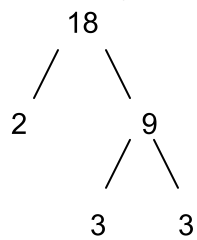
Understanding Factoring

To factor a quantity means that it is broken up into numbers or expressions that could be multiplied to produce the original problem. Those numbers are called factors of the original problem.

Example: 6 and 4 are called factors of 24 because 6 * 4 = 24. However, usually these problems are broken up (factored) until they will not break up (factor) any further. They are then said to be factored completely so 24 completely factors into 2*2*2*3. You might recall that numbers like 2, 3, 5, 7, 11, etc. are called prime numbers which means they can't be factored any further.

You probably have worked with "factor trees" before. Look at the example below.



so 18 = 2 * 3 * 3 (since 2 and 3 are prime numbers, 18 won't factor any further and is said to be completely factored)

Test your understanding: factor each of the following. 1) 108 2) 51 3) 750

In algebra II, we typically work with expressions that need to be factored. Sometimes they are just single terms (no addition or subtraction is shown) and so they are called Monomials. An example is $72x^2y^3 = 2*2*2*3*3*x*x*y*y*y$.

Test your understanding: factor $900x^3yz^2$

Sometimes we work with expressions that involve 2 or more terms (which means expressions that are separated by either a subtraction sign or an addition sign that is **NOT** located inside of any parentheses). Examples are listed below.

1) $x - 5y$ (since this problem has one term of x and another of 5y joined by a subtraction sign, it is called a Binomial

2) $3x^2 - 43x + 11$ (since this problem has one term of $3x^2$, another of $-43x$, and yet a third one that's $+11$, it has three terms and it is called a Trinomial).

3) $-5x^3 + 22x^2 - 34x + 9$ (since this problem has 4 terms, it is just called a polynomial with 4 terms). Note that all examples given above are called polynomials too, but for problems that have as many as 4 terms we just don't give them special names.

Test your understanding: Complete the table below. Remember that terms have to be separated by either addition or subtraction signs that are **NOT** inside of parentheses.

The Given Problem	How many terms does it have?	The name of this type of polynomial is what?
$32x^3 - 14x$		
55		
$19x^3 + 20x^2 + 9x - 3$		
$13x^3 - 5x + 76$		

$(x - 5)(3x + 87)$		

A critical issue in learning to factor polynomials is an understanding of Greatest Common Factor (often abbreviated as GCF). A GCF is an expression (monomial) that consists of the product of all factors that each term in a polynomial has in common.

For example: $12x^2y^3$ and $8xy^2$ could each be separately factored into $12x^2y^3$ =2*2*3*x*x*y*y*y and $8xy^2$ = 2*2*2*x*y*y. What is the most that these problems have in common? They both have 2's, at least one x, and at least two y's. That makes the GCF = 2*2*x*y*y = $4xy^2$. So $4xy^2$ is called the GCF.

Test your understanding: Find the GCF of $36x^3y$ and $45x^2y^2$.

One thing that you might find difficult about learning to factor polynomials is that there is no magic formula and only minimal help available from an appropriate calculator. However, there is **ONE RULE** which **ALWAYS** takes precedence over all other factoring strategies. It is: **ALWAYS FACTOR OUT THE GREATEST COMMON FACTOR WHENEVER POSSIBLE.**

What is meant by "factor out"? Let's take that last problem and join them together with an addition sign to make a binomial. It would be $36x^3y+45x^2y^2$. To factor out the GCF from this expression means to put outside a set of parentheses the GCF of the two terms and to put inside the parentheses whatever is left over when each of the factors in the GCF is removed from the original terms. That makes it look like this.

$$\begin{aligned} 36x^3y+45x^2y^2 &= 2*2*3*3*x*x*x*y + 3*3*5*x*x*y*y \\ &= 3*3*x*x*y\ (2*2*x + 5*y) \\ &= 9x^2y(4x+5y) \end{aligned}$$

This last step shows the answer with the GCF "factored out".

Test your understanding: Factor out the GCF from $90x^2y^3z-105x^3yz^2$

As you learn other problem patterns later, keep in mind that there is one thing you should ALWAYS do and that is to factor out the GCF first!

Problems that contain four terms are usually factored by a method called "factoring by grouping".

