Name _____ Date _____ Class _____

LESSON 14-3

# Practice C

## *Fundamental Trigonometric Identities*

**Prove each trigonometric identity.**

**1.** $\frac{\cos\theta}{1 + \sin\theta} = \frac{1 - \sin\theta}{\cos\theta}$

**2.** $\tan\theta \sec\theta = \frac{\sec\theta + \tan\theta}{\cos\theta + \cot\theta}$

**3.** $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$

**4.** $1 + \cot^4\theta = \csc^4\theta - 2\cot^2\theta$

**Rewrite each expression in terms of $\sin\theta$ and $\cos\theta$. Then simplify.**

**5.** $\frac{\cot\theta}{\sin\theta}$

**6.** $\cot^2\theta - \cos\theta \cot^2\theta$

**7.** $\tan^2\theta + 4\sec^2\theta + 1$

**Solve.**

**8.** Alan is using the equation $mg \sin\theta = \mu mg \cos\theta$ to determine the coefficient of friction, $\mu$, between a flat rock and a metal ramp. Find $\mu$ to the nearest hundredth if the rock begins to slide at 19°.

LESSON 14-3

## Practice A

### Fundamental Trigonometric Identities

**Prove each trigonometric identity.**

**1.** $\frac{\sin\theta}{1-\cos^2\theta} = \csc\theta$

**a.** Modify the left-hand side. Replace 1 with a known identify.

**b.** Simplify the denominator.

**c.** Keep simplifying and substituting identities until the left side matches the right side.

$$\frac{\sin\theta}{\frac{\cos^2\theta+\sin^2\theta-\cos^2\theta}{\sin\theta}}$$

$$\frac{\sin^2\theta}{\sin\theta} = \frac{1}{\sin\theta};\ \csc\theta = \csc\theta$$

**2.** $\cos^4\theta + \cos^2\theta\sin^2\theta + \sin^2\theta = 1$

$$\cos^4\theta + \cos^2\theta\sin^2\theta + \sin^2\theta = 1$$
$$\cos^2\theta(\cos^2\theta + \theta\sin^2\theta) + \sin^2\theta = 1$$
$$\cos^2\theta(\cos^2\theta + \sin^2\theta) + \sin^2\theta = 1$$
$$\cos^2\theta\,(1) + \sin^2\theta = 1$$
$$\cos^2\theta + \sin^2\theta = 1$$
$$1 = 1$$

**3.** $\cot\theta = \frac{\csc\theta}{\sec\theta}$

$$\cot\theta = \frac{\csc\theta}{\sec\theta};\ \cot\theta = \frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}},$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta};\ \cot\theta = \cot\theta$$

**Rewrite each expression in terms of $\cos\theta$. Then simplify.**

**4.** $\frac{\cot\theta}{2\csc\theta}$

**a.** Rewrite each function in terms of $\sin\theta$ or $\cos\theta$.

**b.** Rewrite the fraction in simplest form.

$$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{2}{\sin\theta}}$$

$$\frac{\sin\theta\cos\theta}{2\sin\theta} = \frac{\cos\theta}{2}$$

**5.** $\sin\theta\cot\theta$

$$\sin\theta\cot\theta =$$
$$\sin\theta \cdot \frac{\cos\theta}{\sin\theta} =$$
$$\cos\theta$$

**6.** $\frac{\cos\theta - \cos^2\theta}{\sin^2\theta}$

$$\frac{\cos\theta - \cos^2\theta}{\sin^2\theta} =$$
$$\frac{\cos\theta - \cos^2\theta}{1-\cos^2\theta} =$$
$$\frac{\cos\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} =$$
$$\frac{\cos\theta}{1+\cos\theta}$$

**7.** $\frac{\sin^2\theta + 2\cos^2\theta}{\sec^2\theta - \tan^2\theta}$

$$\frac{\sin^2\theta + 2\cos^2\theta}{\sec^2\theta - \tan^2\theta} =$$
$$\frac{1-\cos^2\theta + 2\cos^2\theta}{\frac{1}{\cos^2\theta} - (\sec^2\theta - 1)} =$$
$$\frac{1+\cos^2\theta}{\frac{1}{\cos^2\theta} - \frac{1}{\cos^2\theta} + 1} =$$
$$1+\cos^2\theta$$

**Solve.**

**8.** Use the equation $mg\sin\theta = \mu mg\cos\theta$ to determine the angle at which a desk can be tilted before a paperback book on the desk begins to slide. Assume $\mu = 1.11$.

