Name ______________________ Date __________ Class __________

LESSON 14-3

Practice A

Fundamental Trigonometric Identities

Prove each trigonometric identity.

1. $\dfrac{\sin\theta}{1 - \cos^2\theta} = \csc\theta$

a. Modify the left-hand side. Replace 1 with a known identify. ____________

b. Simplify the denominator. ____________

c. Keep simplifying and substituting identities until the left side matches the right side. ____________

2. $\cos^4\theta + \cos^2\theta\sin^2\theta + \sin^2\theta = 1$

3. $\cot\theta = \dfrac{\csc\theta}{\sec\theta}$

____________ ____________

Rewrite each expression in terms of cosθ. Then simplify.

4. $\dfrac{\cot\theta}{2\csc\theta}$

a. Rewrite each function in terms of $\sin\theta$ or $\cos\theta$. ____________

b. Rewrite the fraction in simplest form. ____________

5. $\sin\theta\cot\theta$

6. $\dfrac{\cos\theta - \cos^2\theta}{\sin^2\theta}$

7. $\dfrac{\sin^2\theta + 2\cos^2\theta}{\sec^2\theta - \tan^2\theta}$

____________ ____________ ____________

Solve.

8. Use the equation $mg\sin\theta = \mu mg\cos\theta$ to determine the angle at which a desk can be tilted before a paperback book on the desk begins to slide. Assume $\mu = 1.11$. ____________

LESSON 14-3

Practice A

Fundamental Trigonometric Identities

Prove each trigonometric identity.

1. $\frac{\sin\theta}{1-\cos^2\theta} = \csc\theta$

   a. Modify the left-hand side. Replace 1 with a known identify. $\frac{\sin\theta}{\cos^2\theta + \sin^2\theta - \cos^2\theta}$

   b. Simplify the denominator. $\frac{\sin\theta}{\sin^2\theta}$

   c. Keep simplifying and substituting identities until the left side matches the right side. $\frac{1}{\sin\theta} = \csc\theta;\ \csc\theta = \csc\theta$

2. $\cos^4\theta + \cos^2\theta\sin^2\theta + \sin^2\theta = 1$

   $\cos^4\theta + \cos^2\theta\sin^2\theta + \sin^2\theta = 1$
   $\cos^2\theta(\cos^2\theta + \theta\sin^2\theta) + \sin^2\theta = 1$
   $\cos^2\theta(\cos^2\theta + \sin^2\theta) + \sin^2\theta = 1$
   $\cos^2\theta\,(1) + \sin^2\theta = 1$
   $\cos^2\theta + \sin^2\theta = 1$
   $1 = 1$

3. $\cot\theta = \frac{\csc\theta}{\sec\theta}$

   $\cot\theta = \frac{\csc\theta}{\sec\theta};\ \cot\theta = \frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}};$
   $\cot\theta = \frac{\cos\theta}{\sin\theta};\ \cot\theta = \cot\theta$

Rewrite each expression in terms of cosθ. Then simplify.

4. $\frac{\cot\theta}{2\csc\theta}$

   a. Rewrite each function in terms of $\sin\theta$ or $\cos\theta$. $\frac{\frac{\cos\theta}{\sin\theta}}{\frac{2}{\sin\theta}}$

   b. Rewrite the fraction in simplest form. $\frac{\sin\theta\cos\theta}{2\sin\theta} = \frac{\cos\theta}{2}$

5. $\sin\theta\cot\theta$

   $\sin\theta\cot\theta =$
   $\sin\theta\cdot\frac{\cos\theta}{\sin\theta} =$
   $\cos\theta$

6. $\frac{\cos\theta - \cos^2\theta}{\sin^2\theta}$

   $\frac{\cos\theta - \cos^2\theta}{\sin^2\theta} =$
   $\frac{\cos\theta - \cos^2\theta}{1-\cos^2\theta} =$
   $\frac{\cos\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} =$
   $\frac{\cos\theta}{1+\cos\theta}$

7. $\frac{\sin^2\theta + 2\cos^2\theta}{\sec^2\theta - \tan^2\theta}$

   $\frac{\sin^2\theta + 2\cos^2\theta}{\sec^2\theta - \tan^2\theta} =$
   $\frac{1-\cos^2\theta + 2\cos^2\theta}{\frac{1}{\cos^2\theta} - (\sec^2\theta - 1)} =$
   $\frac{1+\cos^2\theta}{\frac{1}{\cos^2\theta} - \frac{1}{\cos^2\theta} + 1} =$
   $1+\cos^2\theta$

Solve.

