

**LESSON**  
**14-3**
**Practice B**
**Fundamental Trigonometric Identities**

Prove each trigonometric identity.

1.  $\sin^2 \theta + \sin^2 \theta \cot^2 \theta = 1$

2.  $\cot^2 \theta \cos^2 \theta = \cot^2 \theta - \cos^2 \theta$

3.  $\tan^2 \theta - \tan^2 \theta \sin^2 \theta = \sin^2 \theta$

4.  $\frac{\sin \theta + \cos \theta}{\sin \theta \cos \theta} = \sec \theta + \csc \theta$

Rewrite each expression in terms of  $\cos \theta$ . Then simplify.

5.  $2 \sin \theta \cos \theta \cot \theta$

6.  $\frac{1 + \cot \theta}{\cot \theta (\sin \theta + \cos \theta)}$

7.  $\cos^4 \theta - \sin^4 \theta + \sin^2 \theta$

Solve.

8. Use the equation  $mg \sin \theta = \mu mg \cos \theta$  to determine the angle at which a waxed wood block on an inclined plane of wet snow begins to slide. Assume  $\mu = 0.17$ .

$$\frac{1 + \cos^2 \theta}{\frac{1}{\cos^2 \theta} - \frac{1}{\cos^2 \theta} + 1} =$$

$$1 + \cos^2 \theta$$

8.  $48^\circ$

### Practice B

1.  $\sin^2 \theta + \sin^2 \theta \cot^2 \theta = 1$

$$\sin^2 \theta (1 + \cot^2 \theta) = 1$$

$$\sin^2 \theta (\csc^2 \theta) = 1$$

$$\sin^2 \theta \left( \frac{1}{\sin^2 \theta} \right) = 1$$

$$1 = 1$$

2.  $\cot^2 \theta \cos^2 \theta = \cot^2 \theta - \cos^2 \theta =$

$$\frac{\cos^2 \theta}{\sin^2 \theta} - \cos^2 \theta = \frac{\cos^2 \theta}{\sin^2 \theta} - \frac{\sin^2 \theta \cos^2 \theta}{\sin^2 \theta} =$$

$$\frac{\cos^2 \theta - \sin^2 \theta \cos^2 \theta}{\sin^2 \theta} = \frac{\cos^2 \theta (1 - \sin^2 \theta)}{\sin^2 \theta} =$$

$$\left( \frac{\cos^2 \theta}{\sin^2 \theta} \right) (1 - \sin^2 \theta) = \cot^2 \theta (1 - \sin^2 \theta) =$$

$$\cot^2 \theta (\cos^2 \theta + \sin^2 \theta - \sin^2 \theta) = \cot^2 \theta \cos^2 \theta$$

3.  $\tan^2 \theta - \tan^2 \theta \sin^2 \theta = \sin^2 \theta$

$$\tan^2 \theta (1 - \sin^2 \theta) = \sin^2 \theta$$

$$\tan^2 \theta (\cos^2 \theta) = \sin^2 \theta$$

$$\frac{\sin^2 \theta}{\cos^2 \theta} (\cos^2 \theta) = \sin^2 \theta$$

$$(\sin^2 \theta) \frac{\cos^2 \theta}{\cos^2 \theta} = \sin^2 \theta$$

$$\sin^2 \theta = \sin^2 \theta$$

4.  $\frac{\sin \theta + \cos \theta}{\sin \theta \cos \theta} = \sec \theta + \csc \theta$

$$\frac{\sin \theta}{\sin \theta \cos \theta} + \frac{\cos \theta}{\sin \theta \cos \theta} = \sec \theta + \csc \theta$$

$$\left( \frac{\sin \theta}{\sin \theta} \right) \frac{1}{\cos \theta} + \frac{1}{\sin \theta} \left( \frac{\cos \theta}{\cos \theta} \right) = \sec \theta + \csc \theta$$

$$(1) \frac{1}{\cos \theta} + \frac{1}{\sin \theta} (1) = \sec \theta + \csc \theta$$

$$\frac{1}{\cos \theta} + \frac{1}{\sin \theta} = \sec \theta + \csc \theta$$

$$\sec \theta + \csc \theta = \sec \theta + \csc \theta$$

5.  $2 \sin \theta \cos \theta \cot \theta =$

$$2 \sin \theta \cos \theta \cdot \frac{\cos \theta}{\sin \theta} = 2 \cos^2 \theta$$

6.  $\frac{1 + \cot \theta}{\cot \theta (\sin \theta + \cos \theta)} =$

$$\frac{1 + \frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta}{\sin \theta} (\sin \theta + \cos \theta)} =$$

$$\frac{\frac{\sin \theta + \cos \theta}{\sin \theta}}{\frac{\cos \theta (\sin \theta + \cos \theta)}{\sin \theta}} =$$

$$\frac{\sin \theta + \cos \theta}{\cos \theta (\sin \theta + \cos \theta)} = \frac{1}{\cos \theta}$$

7.  $\cos^4 \theta - \sin^4 \theta + \sin^2 \theta = (\cos^2 \theta + \sin^2 \theta)$   
 $(\cos^2 \theta - \sin^2 \theta) + \sin^2 \theta (1) (\cos^2 \theta - \sin^2 \theta)$   
 $+ \sin^2 \theta = \cos^2 \theta$

8.  $9.6^\circ$

### Practice C

1.  $\frac{\cos \theta}{1 + \sin \theta} = \frac{1 - \sin \theta}{\cos \theta}$

$$\frac{\cos \theta}{1 + \sin \theta} \left( \frac{1 - \sin \theta}{1 - \sin \theta} \right) = \frac{1 - \sin \theta}{\cos \theta}$$

$$\frac{\cos \theta (1 - \sin \theta)}{1 - \sin^2 \theta} = \frac{1 - \sin \theta}{\cos \theta}$$

$$\frac{\cos \theta (1 - \sin \theta)}{\cos^2 \theta} = \frac{1 - \sin \theta}{\cos \theta}$$

$$\frac{1 - \sin \theta}{\cos \theta} = \frac{1 - \sin \theta}{\cos \theta}$$

2.  $\tan \theta \sec \theta = \frac{\sec \theta + \tan \theta}{\cos \theta + \cot \theta} = \frac{\sec \theta + \frac{\sin \theta}{\cos \theta}}{\cos \theta + \frac{\cos \theta}{\sin \theta}}$