48°





LESSON 14-3

## Practice B

### Fundamental Trigonometric Identities

**Prove each trigonometric identity.**

**1.** $\sin^2\theta + \sin^2\theta\cot^2\theta = 1$

$$\sin^2\theta + \sin^2\theta\cot^2\theta = 1$$
$$\sin^2\theta\,(1 + \cot^2\theta) = 1$$
$$\sin^2\theta(\csc^2\theta) = 1$$
$$\sin^2\theta\left(\frac{1}{\sin^2\theta}\right) = 1$$
$$1 = 1$$

**2.** $\cot^2\theta\cos^2\theta = \cot^2\theta - \cos^2\theta$

$$\cot^2\theta\cos^2\theta = \cot^2\theta - \cos^2\theta =$$
$$\frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta = \frac{\cos^2\theta}{\sin^2\theta} - \frac{\sin^2\theta\cos^2\theta}{\sin^2\theta} =$$
$$\frac{\cos^2\theta - \sin^2\theta\cos^2\theta}{\sin^2\theta} = \frac{\cos^2\theta\,(1-\sin^2\theta)}{\sin^2\theta} =$$
$$\left(\frac{\cos^2\theta}{\sin^2\theta}\right)(1-\sin^2\theta) = \cot^2\theta\,(1-\sin^2\theta) =$$
$$\cot^2\theta\,(\cos^2\theta + \sin^2\theta - \sin^2\theta) = \cot^2\theta\cos^2\theta$$

**3.** $\tan^2\theta - \tan^2\theta\sin^2\theta = \sin^2\theta$

$$\tan^2\theta - \tan^2\theta\sin^2\theta = \sin^2\theta$$
$$\tan^2\theta(1-\sin^2\theta) = \sin^2\theta$$
$$\tan^2\theta(\cos^2\theta) = \sin^2\theta$$
$$\frac{\sin^2\theta}{\cos^2\theta}(\cos^2\theta) = \sin^2\theta$$
$$(\sin^2\theta)\frac{\cos^2\theta}{\cos^2\theta} = \sin^2\theta$$
$$\sin^2\theta = \sin^2\theta$$

**4.** $\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = \sec\theta + \csc\theta$

$$\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = \sec\theta + \csc\theta$$
$$\frac{\sin\theta}{\sin\theta\cos\theta} + \frac{\cos\theta}{\sin\theta\cos\theta} = \sec\theta + \csc\theta$$
$$\left(\frac{\sin\theta}{\sin\theta}\right)\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\left(\frac{\cos\theta}{\cos\theta}\right) = \sec\theta + \csc\theta$$
$$(1)\frac{1}{\cos\theta} + \frac{1}{\sin\theta}(1) = \sec\theta + \csc\theta$$
$$\frac{1}{\cos\theta} + \frac{1}{\sin\theta} = \sec\theta + \csc\theta$$
$$\sec\theta + \csc\theta = \sec\theta + \csc\theta$$

**Rewrite each expression in terms of $\cos\theta$. Then simplify.**

**5.** $2\sin\theta\cos\theta\cot\theta$

$$2\sin\theta\cos\theta\cot\theta =$$
$$2\sin\theta\cos\theta \cdot \frac{\cos\theta}{\sin\theta} =$$
$$2\cos^2\theta$$

**6.** $\frac{1+\cot\theta}{\cot\theta(\sin\theta+\cos\theta)}$

$$\frac{1+\cot\theta}{\cot\theta(\sin\theta+\cos\theta)} =$$
$$\frac{1+\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}(\sin\theta+\cos\theta)} =$$
$$\frac{\frac{\sin\theta+\cos\theta}{\sin\theta}}{\frac{\cos\theta(\sin\theta+\cos\theta)}{\sin\theta}} =$$
$$\frac{\sin\theta+\cos\theta}{\cos\theta(\sin\theta+\cos\theta)} = \frac{1}{\cos\theta}$$