8. Use the equation $mg\sin\theta = \mu mg\cos\theta$ to determine the angle at which a desk can be tilted before a paperback book on the desk begins to slide. Assume $\mu = 1.11$. **48°**

LESSON 14-3

Practice B

Fundamental Trigonometric Identities

Prove each trigonometric identity.

1. $\sin^2\theta + \sin^2\theta\cot^2\theta = 1$

   $\sin^2\theta + \sin^2\theta\cot^2\theta = 1$
   $\sin^2\theta\,(1+\cot^2\theta) = 1$
   $\sin^2\theta(\csc^2\theta) = 1$
   $\sin^2\theta\left(\frac{1}{\sin^2\theta}\right) = 1$
   $1 = 1$

2. $\cot^2\theta\cos^2\theta = \cot^2\theta - \cos^2\theta$

   $\cot^2\theta\cos^2\theta = \cot^2\theta - \cos^2\theta =$
   $\frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta = \frac{\cos^2\theta}{\sin^2\theta} - \frac{\sin^2\theta\cos^2\theta}{\sin^2\theta} =$
   $\frac{\cos^2\theta - \sin^2\theta\cos^2\theta}{\sin^2\theta} = \frac{\cos^2\theta\,(1-\sin^2\theta)}{\sin^2\theta} =$
   $\left(\frac{\cos^2\theta}{\sin^2\theta}\right)(1-\sin^2\theta) = \cot^2\theta\,(1-\sin^2\theta) =$
   $\cot^2\theta\,(\cos^2\theta + \sin^2\theta - \sin^2\theta) = \cot^2\theta\cos^2\theta$

3. $\tan^2\theta - \tan^2\theta\sin^2\theta = \sin^2\theta$

   $\tan^2\theta - \tan^2\theta\sin^2\theta = \sin^2\theta$
   $\tan^2\theta(1-\sin^2\theta) = \sin^2\theta$
   $\tan^2\theta(\cos^2\theta) = \sin^2\theta$
   $\frac{\sin^2\theta}{\cos^2\theta}(\cos^2\theta) = \sin^2\theta$
   $(\sin^2\theta)\frac{\cos^2\theta}{\cos^2\theta} = \sin^2\theta$
   $\sin^2\theta = \sin^2\theta$

4. $\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = \sec\theta + \csc\theta$

   $\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = \sec\theta + \csc\theta$
   $\frac{\sin\theta}{\sin\theta\cos\theta} + \frac{\cos\theta}{\sin\theta\cos\theta} = \sec\theta + \csc\theta$
   $\left(\frac{\sin\theta}{\sin\theta}\right)\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\left(\frac{\cos\theta}{\cos\theta}\right) = \sec\theta + \csc\theta$
   $(1)\frac{1}{\cos\theta} + \frac{1}{\sin\theta}(1) = \sec\theta + \csc\theta$
   $\frac{1}{\cos\theta} + \frac{1}{\sin\theta} = \sec\theta + \csc\theta$
   $\sec\theta + \csc\theta = \sec\theta + \csc\theta$

Rewrite each expression in terms of cosθ. Then simplify.