In this type of problem we begin by putting parentheses around (grouping) the first two terms and around the second two terms leaving either an addition or a subtraction sign between the parentheses.
Example: $6x^2+10x+21x+35$ has 4 terms so we'll insert the parentheses to make it look like this: $\left(3x^2+10x\right)+(21x+35)$. Next we factor out the GCF from each of the parentheses separately. Since the GCF of $6x^2$ and $10x$ is 2x and since the GCF of 21x and 35 is 7 we can rewrite the problem as $\left(6x^2+10x\right)+\left(21x+35\right) = 2x\left(3x+5\right)+7\left(3x+5\right)$.
Notice how the expressions inside the parentheses match? If they didn't match we've either made a mistake or else this method of "factoring by grouping" is not useful on this problem. The fact that they do match leads us to the last step on this problem. Inside one set of parentheses put the terms that match and inside the other parentheses put the other terms which are outside the parentheses. This gives our final answer of $\left(3x+5\right)\left(2x+7\right)$.

<u>Important!</u> If it happens that the subtraction sign falls between the 2 parentheses when we first add them at the beginning of the problem, then <u>we must change the sign inside the second parentheses!</u> Look at the example below to notice this and how the other steps are the same as the example above.

Example: $9x^2-9x-4x+4=(9x^2-9x)-(4x-4)$

$$=9x(x-1)-4(x-1)$$

$$=(x-1)(9x-4)$$

Problems that contain three terms are factored by either "trial and error" or using a variation of "factoring by grouping". Your instructor will give you information about this last type of problem.

<u>No matter what type of problem you are trying to factor, always remember to factor out the GCF first if it's possible,</u> then, and only then try to use the patterns shown above. Also remember this sheet is just a summary intended to get you started in recognizing the various patterns of factoring. After you have a few days to practice all the problem types you will be on your own so no notes will be allowed on quizzes or tests.

<u>THE ONLY WAY TO LEARN HOW TO FACTOR IS TO PRACTICE!</u>

<u>Special Patterns for Factoring</u>: A key to success at factoring polynomials is recognition of the pattern that a problem fits. This sheet is a summary of a few basic patterns (there are others). After practice these patterns should become easy to recognize and will need to be memorized.

Problems that contain <u>two</u> terms usually fit one of the 4 patterns below.

1. The **<u>"difference of two squares"</u>** consists of 2 terms that are separated by a subtraction sign. Additionally, each is the square of some expression. Example: $a^2 - b^2$ has two terms separated by a subtraction sign and each term is something squared. This pattern always factors into two binomials in parentheses with different signs in the middle. That is, $a^2 - b^2 = (a-b)(a+b)$.

<u>Note</u>: Sometimes you'll have to write the terms as squares first!

Example: $4a^2 - 25b^2$ can first be written as $(2a)^2 - (5b)^2$. Then that can be factored into (2a – 5b) (2a + 5b).

2. The **"<u>sum of two squares</u>"** consists of 2 terms that are separated by an addition sign. Additionally, each is the square of some expression. <u>Example:</u> a^2+b^2 has two terms separated by an addition sign and each term is something squared. This pattern does not factor so is said to be **prime**.

3. The **"<u>difference of two cubes</u>"** consists of 2 terms that are separated by a subtraction sign. Additionally, each is the cube of some expression.

<u>Example</u>: a^3-b^3 has two terms separated by a subtraction sign and each term is something cubed. This pattern always factors into the pattern shown. $a^3-b^3=(a-b)(a^2+ab+b^2)$.

<u>Note</u>: Sometimes you'll have to write the terms as cubes first!

<u>Example</u>: $8a^3-125b^3$ can be first written as $(2a)^3-(5b)^3$. Then that can be factored into $(2a-5b)\left((2a)^2+(2a)(5b)+(5b)^2\right)$ which simplifies to make $(2a-5b)(4a^2+10ab+25b^2)$.

4. The **“<u>sum of two cubes</u>”** consists of 2 terms that are separated by an addition sign. Additionally, each is the cube of some expression. Example: a^3+b^3 has two terms separated by an addition sign and each term is something cubed. This pattern always factors into the pattern shown. $a^3+b^3=(a+b)(a^2-ab+b^2)$.

<u>Note</u>: Sometimes you’ll have to write the terms as cubes first!
Example: $8a^3+125b^3$ can be first written as $(2a)^3+(5b)^3$. Then that can be factored into $(2a+5b)\left((2a)^2-(2a)(5b)+(5b)^2\right)$ which simplifies to make $(2a+5b)(4a^2-10ab+25b^2)$.