**7.** $\cos^4\theta - \sin^4\theta + \sin^2\theta$

$$\cos^4\theta - \sin^4\theta + \sin^2\theta$$
$$= (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) + \sin^2\theta$$
$$(1)(\cos^2\theta - \sin^2\theta) + \sin^2\theta = \cos^2\theta$$

**Solve.**

**8.** Use the equation $mg\sin\theta = \mu mg\cos\theta$ to determine the angle at which a waxed wood block on an inclined plane of wet snow begins to slide. Assume $\mu = 0.17$.

9.6°





LESSON 14-3

## Practice C

### Fundamental Trigonometric Identities

**Prove each trigonometric identity.**

**1.** $\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$

$$\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$$
$$\frac{\cos\theta}{1+\sin\theta}\left(\frac{1-\sin\theta}{1-\sin\theta}\right) = \frac{1-\sin\theta}{\cos\theta}$$
$$\frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta} = \frac{1-\sin\theta}{\cos\theta}$$
$$\frac{\cos\theta(1-\sin\theta)}{\cos^2\theta} = \frac{1-\sin\theta}{\cos\theta}$$
$$\frac{1-\sin\theta}{\cos\theta} = \frac{1-\sin\theta}{\cos\theta}$$

**2.** $\tan\theta\sec\theta = \frac{\sec\theta+\tan\theta}{\cos\theta+\cot\theta}$

$$\tan\theta\sec\theta = \frac{\sec\theta+\tan\theta}{\cos\theta+\cot\theta} = \frac{\sec\theta+\frac{\sin\theta}{\cos\theta}}{\cos\theta+\frac{\cos\theta}{\sin\theta}}$$
$$= \frac{\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\cos\theta+\frac{\cos\theta}{\sin\theta}} = \frac{\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta\cos\theta}{\sin\theta}+\frac{\cos\theta}{\sin\theta}}$$
$$= \frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{\sin\theta\cos\theta+\cos\theta}{\sin\theta}} = \frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{\cos\theta(\sin\theta+1)}{\sin\theta}}$$
$$= \frac{\sin\theta(1+\sin\theta)}{\cos^2\theta(\sin\theta+1)} = \frac{\sin\theta}{\cos^2\theta}$$
$$= \left(\frac{\sin\theta}{\cos\theta}\right)\frac{1}{\cos\theta} = (\tan\theta)\frac{1}{\cos\theta}$$
$$= \tan\theta\sec\theta$$

**3.** $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$

$$\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$$
$$\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta = \tan^2\theta\sin^2\theta$$
$$\frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta} = \tan^2\theta\sin^2\theta$$
$$\frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} = \tan^2\theta\sin^2\theta$$
$$\frac{\sin^2\theta(\sin^2\theta)}{\cos^2\theta} = \tan^2\theta\sin^2\theta$$
$$\left(\frac{\sin^2\theta}{\cos^2\theta}\right)\sin^2\theta = \tan^2\theta\sin^2\theta$$
$$\tan^2\theta\sin^2\theta = \tan^2\theta\sin^2\theta$$

**4.** $1 + \cot^4\theta = \csc^4\theta - 2\cot^2\theta$

$$1 + \cot^4\theta = \csc^4\theta - 2\cot^2\theta$$
$$1 + \cot^4\theta = (\cot^2\theta + 1)^2 - 2\cot^2\theta$$
$$1 + \cot^4\theta = \cot^4\theta + 2\cot^2\theta + 1 - 2\cot^2\theta$$
$$1 + \cot^4\theta = \cot^4\theta + 1$$
$$1 + \cot^4\theta = 1 + \cot^4\theta$$

**Rewrite each expression in terms of $\sin\theta$ and $\cos\theta$. Then simplify.**

**5.** $\frac{\cot\theta}{\sin\theta}$

$$\frac{\cot\theta}{\sin\theta} = \frac{\frac{\cos\theta}{\sin\theta}}{\sin\theta} =$$
$$\frac{\cos\theta}{\sin^2\theta} = \frac{\cos\theta}{1-\cos^2\theta}$$

