Name ______________________ Date ___________ Class ___________

LESSON 12-5

Practice B

Mathematical Induction and Infinite Geometric Series

Determine whether each geometric series converges or diverges.

1. $\frac{81}{625} + \frac{27}{125} + \frac{9}{25} + \frac{3}{5} + 1 + \cdots$

2. $1 - \frac{3}{5} + \frac{9}{25} - \frac{27}{125} + \frac{81}{625} - \cdots$

Find the sum of each infinite geometric series, if it exists.

3. $7 + \frac{7}{4} + \frac{7}{16} + \frac{7}{64} + \cdots$

4. $500 - 300 + 180 - 108 + \cdots$

5. $\sum_{k=1}^{\infty} \frac{1}{4}\left(\frac{4}{3}\right)^k$

6. $\sum_{k=1}^{\infty} 99\left(-\frac{4}{9}\right)^k$

Write each repeating decimal as a fraction in simplest form.

7. $0.\overline{16}$

8. $0.\overline{016}$

9. $0.0\overline{16}$

10. $0.0\overline{45}$

11. $0.\overline{1}$

12. $0.\overline{123}$

Identify a counterexample to disprove each statement.

13. $2^{-n} < n^2$

14. $n^3 \geq 3n$

Solve.

15. Ron won a prize that pays $200,000 the first year and half of the previous year's amount each year for the rest of his life.

a. Write the first 4 terms of a series to represent the situation.

b. Write a general rule for a geometric sequence that models his prize each year.

c. Estimate Ron's total prize in the first 10 years.

d. If Ron lives forever, what is the total of his winnings?

LESSON 12-5

Practice A

Mathematical Induction and Infinite Geometric Series

Determine whether the geometric series converges or diverges.

1. $25 + 20 + 16 + 12.8 + \ldots$

a. Find and simplify the common ratio. $\frac{4}{5}$

b. Use the ratio to determine whether the series converges or diverges. Converges

2. $\frac{1}{16} + \frac{1}{8} + \frac{1}{4} + \frac{1}{2} + \cdots$

Diverges

3. $35.2 + 8.8 + 2.2 + 0.55 + \cdots$

Converges

Find the sum of each infinite geometric series, if it exists.

4. $864 + 576 + 384 + 256 + \cdots$

a. Test the series for convergence. Converges

b. Use the formula $S = \frac{a_1}{1 - r}$ to find the sum. 2592

5. $200 + 150 + 112.5 + 84.375 + \cdots$

800

6. $\frac{2}{5} - \frac{4}{25} + \frac{8}{125} - \frac{16}{625} + \cdots$

$\frac{2}{7}$

7. $\frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \frac{16}{625} + \cdots$

$\frac{2}{3}$

8. $\frac{16}{625} - \frac{8}{125} + \frac{4}{25} - \frac{2}{5} + \cdots$

Does not exist

Write each repeating decimal as a fraction in simplest form.

9. $0.\overline{57}$

a. Write the decimal as an infinite series.

$0.57 + 0.0057 + 0.000057 + \ldots$

b. Find the common ratio.

0.01

c. Use $S = \frac{a_1}{1 - r}$ to write the fraction.

$\frac{19}{33}$

10. $0.\overline{7}$

$\frac{7}{9}$

11. $0.8\overline{3}$

$\frac{5}{6}$

12. $0.\overline{23}$

$\frac{23}{99}$

LESSON 12-5

Practice B

Mathematical Induction and Infinite Geometric Series

Determine whether each geometric series converges or diverges.

1. $\frac{81}{625} + \frac{27}{125} + \frac{9}{25} + \frac{3}{5} + 1 + \cdots$

Diverges

2. $1 - \frac{3}{5} + \frac{9}{25} - \frac{27}{125} + \frac{81}{625} - \cdots$

Converges

Find the sum of each infinite geometric series, if it exists.

3. $7 + \frac{7}{4} + \frac{7}{16} + \frac{7}{64} + \cdots$

$\frac{28}{3}$

4. $500 - 300 + 180 - 108 + \cdots$

312.5

5. $\sum_{k=1}^{\infty} \frac{1}{4}\left(\frac{4}{3}\right)^k$

Does not exist

6. $\sum_{k=1}^{\infty} 99\left(-\frac{4}{9}\right)^k$

≈ -30.46

Write each repeating decimal as a fraction in simplest form.

7. $0.\overline{16}$

$\frac{16}{99}$

8. $0.\overline{016}$

$\frac{16}{999}$

9. $0.0\overline{16}$

$\frac{16}{990}$

10. $0.0\overline{45}$

$\frac{1}{22}$

11. $0.\overline{1}$

$\frac{1}{9}$

12. $0.\overline{123}$

$\frac{41}{333}$

Identify a counterexample to disprove each statement.

13. $2^{-n} < n^2$

Possible answer: $n = -5$

14. $n^3 \geq 3n$

Possible answer: $n = -2$

Solve.

15. Ron won a prize that pays $200,000 the first year and half of the previous year's amount each year for the rest of his life.

a. Write the first 4 terms of a series to represent the situation.

