Lesson 12-3

Objective - To write rules and find sums for arithmetic sequences.

An arithmetic sequence has a common difference between consecutive terms.

Rule for an Arithmetic Sequence

$a_n = a_1 + (n-1)d$

a_n represents the nth term of the sequence.

a_1 represents the first term.

n is the number of terms.

d is the difference.

Decide if each series is an arithmetic series.

1) $-5, -1, 3, 7, 11, \ldots$ Yes, difference = 4.

2) $4, 5, 7, 10, 14, \ldots$ No common difference.

3) $1, 4, 8, 12, 16, \ldots$ No common difference.

4) $-4, -8, -12, -16, -20, \ldots$ Yes, difference = -4.

Write a rule for the nth term of each sequemce. Then find a_{25}.

1) $48, 53, 58, 63, \ldots$ $a_n = a_1 + (n-1)d$

$a_n = 48 + (n-1)(53-48)$

$a_n = 5n + 43$ $a_{25} = 5(25) + 43 = 168$

2) $-21, -39, -57, -75, \ldots$ $a_n = a_1 + (n-1)d$

$a_n = -21 + (n-1)(-39-(-21))$

$a_n = -18n - 3$ $a_{25} = -18(25) - 3 = -453$

Write a rule for the nth term.

1) $a_{15} = 10, \ a_{20} = 25$ $d = \dfrac{25-10}{20-15} = 3$

$a_{15} = a_1 + (15-1)3$

$10 = a_1 + 42$

$a_1 = -32$

$a_n = -32 + (n-1)3$

$a_n = 3n - 35$

Write a rule for the nth term.

1) $a_{12} = -23, \ a_{27} = 37$ $d = \dfrac{37-(-23)}{27-12} = 4$

$a_{12} = a_1 + (12-1)4$

$-23 = a_1 + 44$

$a_1 = -67$

$a_n = -67 + (n-1)4$

$a_n = 4n - 71$

Write a rule for the nth term.

1) $a_{17} = 22 \quad d = -4$

$a_{17} = a_1 + (17-1)(-4)$

$22 = a_1 - 64$

$a_1 = 86$

$a_n = 86 + (n-1)(-4)$

$a_n = -4n + 90$

Lesson 12-3 (cont.)

Sum of a Finite Arithmetic Sequence

Sum of the first n terms of an arithmetic sequence

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

Find the sum of the first 20 terms

12, 18, 24, 30, 36,...

$$a_{20} = 12 + (20-1)6 = 126$$

$$S_{20} = n\left(\frac{a_1 + a_{20}}{2}\right) = 20\left(\frac{12+126}{2}\right) = 1380$$

Find the sum of the first 50 terms of this series,

$$34 + 45 + 56 + 67 + 78 + \ldots$$

$$a_{50} = 34 + (50-1)11$$

$$a_{50} = 573$$

$$S_{50} = n\left(\frac{a_1 + a_{50}}{2}\right)$$

$$S_{50} = 50\left(\frac{34+573}{2}\right)$$

$$S_{50} = 15{,}175$$

Find n if $S_n = 20$ for the following series,

$$-16 + (-12) + (-8) + (-4) + 0 + \ldots$$

$$a_n = a_1 + (n-1)d \qquad S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

$$a_n = -16 + (n-1)4$$

$$a_n = 4n - 20 \qquad 20 = n\left(\frac{-16 + 4n - 20}{2}\right)$$

$$20 = 2n^2 - 18n$$

$$0 = 2n^2 - 18n - 20$$

$$0 = n^2 - 9n - 10$$

$$0 = (n-10)(n+1)$$

$$n = 10 \quad n = -1$$