5. $2\sin\theta\cos\theta\cot\theta$

   $2\sin\theta\cos\theta\cot\theta =$
   $2\sin\theta\cos\theta\bullet\frac{\cos\theta}{\sin\theta} =$
   $2\cos^2\theta$

6. $\frac{1+\cot\theta}{\cot\theta(\sin\theta + \cos\theta)}$

   $\frac{1+\cot\theta}{\cot\theta(\sin\theta+\cos\theta)} =$
   $\frac{1+\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}(\sin\theta+\cos\theta)} =$
   $\frac{\frac{\sin\theta+\cos\theta}{\sin\theta}}{\frac{\cos\theta(\sin\theta+\cos\theta)}{\sin\theta}} =$
   $\frac{\sin\theta+\cos\theta}{\cos\theta(\sin\theta+\cos\theta)} = \frac{1}{\cos\theta}$

7. $\cos^4\theta - \sin^4\theta + \sin^2\theta$

   $\cos^4\theta - \sin^4\theta + \sin^2\theta$
   $= (\cos^2\theta + \sin^2\theta)$
   $(\cos^2\theta - \sin^2\theta) + \sin^2\theta$
   $(1)(\cos^2\theta - \sin^2\theta) +$
   $\sin^2\theta = \cos^2\theta$

Solve.

8. Use the equation $mg\sin\theta = \mu mg\cos\theta$ to determine the angle at which a waxed wood block on an inclined plane of wet snow begins to slide. Assume $\mu = 0.17$. **9.6°**

LESSON 14-3

Practice C

Fundamental Trigonometric Identities

Prove each trigonometric identity.

1. $\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$

   $\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$
   $\frac{\cos\theta}{1+\sin\theta}\left(\frac{1-\sin\theta}{1-\sin\theta}\right) = \frac{1-\sin\theta}{\cos\theta}$
   $\frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta} = \frac{1-\sin\theta}{\cos\theta}$
   $\frac{\cos\theta(1-\sin\theta)}{\cos^2\theta} = \frac{1-\sin\theta}{\cos\theta}$
   $\frac{1-\sin\theta}{\cos\theta} = \frac{1-\sin\theta}{\cos\theta}$

2. $\tan\theta\sec\theta = \frac{\sec\theta + \tan\theta}{\cos\theta + \cot\theta}$

   $\tan\theta\sec\theta = \frac{\sec\theta+\tan\theta}{\cos\theta+\cot\theta} = \frac{\sec\theta + \frac{\sin\theta}{\cos\theta}}{\cos\theta + \frac{\cos\theta}{\sin\theta}}$
   $= \frac{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}{\cos\theta + \frac{\cos\theta}{\sin\theta}} = \frac{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta\cos\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}}$
   $= \frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{\sin\theta\cos\theta+\cos\theta}{\sin\theta}} = \frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{\cos\theta(\sin\theta+1)}{\sin\theta}}$
   $= \frac{\sin\theta(1+\sin\theta)}{\cos^2\theta(\sin\theta+1)} = \frac{\sin\theta}{\cos^2\theta}$
   $= \left(\frac{\sin\theta}{\cos\theta}\right)\frac{1}{\cos\theta} = (\tan\theta)\frac{1}{\cos\theta}$
   $= \tan\theta\sec\theta$

3. $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$

   $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$
   $\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta = \tan^2\theta\sin^2\theta$
   $\frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta} = \tan^2\theta\sin^2\theta$
   $\frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} = \tan^2\theta\sin^2\theta$
   $\frac{\sin^2\theta(\sin^2\theta)}{\cos^2\theta} = \tan^2\theta\sin^2\theta$
   $\left(\frac{\sin^2\theta}{\cos^2\theta}\right)\sin^2\theta = \tan^2\theta\sin^2\theta$
   $\tan^2\theta\sin^2\theta = \tan^2\theta\sin^2\theta$

4. $1 + \cot^4\theta = \csc^4\theta - 2\cot^2\theta$

   $1 + \cot^4\theta = \csc^4\theta - 2\cot^2\theta$
   $1 + \cot^4\theta = (\cot^2\theta + 1)^2 - 2\cot^2\theta$
   $1 + \cot^4\theta = \cot^4\theta + 2\cot^2\theta + 1 - 2\cot^2\theta$
   $1 + \cot^4\theta = \cot^4\theta + 1$
   $1 + \cot^4\theta = 1 + \cot^4\theta$

Rewrite each expression in terms of sinθ and cosθ. Then simplify.

