Exam VI Review Sections 6.5 – 6.7; 6.9 KEY **Algebra 2**

1. Identify all of the roots of the equation by factoring, using the Rational Root Theorem and synthetic division to find a depressed polynomial that can be solved by using the quadratic formula. State the multiplicity of each root, if more that one.

a. $x^4 - 36 = 0$

$(x^2-6)(x^2+6)$ $\pm\sqrt{6}$ $\pm\sqrt{6}i$

$a=1$, $b=0$, $c=-6$: $\frac{-0\pm\sqrt{0^2-4(1)(-6)}}{2(1)}$ $a=1$, $b=0$, $c=6$: $\frac{-0\pm\sqrt{0^2-4(1)(6)}}{2(1)}$

b. $x^3 - 8 = 0$

$(x-2)(x^2+2x+4)$ $2, -1\pm\sqrt{3}i$

$a=1$, $b=2$, $c=4$: $\frac{-2\pm\sqrt{2^2-4(1)(4)}}{2(1)}$

c. $x^4 - 6x^3 = -9x^2$

$x^2(x-3)^2$

Roots = 0, 3

Multiplicity = 2, 2

d. $x^3 - 5x^2 - 4x + 20 = 0$

$(x-5)(x-2)(x+2)$

$5, \pm 2$

2. Solve by finding all roots by using the calculator to find as many rational roots as possible, then use synthetic division and the quadratic formula to find the remaining roots. Show all work!

$x^4 - 5x^3 - 2x^2 - 20x - 24 = 0$

$(x-6)(x+1)(x^2+4)$

$a=1$, $b=0$, $c=4$: $\frac{-0\pm\sqrt{0^2-4(1)(4)}}{2(1)}$

$6, -1, \pm 2i$

3. Write factors of the simplest polynomial function with rational coefficients that has the given zeros, then multiply out the factors to give the polynomial function in standard form.

a. 5, 1, -1

a. factors: $(x-5)(x-1)(x+1)$

polynomial function: f(x) = $x^3 - 5x^2 - x + 5$

b. 1, $5i$ $-5i$

b. factors: $(x-1)(x+5i)(x-5i)$

polynomial function: f(x) = $x^3 - x^2 + 25x - 25$

c. -3, $-\sqrt{2}$ $+\sqrt{2}$

c. factors: $(x+3)(x-\sqrt{2})(x+\sqrt{2})$

polynomial function: f(x) = $x^3 + 3x^2 - 2x - 6$

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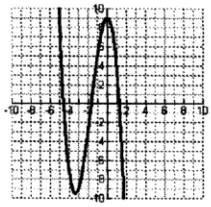
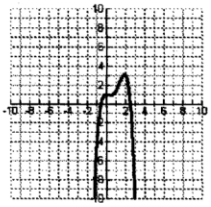
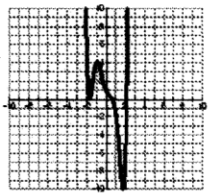
Classify each of the graphs of the polynomial functions below by their degree and leading coefficient. Then determine the end behavior of each graph.

4. 

5. 

6. 

7. 

Circle the correct choice.

4	5	6	7
Degree: Even *or* (Odd)	Degree: (Even) *or* Odd	Degree: Even *or* (Odd)	Degree: (Even) *or* Odd
Leading Coefficient is Positive *or* (Negative)	Leading Coefficient is Positive *or* (Negative)	Leading Coefficient is (Positive) *or* Negative	Leading Coefficient is (Positive) *or* Negative
As $x \to -\infty, f(x) \to$ $+\infty$	As $x \to -\infty, f(x) \to$ $-\infty$	As $x \to -\infty, f(x) \to$ $-\infty$	As $x \to -\infty, f(x) \to$ $+\infty$
As $x \to +\infty, f(x) \to$ $-\infty$	As $x \to +\infty, f(x) \to$ $-\infty$	As $x \to +\infty, f(x) \to$ $+\infty$	As $x \to +\infty, f(x) \to$ $+\infty$

Match the polynomial equation with its end behavior.

8. D $f(x) = 4x^3 + 5x^2 + 1$

9. C $f(x) = 2x^4 - 5x + 3$

10. B $f(x) = -2x^3 + x - 5$

11. A $f(x) = -3x^4 - 7x^2 + 1$

A. As $x \to -\infty, f(x) \to -\infty$; As $x \to +\infty, f(x) \to -\infty$

B. As $x \to -\infty, f(x) \to +\infty$; As $x \to +\infty, f(x) \to -\infty$

C. As $x \to -\infty, f(x) \to +\infty$; As $x \to +\infty, f(x) \to +\infty$

D. As $x \to -\infty, f(x) \to -\infty$; As $x \to +\infty, f(x) \to +\infty$

12. (Solve) each polynomial equation by (factoring).

a. $x^3 - x^2 - 9x + 9 = 0$

$(x-3)(x-1)(x+3)$

$x = 1, \pm 3$

b. $x^4 - 11x^2 + 28 = 0$

$(x-2)(x+2)(x^2-7)$

$x^2 - 7 = 0$

$x^2 = 7$

$x = \pm\sqrt{7}$

$x = \pm 2, \pm\sqrt{7}$

c. $x^4 - 6x^3 + 9x^2 = 0$

$x^2(x-3)^2$

$x = 0, 3$

13. Use the Rational Root Theorem to list all the *possible* rational roots of the polynomial equation.

$7x^3 + 6x^2 - 4x - 6 = 0$

-6: p: $\pm 1, \pm 2, \pm 3, \pm 6$

7: q: $\pm 1, \pm 7$

$\frac{p}{q}$: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}, \pm\frac{1}{7}, \pm\frac{2}{7}, \pm\frac{3}{7}, \pm\frac{6}{7}$

Exam VI Review Sections 6.5 – 6.7; 6.9 KEY **Algebra 2**

14. Use your graphing calculator to estimate the real zeros, local maximum, and local minimum of $f(x) = x^3 - 10x + 1$. Round your answers to the nearest thousandth.

real zeros -3.211, 0.100, 3.111 maximum 13.172 minimum -13.172

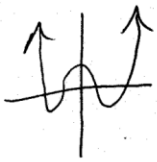
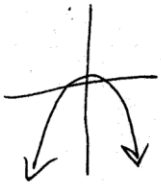
15. Which polynomial function with rational coefficients has zeros $1+\sqrt{5}$ *and* $1-\sqrt{5}$?

A. $f(x) = x^2 + 2x - 4$ B. $f(x) = x^2 - 2x - 4$ C. $f(x) = x^2 + 4$ D. $f(x) = x^2 - 3x - 6$

$(x - 1 - \sqrt{5})(x - 1 + \sqrt{5}) = x^2 - 2x - 4$

16. A polynomial function of degree ***n*** has at most $N-1$ turning points (local maxima and local minima) and at most N x-intercepts.

17. In each box, sketch a graph of a polynomial function that fits the description.

Leading coefficient	Odd degree	Even degree
Positive		
Negative		

18. Determine the best Polynomial Function for the given data using Finite Differences and give the r^2 value.

Time	2	4	6	8	10	12	14	16	18
Velocity	25.51	16.44	-8.91	-59.70	-142.56	-262.57	-423.28	-626.71	-873.34

4th degree

$0.064x^4 - 0.25x^3 + 0.44x^2 - 0.156x + 2500$

$R^2 = 0.999999999999999$