Lesson 12-4

Objective - To write rules and find sums for geometric sequences.

A geometric sequence has a common ratio between consecutive terms.

Rule for a Geometric Sequence

$a_n = a_1 r^{n-1}$

a_n represents the nth term of the sequence.

a_1 represents the first term.

n is the number of terms.

r is the common ratio.

Decide if each series is a geometric series.

1) $-5, -10, -20, -40, \ldots$ Yes, ratio = 2.

2) $\frac{4}{7}, \frac{8}{49}, \frac{16}{343}, \frac{32}{2401}, \ldots$ Yes, ratio = 2/7.

3) $1, 4, 8, 12, 16, \ldots$ No common ratio.

4) $128, 64, 32, 16, 8, \ldots$ Yes, ratio = 1/2.

Write a rule for the nth term of each sequemce. Then find a_{10}.

1) $2, 16, 128, 1024, \ldots$ $a_n = a_1 r^{n-1}$

$a_n = 2 \bullet \left(\frac{16}{2}\right)^{n-1}$ $a_n = 2 \bullet 8^{n-1}$

$a_{10} = 2 \bullet 8^9 = 268,435,456$

2) $\frac{2}{5}, \frac{6}{25}, \frac{18}{125}, \frac{54}{625}, \ldots$

$a_n = \frac{2}{5} \bullet \left(\frac{\frac{6}{25}}{\frac{2}{5}}\right)^{n-1}$ $a_n = \frac{2}{5} \bullet \left(\frac{3}{5}\right)^{n-1}$ $a_{10} = \frac{2}{5} \bullet \frac{3^9}{5} = 0.004$

Write a rule for the nth term.

1) $a_2 = -1, \ r = 2$ Write the first few terms.

-0.5, -1, -2, -4, -8,…

$a_n = a_1 \bullet r^{n-1}$

$a_n = (-0.5) \bullet 2^{n-1}$

Write a rule for the nth term.

1) $a_{12} = \frac{2}{3}, \ r = 3$

$a_{12} = a_1 (3)^{12-1}$

$\frac{2}{3} = a_1 (3)^{11}$

$a_1 = \frac{2}{3} \div 3^{11}$

$a_1 = 0.0000038$

$a_n = 0.0000038 \bullet (3)^{n-1}$

Write a rule for the nth term.

1) $a_2 = 8 \quad a_5 = 512$

$a_2 = a_1 \bullet r^{2-1} \longrightarrow 8 = a_1 \bullet r \qquad a_1 = \frac{8}{r}$

$a_5 = a_1 \bullet r^{5-1} \longrightarrow 512 = a_1 \bullet r^4$

$512 = \frac{8}{r} \bullet r^4$

$512 = 8 \bullet r^3$

$64 = r^3$

$r = 4$

$a_1 = \frac{8}{4} = 2$

$a_n = 2 \bullet (4)^{n-1}$

Lesson 12-4 (cont.)

Sum of a Finite Geometric Sequence

Sum of the first n terms of a geometric sequence

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

Find the sum of the first 6 terms

2, 10, 50, 250,... $r = \frac{10}{2} = 5$

$$S_6 = 2\left(\frac{1-5^6}{1-5}\right) = 2\left(\frac{-15,624}{-4}\right) = 7812$$

Find the sum of the first 6 terms of this seires,

$1+7+49+343+...$

$$r = \frac{7}{1} = 7$$

$$S_6 = 1\left(\frac{1-7^6}{1-7}\right)$$

$$S_6 = \frac{-117,648}{-6}$$

$$S_6 = 19,608$$

Find n if S_n =55,987 for the following series,

$1+6+36+216+...$

$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$ $\quad 55,987 = 1\left(\frac{1-6^n}{1-6}\right)$

$$55,987 = \frac{1-6^n}{-5}$$

$$-279,935 = 1-6^n$$

$$279,936 = 6^n$$

$$\log(279,936) = \log(6^n)$$

$\log(279,936) = n\log 6$ $\quad n = \frac{\log(279,936)}{\log 6} = 7$