5. $\frac{\cot\theta}{\sin\theta}$

   $\frac{\cot\theta}{\sin\theta} = \frac{\frac{\cos\theta}{\sin\theta}}{\sin\theta} =$
   $\frac{\cos\theta}{\sin^2\theta} = \frac{\cos\theta}{1-\cos^2\theta}$

6. $\cot^2\theta - \cos\theta\cot^2\theta$

   $\cot^2\theta(1-\cos\theta) =$
   $\frac{\cos^2\theta}{\sin^2\theta}(1-\cos\theta) =$
   $\frac{\cos^2\theta(1-\cos\theta)}{1-\cos^2\theta} =$
   $\frac{\cos^2\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{\cos^2\theta}{1+\cos\theta}$

7. $\tan^2\theta + 4\sec^2\theta + 1$

   $\tan^2\theta + 4\sec^2\theta + 1 =$
   $\sec^2\theta - 1 + 4\sec^2\theta + 1 =$
   $5\sec^2\theta = 5\left(\frac{1}{\cos^2\theta}\right) = \frac{5}{\cos^2\theta}$

Solve.

8. Alan is using the equation $mg\sin\theta = \mu mg\cos\theta$ to determine the coefficient of friction, μ, between a flat rock and a metal ramp. Find μ to the nearest hundredth if the rock begins to slide at 19°. **0.34**

LESSON 14-3

Reteach

Fundamental Trigonometric Identities

To prove a trigonometric identity, use the fundamental identities to make one side of the equation resemble the other side.

Reciprocal and Ratio Identities		
$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$
$\tan\theta = \frac{\sin\theta}{\cos\theta}$		$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Negative-Angle Identities	Pythagorean Identities
$\sin(-\theta) = -\sin\theta$	$\sin^2\theta + \cos^2\theta = 1$
$\cos(-\theta) = \cos\theta$	$\tan^2\theta + 1 = \sec^2\theta$
$\tan(-\theta) = -\tan\theta$	$1 + \cot^2\theta = \csc^2\theta$

As long as you know this identity, you can divide to derive the other two.

Prove: $\tan\theta\sin\theta = \sec\theta - \cos\theta$

Try the right side since you can use reciprocal identities

$\tan\theta\sin\theta$	$= \sec\theta - \cos\theta$	*Modify the right side.*
	$= \frac{1}{\cos\theta} - \cos\theta$	*Definition of secant*
	$= \frac{1-\cos^2\theta}{\cos\theta}$	*Subtract. Use a common denominator.*
	$= \frac{\sin^2\theta}{\cos\theta}$	*Pythagorean identity*
	$= \frac{\sin\theta}{\cos\theta}\sin\theta$	*Rewrite the product.*
	$= \tan\theta\sin\theta$	*Definition of tangent*

$\sin^2\theta + \cos^2\theta = 1$, so $\sin^2\theta = 1 - \cos^2\theta$.

Write the missing step or reason to prove each trigonometric identity.

1. $\cot(-\theta) = -\cot\theta$ — Modify the left side.

   $\frac{\cos(-\theta)}{\sin(-\theta)} = -\cot\theta$ — **Definition of cotangent**

   $\frac{\cos\theta}{\sin(-\theta)} = -\cot\theta$ — Negative-angle identity

   $\frac{\cos\theta}{-\sin\theta} = -\cot\theta$ — **Negative-angle identity**

   $-\cot\theta = -\cot\theta$ — **Definition of cotangent**

2. $\sec\theta\cot\theta = \csc\theta$ — Modify the left side.

   $\left(\frac{1}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) = \csc\theta$ — **Definitions of secant and cotangent**

   $\frac{1}{\sin\theta}$ — Simplify.

   $\csc\theta = \csc\theta$ — **Definition of cosecant**