**6.** $\cot^2\theta - \cos\theta\cot^2\theta$

$$\cot^2\theta(1-\cos\theta) =$$
$$\frac{\cos^2\theta}{\sin^2\theta}(1-\cos\theta) =$$
$$\frac{\cos^2\theta(1-\cos\theta)}{1-\cos^2\theta} =$$
$$\frac{\cos^2\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{\cos^2\theta}{1+\cos\theta}$$

**7.** $\tan^2\theta + 4\sec^2\theta + 1$

$$\tan^2\theta + 4\sec^2\theta + 1 =$$
$$\sec^2\theta - 1 + 4\sec^2\theta + 1 =$$
$$5\sec^2\theta = 5\left(\frac{1}{\cos^2\theta}\right) = \frac{5}{\cos^2\theta}$$

**Solve.**

**8.** Alan is using the equation $mg\sin\theta = \mu mg\cos\theta$ to determine the coefficient of friction, $\mu$, between a flat rock and a metal ramp. Find $\mu$ to the nearest hundredth if the rock begins to slide at 19°.

0.34





LESSON 14-3

## Reteach

### Fundamental Trigonometric Identities

To prove a trigonometric identity, use the fundamental identities to make one side of the equation resemble the other side.

| Reciprocal and Ratio Identities | | |
|---|---|---|
| $\csc\theta = \frac{1}{\sin\theta}$ | $\sec\theta = \frac{1}{\cos\theta}$ | $\cot\theta = \frac{1}{\tan\theta}$ |
| $\tan\theta = \frac{\sin\theta}{\cos\theta}$ | | $\cot\theta = \frac{\cos\theta}{\sin\theta}$ |

| Negative-Angle Identities | Pythagorean Identities |
|---|---|
| $\sin(-\theta) = -\sin\theta$ | $\sin^2\theta + \cos^2\theta = 1$ |
| $\cos(-\theta) = \cos\theta$ | $\tan^2\theta + 1 = \sec^2\theta$ |
| $\tan(-\theta) = -\tan\theta$ | $1 + \cot^2\theta = \csc^2\theta$ |

As long as you know this identity, you can divide to derive the other two.

Try the right side since you can use reciprocal identities

**Prove:** $\tan\theta\sin\theta = \sec\theta - \cos\theta$

| | |
|---|---|
| $\tan\theta\sin\theta$ | *Modify the right side.* |
| $= \frac{1}{\cos\theta} - \cos\theta$ | *Definition of secant* |
| $= \frac{1-\cos^2\theta}{\cos\theta}$ | *Subtract. Use a common denominator.* |
| $= \frac{\sin^2\theta}{\cos\theta}$ | *Pythagorean identity* |
| $= \frac{\sin\theta}{\cos\theta}\sin\theta$ | *Rewrite the product.* |
| $= \tan\theta\sin\theta$ | *Definition of tangent* |

$\sin^2\theta + \cos^2\theta = 1$, so $\sin^2\theta = 1 - \cos^2\theta$.

**Write the missing step or reason to prove each trigonometric identity.**

**1.** $\cot(-\theta) = -\cot\theta$ — Modify the left side.

$$\frac{\cos(-\theta)}{\sin(-\theta)} = -\cot\theta$$ — **Definition of cotangent**

$$\frac{\cos\theta}{\sin(-\theta)} = -\cot\theta$$ — Negative-angle identity

$$\frac{\cos\theta}{-\sin\theta} = -\cot\theta$$ — **Negative-angle identity**

$$-\cot\theta = -\cot\theta$$ — **Definition of cotangent**

**2.** $\sec\theta\cot\theta = \csc\theta$ — Modify the left side.

$$\left(\frac{1}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) = \csc\theta$$ — **Definitions of secant and cotangent**

$$\frac{1}{\sin\theta}$$ — Simplify.

$$\csc\theta = \csc\theta$$ — **Definition of cosecant**