Practice B
Finding Real Roots of Polynomial Equations

Solve each polynomial equation by factoring.

1. \(9x^3 - 3x^2 - 3x + 1 = 0\)
2. \(x^5 - 2x^4 - 24x^3 = 0\)
3. \(3x^5 + 18x^4 - 21x^3 = 0\)
4. \(-x^4 + 2x^3 + 8x^2 = 0\)

Identify the roots of each equation. State the multiplicity of each root.

5. \(x^3 + 3x^2 + 3x + 1 = 0\)
6. \(x^3 + 5x^2 - 8x - 48 = 0\)

Identify all the real roots of each equation.

7. \(x^3 + 10x^2 + 17x = 28\)
8. \(3x^3 + 10x^2 - 27x = 10\)

Solve.

9. An engineer is designing a storage compartment in a spacecraft. The compartment must be 2 meters longer than it is wide and its depth must be 1 meter less than its width. The volume of the compartment must be 8 cubic meters.
   a. Write an equation to model the volume of the compartment.

   b. List all possible rational roots.

   c. Use synthetic division to find the roots of the polynomial equation. Are the roots all rational numbers?

   d. What are the dimensions of the storage compartment?
8. \(x^3 - 1^3\)
   \((x - 1)(x^2 + x + 1)\)

**Challenge**

1. \{((6)x + 8|x - 5)x + 1\}
2. \{((−2)x + 5|x - 1)x + 3\}x - 4
3. \(N(3) = 128\)
4. \(N(3) = 128\)
5. \(N(x) = 5x^3 - 3x^2 + 7x - 1\)
6. \(N(3) = 5 \cdot 3^3 - 3 \cdot 3^2 + 7 \cdot 3 - 1 = 128\)
7. \(P(x) = \{(2)x - 6|x - 2\}x - 30, P(4) = \{(2)4 - 6\}4 - 30 = \{(2)4 - 2\}4 - 30 = (6)4 - 30 = -6; \) The numbers in the innermost nests are the coefficients of the quotient, and the last number is the remainder. So the quotient is \(2x^2 + 2x + 6\) and the remainder is \(-6\).
8. The quotient is \(2x^2 + 2x + 6\) and the remainder is \(-6\).

**Problem Solving**

1. a. \(-2, 4, 6\)
   b. \((x + 2)(x - 4)(x - 6)\)
2. a. \(-4, -1, 0\)
   b. \((2x + 2)(x + 4)(x)\)
3. a. 5, 8, 9
   b. \((x - 5)(x - 8)(x - 9)\)
4. \[\begin{array}{|c|c|c|c|} 
\hline
\text{Basket} & \text{Dimensions (in terms of } x) & \text{Actual Dimensions} & \text{Volume} \\
\hline
A & (x - 5), (x - 8), (x - 9) & 7 \text{ by } 4 \text{ by } 3 & 84 \text{ cubic units} \\
B & (x + 2), (x - 4), (x - 6) & 14 \text{ by } 8 \text{ by } 6 & 672 \text{ cubic units} \\
C & (2x + 2), (x + 4), (x) & 26 \text{ by } 16 \text{ by } 12 & 4992 \text{ cubic units} \\
\hline
\end{array}\]
5. No; the dimensions of each basket are doubled from one size to the next except for 14 to 26.
6. No; \(\frac{84}{672} \neq \frac{672}{4992}\)

**Reading Strategies**

1. Multiply \((x + 3)\) and \((x^2 + 2)\).
2. No; there are no two factors that have \(x^2 + 2\) as their product.

3. 5
4. a. \((x^3 - 8x^2) + (-x + 8)\)
   b. \(x^2\)
   c. \(-1\)
   d. \(x^2(x - 8) - 1(x - 8) = (x - 8)(x^2 - 1)\)
   e. \(x^2 - 1; (x + 1)(x - 1)\)
   f. \((x - 8)(x + 1)(x - 1)\)

**LESSON 6-5**

**Practice A**

1. \(x^2 + 2x + 1; -1, 0\)
2. \(2x^2 - 18; -3, -2, 3\)
3. \(-5, 0\)
4. \(-1, 0, 7\)
5. \(x = -3\) with multiplicity 1; \(x = 0\) with multiplicity 1
6. \(x = 0\) with multiplicity 1; \(x = 1\) with multiplicity 2
7. \(x = 1\) with multiplicity 2; \(x = 5\) with multiplicity 1
8. \(x = -6\) with multiplicity 2; \(x = 0\) with multiplicity 1
9. \(\pm \frac{1}{3}, \pm 1\)
10. \(\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12\)
11. \(\pm \frac{1}{2}, \pm 1, \pm 2, \pm \frac{5}{2}, \pm 5, \pm 10\)
12. \(\pm 1, \pm 3, \pm 9\)
13. Stefan is correct. The roots of the expression are 2 and \(-2\), both of which have multiplicity 1.