$200{,}000 + 100{,}000 + 50{,}000 + 25{,}000 + \ldots$

b. Write a general rule for a geometric sequence that models his prize each year. $a_n = 200{,}000(0.5)^{n-1}$

c. Estimate Ron's total prize in the first 10 years. ≈$399,609.38

d. If Ron lives forever, what is the total of his winnings? $400,000

LESSON 12-5

Practice C

Mathematical Induction and Infinite Geometric Series

Find the sum of each infinite geometric series, if it exists.

1. $-10 + 0.4 - 0.016 + 0.00064 - \ldots$

$\frac{250}{26}$

2. $54 + 18 + 6 + 2 + \ldots$

81

3. $\sum_{k=1}^{\infty} \frac{2}{3}\left(\frac{3}{2}\right)^k$

Does not exist

4. $\sum_{k=1}^{\infty} 100\left(-\frac{3}{4}\right)^k$

$-\frac{300}{7}$

Write each repeating decimal as a fraction in simplest form.

5. $0.\overline{72}$

$\frac{8}{11}$

6. $0.0\overline{72}$

$\frac{4}{55}$

7. $0.\overline{072}$

$\frac{8}{111}$

8. $3.6\overline{552}$

$\frac{6086}{1665}$

9. $1.4\overline{09}$

$\frac{31}{22}$

10. $0.\overline{714285}$

$\frac{5}{7}$

Use mathematical induction to prove.

11. $\sum_{k=1}^{n} \frac{2}{k(k+1)} = 1 + \frac{1}{3} + \ldots + \frac{2}{n(n+1)} = \frac{2n}{n+1}$

For $n = 1$: $\frac{2(1)}{(1)+1} = 1$ If the statement is true for $n = a$, then $\sum_{k=1}^{a} \frac{2}{k(k+1)} = \frac{2a}{a+1}$.

Let $n = a + 1$. $\sum_{k=1}^{a+1} \frac{2}{k(k+1)} = \frac{2a}{a+1} + \frac{2}{(a+1)(a+2)} = \frac{2a(a+2)+2}{(a+1)(a+2)} =$

$\frac{2(a+1)^2}{(a+1)(a+2)} = \frac{2(a+1)}{(a+1)+1}$; So $\sum_{k=1}^{n} \frac{2}{k(k+1)} = \frac{2n}{n+1}$.

Identify a counterexample to disprove each statement.

12. $(2n + 1)^3 > n^2$

Possible answer: $n = -1$

13. $4n^2 \leq 2n^4$

Possible answer: $n = 1$

Solve.

14. A movie earned $60 million in the first week that it was released. In each successive week, sales declined by about 20%.

a. Write a general rule for a geometric sequence that models the movie's sales each week. $a_n = 60(0.8)^{n-1}$, where a_n represents millions of dollars

b. Estimate the movie's total sales in the first 8 weeks. ≈$249.67 million

c. If this pattern continued indefinitely, what would be the movie's total sales? $300 million

LESSON 12-5

Reteach

Mathematical Induction and Infinite Geometric Series

An **infinite geometric series** has infinitely many terms.

You add some of the terms of an infinite geometric series to find a partial sum of the series.

When the partial sum approaches a fixed number, the series is said to **converge**. — A geometric series converges when $|r| < 1$.

When the partial sum does not approach a fixed number, the series is said to **diverge**. — A geometric series diverges when $|r| \geq 1$.

To determine whether a geometric series converges or diverges, compare the absolute value of the common ratio of the terms of the series, denoted $|r|$, to 1.

$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} + \cdots$	$2 - 4 + 8 - 16 + 32 + \cdots$
Step 1 Find the common ratio. $r = \frac{-\frac{1}{2}}{1} = -\frac{1}{2}$	**Step 1** Find the common ratio. $r = \frac{-4}{2} = -2$
Step 2 Compare $\lvert r \rvert$ to 1. $\lvert r \rvert = \left\lvert -\frac{1}{2} \right\rvert = \frac{1}{2}$ Since $\frac{1}{2} < 1$, the series converges.	**Step 2** Compare $\lvert r \rvert$ to 1. $\lvert r \rvert = \lvert -2 \rvert = 2$ Since $2 \geq 1$, the series diverges.

Determine whether each geometric series converges or diverges.

1. $1 + \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \frac{81}{16} + \cdots$

$r =$ $\frac{3}{2}$

Compare $|r|$ to 1. $\left|\frac{3}{2}\right| \geq 1$

diverges

2. $256 + 64 + 16 + 4 + 1 + \cdots$

$r =$ $\frac{1}{4}$

Compare $|r|$ to 1. $\left|\frac{1}{4}\right| < 1$

converges

3. $1 - \frac{1}{5} + \frac{1}{25} - \frac{1}{125} + \frac{1}{625} + \cdots$

$r =$ $-\frac{1}{5}$

Compare $|r|$ to 1. $\left|-\frac{1}{5}\right| < 1$

converges

4. $6 + 9 + 13.5 + 20.25 + \cdots$

$r =$ 1.5

Compare $|r|$ to 1. $|1.5| \geq 1$

diverges