If $\tan A = 0.8$, find $\cot A$.

$$\begin{aligned} \cot A &= \frac{1}{\tan A} \\ &= \frac{1}{0.8} \\ &= 1.25 \end{aligned}$$

Quotient Identities:

$$\begin{array}{ll} \frac{\sin A}{\cos A} = \tan A & \rightarrow \sin A = \cos A \tan A \\ \frac{\cos A}{\sin A} = \cot A & \rightarrow \cos A = \sin A \cot A \end{array}$$

Pythagorean Identities:

$$\begin{aligned} \sin^2 A + \cos^2 A &= 1 \\ \tan^2 A + 1 &= \sec^2 A \\ 1 + \cot^2 A &= \csc^2 A \end{aligned}$$

Example 2: If $\tan A = \frac{2}{5}$, find $\cos A$.

To find $\cos A$, first find $\sec A$.

$$\tan^2 A + 1 = \sec^2 A$$

$$\left(\frac{2}{5}\right)^2 + 1 = \sec^2 A$$

$$\frac{29}{25} = \sec^2 A$$

$$\pm \frac{\sqrt{29}}{5} = \sec A$$

Then, find $\cos A$

$$\cos A = \frac{1}{\sec A}$$

$$\cos A = \pm \frac{5}{\sqrt{29}} = \pm \frac{5\sqrt{29}}{29}$$

Example 3: Simplify $\frac{\sin^2 x + \sin^2 x \tan^2 x}{\sin^2 x(1 + \tan^2 x)}$.

$$\sin^2 x(\sec^2 x)$$

$$\sin^2 x \cdot \frac{1}{\cos^2 x}$$

$$\frac{\sin^2 x}{\cos^2 x}$$

$$\tan^2 x$$

Example 4: Simplify $\frac{\csc^2 x - \cot^2 x}{1}$ (using $1 + \cot^2 x = \csc^2 x$)

Example 5: Simplify $\frac{\sin^2 x + \cos^2 x}{\cos^2 x}$

$$\frac{1}{\cos^2 x}$$

$$\sec^2 x$$

Example 6: Simplify $\cos y \csc y$

$$\cos y \cdot \frac{1}{\sin y}$$

$$\frac{\cos y}{\sin y}$$

$$\cot y$$

Example 7: Simplify $2 \csc^2 x - \csc^4 x + \cot^4 x$

$$2 \csc^2 x + \cot^4 x - \csc^4 x$$

$$2 \csc^2 x + (\cot^2 x - \csc^2 x)(\cot^2 x + \csc^2 x)$$

$$2 \csc^2 x + (-1)(\cot^2 x + \csc^2 x)$$

$$2 \csc^2 x - \cot^2 x - \csc^2 x$$

$$\csc^2 x - \cot^2 x$$

$$1$$

Example 8: Simplify $\frac{\tan x}{\sin x}$

$$\frac{\frac{\sin x}{\cos x}}{\sin x}$$

$$\frac{\sin x}{\cos x} \div \sin x$$

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}$$

$$\frac{1}{\cos x}$$

$$\sec x$$