**Practice B**

1. \(\frac{1}{3}, \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}\)
2. \(-4, 0, 6\)
3. \(-7, 0, 1\)
4. \(-2, 0, 4\)
5. \(x = -1\) with multiplicity 3
6. \(x = 3\) with multiplicity 1; \(x = -4\) with multiplicity 2
7. \(-4, 1, -7\)
8. \(-5, -\frac{1}{3}, 2\)
9. a. \(x^3 + x^2 - 2x - 8 = 0\)
b. $\pm 1, \pm 2, \pm 4, \pm 8$

c. $2, \frac{-3 \pm i\sqrt{7}}{2}$; no, 2 of the roots are irrational numbers.

d. 2 m wide, 4 m long, and 1 m deep

**Practice C**
1. $-5, 0, 7$  
2. $0, 3, 4$
3. $x = 2$ with multiplicity 3
4. $x = -4$ with multiplicity 2; $x = -2$ with multiplicity 1
5. $-8, 0, 6$
6. $3, 6, 2 \pm \sqrt{3}$
7. $-3, 0, 1$
8. $-3, 1, -3 \pm \sqrt{11}$
9. a. $2x^3 - 4x^2 - 64 = 0$
   b. $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64$
   c. $4, -1 \pm i\sqrt{7}$; no, 2 of the roots are irrational numbers.
   d. 4 in. wide, 8 in. long, and 2 in. deep

**Reteach**
1. $3x^4(x - 5)(x + 2); -2, 0, 5$
2. $x^2(x^2 - 5x + 6); x^2(x - 2)(x - 3); 0, 2, 3$
3. $2x(x^2 - 3x - 18); 2x(x - 6)(x + 3); -3, 0, 6$
4. $2x^4(x^2 - 16); 2x^4(x + 4)(x - 4); -4, 0, 4$
5. a. $\pm 1, \pm 3, \pm 5, \pm 15$
   b. 3 or 5

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c. $(x - 3)(x^2 - 4x - 5) = 0; (x - 3)(x - 5)(x + 1) = 0$

d. $x = 3$ or $x = 5$ or $x = -1$

**Challenge**
1. $y = (x + 3)(x)(x - 4)$
2. $y = -(x + 1)^2(x - 1)(x - 3)$

3. $y = (x + 5)(x + 2)^2 \left( x - \frac{1}{2} \right)(x - 3)$
4. $y = (x + 6)^2(x)(x - 1) \left( x - 3 \right)^2(x - 4)^2$
5. $y = (x + 3i)(x - 3i)(x - 1)^2 \left( x - \left( 1 + \sqrt{3} \right) \right)$
   \[ \left( x + \left( 1 + \sqrt{3} \right) \right) \]

**Problem Solving**
1. $V = w(w + 10)(w - 14)$
2. $w^3 - 4w^2 - 140w - 76,725 = 0$
3. No; yes; no
   The constant term is 76,725, which is not a multiple or 4 or 10, but is a multiple of 5.
4. Students should test possible roots that are multiples of 5 but not multiples of 10, such as 35, 45, and 55.
5. C  
6. A

**Reading Strategies**
1. Substitute the value of the root in the function and see if it equals 0.
2. $(x - 3)$ and $(x + 2)$
3. a. $(x + 4)$
   b. 3 times
4. $4x, (x - 3), (x + 3); -3, 0, 3$
5. $-x, (x - 5), (x - 1); 0, 1, 5$
6. $(x + 2), (x + 2), (x - 2); -2, 2$

**Lesson 6-6**

**Practice A**
1. 3  
2. 5  
3. 4
4. a. $P(x) = (x + 1)(x - 2)$
   b. $P(x) = x^2 + x(x - 2)$
   c. $P(x) = x^3 - 2x^2 + x^2 - 2x$
   d. $P(x) = x^3 - x^2 - 2x$
5. $P(x) = x^3 - 3x^2 - 13x + 15$
6. $P(x) = x^3 + 4x^2 - x - 4$
7. a. 2  
   b. $2i, -2i$