Chapter 8 – 2: NOTES   *Key*

Simplifying Rational Expressions

Definitions and General Properties of
Rational Numbers and Rational Expressions

A **rational number** can be written as a quotient of two integers, in the form $\frac{a}{b}$, where the denominator, b, is not 0.

A **rational expression** is the indicated quotient of two polynomials where the value of the denominator is assumed to be nonzero.

Sign rules of rational numbers and expressions:

1. $\dfrac{-a}{b} = \dfrac{a}{-b} = -\dfrac{a}{b}$

2. $\dfrac{-a}{-b} = \dfrac{a}{b}$

Fundamental Principle of Fractions:

If b and k are nonzero integers and a is any integer, then $\dfrac{a \bullet k}{b \bullet k} = \dfrac{a}{b}$.

Simplifying a rational expression:

1. Completely factor the polynomial given in the numerator and
2. Apply the fundamental principle of fractions by dividing the common
3. The simplest form will be the quotient of the product of remaining

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Warm-up 1. Reduce to lowest terms:

a) $\dfrac{-80}{-96} = \dfrac{2 \bullet 2 \bullet 2 \bullet 2 \bullet 5}{2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 3} = \dfrac{5}{6}$

b) $\dfrac{-12yz}{20y} = -\dfrac{2 \bullet 2 \bullet 3 \bullet y \bullet z \bullet}{2 \bullet 2 \bullet 5 \bullet y} = -\dfrac{3z}{5}$

c) $\dfrac{2m^2 - 18}{m^2 - 6m + 9} = \dfrac{2(m+3)(m-3)}{(m-3)(m-3)} = \dfrac{2m+4}{m-3}$

d) $\dfrac{x^2 - 4}{10 - 5x} = \dfrac{(x-2)(x+2)}{5(2-x)} = (-1)\dfrac{(x-2)(x+2)}{5(x-2)}$

$= -\left(\dfrac{x+2}{5}\right) \ or \ \dfrac{-x-2}{5}$

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Problems - Simplify:

1. $\dfrac{-90\,a^3 c}{-96\,abc^2}$ $\quad \dfrac{15a^2}{16\,bc}$

2. $\dfrac{9y^2 + 27y}{y^3 + 27} = \dfrac{9y}{y^2 - 3y + 9}$

3. $\dfrac{ay + 3a - 4y - 12}{2ay + 4a - 8y - 16} = \dfrac{y + 3}{2(y + 2)}$

4. $\dfrac{a^2 - 36}{18 - 3a} = \dfrac{a + 6}{-3} = -\left(\dfrac{a + 6}{3}\right)$

Multiplying and Dividing Rational Expressions

Multiplying Rational Expressions
Basic definition for multiplying rational numbers: If a, b, c, and d are integers with b and d not equal to zero,   then $\dfrac{a}{b} \bullet \dfrac{c}{d} = \dfrac{a \bullet c}{b \bullet d} = \dfrac{ac}{bd}$.   **Multiplying rational expressions:**   1. Completely factor each numerator and denominator.   2. Apply the basic definition for multiplying rational numbers by rewriting the numerator as a product of factors and rewrite the denominator as a product of factors.   3. Simplify by dividing common factors.   4. The result is the quotient of the product of remaining factors in the numerator and the product of remaining factors in the denominator.

Warm-up 1. Multiply and simplify:

a) $\dfrac{-5}{14} \bullet \dfrac{7}{-35} = \dfrac{5 \bullet 7}{2 \bullet 7 \bullet 5 \bullet 7} = \dfrac{1}{14}$

b) $\dfrac{5ab}{a + 7} \bullet \dfrac{a^2 - 49}{a^2 - 7a} = \dfrac{5ab(a-7)(a+7)}{a(a+7)(a-7)} = 5b$

c) $\dfrac{2a^2 - a - 15}{a^2 - 9} \bullet \dfrac{3a^2 + 8a - 3}{6a^2 + 17a + 5} = \dfrac{(2a+5)(a-3)(3a-1)(a+3)}{(a+3)(a-3)(2a+5)(3a+1)} = \dfrac{(3a-1)(a+3)}{(2a+5)(3a+1)}$

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Problems - Perform the indicated operation. Express in simplest form:

1. $\dfrac{6}{21} \cdot \dfrac{-7}{36}$ $-\dfrac{1}{18}$

2. $\dfrac{3xy}{x+5} \cdot \dfrac{2x^2 + 20x + 50}{xy - 5y}$ $= \dfrac{6x(x+5)}{(x-5)}$

3. $\dfrac{4n^2 - 4n - 24}{3n^2 + 5n - 2} \cdot \dfrac{3n^2 + 8n - 3}{9 - n^2}$ $= -4$

Dividing Rational Expressions

Basic definition for dividing rational numbers:

If a, b, c, and d are integers with b, c, and d not equal to zero, then

$$\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c} = \dfrac{ad}{bc}.$$

Note: $\dfrac{c}{d}$ and $\dfrac{d}{c}$ are called reciprocals or multiplicative inverses.

Dividing rational expressions:

1. Apply the basic definition for dividing rational numbers.
2. Follow the steps for multiplying rational expressions in summary 1.

Warm-up 2. Divide and simplify:

a) $\dfrac{5}{7} \div \dfrac{10}{-21} = \dfrac{5}{7} \cdot \dfrac{-21}{10} = -\dfrac{3}{2}$

b) $\dfrac{4x^2}{5y^2} \div \dfrac{24x^2y^2}{15xy} = \dfrac{4x^2}{5y^2} \cdot \dfrac{15xy}{24x^2y^2}$

$= \dfrac{4 \cdot 15 \cdot x \cdot y}{5 \cdot 24 \cdot x^2 \cdot y^4} = \dfrac{1}{2xy^3}$

c) $\dfrac{3m^2 + 2m - 1}{3m^2 + 14m - 5} \div \dfrac{-27 + 15m - 2m^2}{2m^2 - 7m - 9} = \dfrac{3m^2 + 2m - 1}{3m^2 + 14m - 5} \cdot \dfrac{2m^2 - 7m - 9}{-27 + 15m - 2m^2}$

$= \dfrac{(3m-1)(m+1)(2m-9)(m+1)}{(3m-1)(m+5)(-3+m)(9-2m)} = -\dfrac{(m+1)(m+1)}{(-3+m)(m+5)} = \dfrac{(m+1)^2}{(m-3)(m+5)}$

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Key

Problems - Perform the indicated operation. Express in simplest form:

4. $\dfrac{5}{12} \cdot \dfrac{8}{15} \div \dfrac{7}{18}$ $= \dfrac{4}{7}$

5. $\dfrac{5xy}{y+7} \div \dfrac{y^2-7y}{y^2-49}$ $= 5x$

6. $\dfrac{xy+xb+cy+cb}{xy-2xb+cy-2cb} \div \dfrac{18x^3+45x^2-27x}{3x^3-27x}$ $= \dfrac{(x-3)(y+b)}{3(2x-1)(y-2b)}$