Transformation Unit NOTES

A function transformation takes the basic parent function $f(x)$ and then "transforms" it or "translates" it, which is a fancy way of saying that you change the equation a bit and it moves the graph around.

Basic Transformations:

ALL basic transformations of functions follow these rules:

Translations ("shifts")

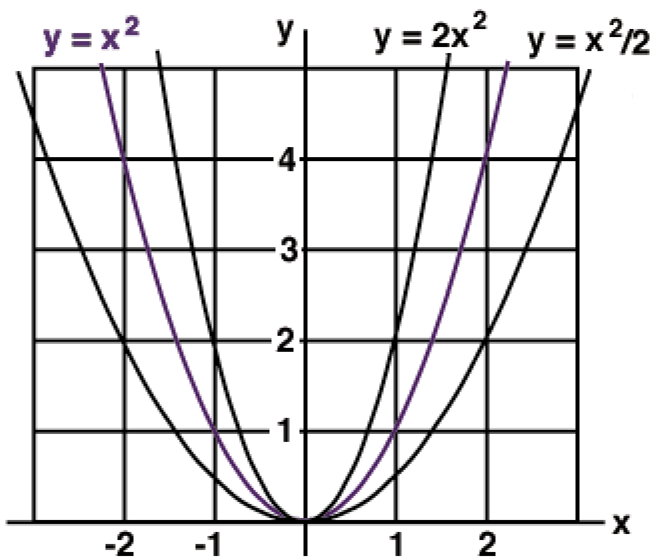
- $f(x) + k$ is $f(x)$ shifted upward k units
- $f(x) - k$ is $f(x)$ shifted downward k units
- $f(x + h)$ is $f(x)$ shifted left h units
- $f(x - h)$ is $f(x)$ shifted right h units

Reflections ("flips")

- $-f(x)$ is $f(x)$ flipped upside down ("reflected about the x-axis")
- $f(-x)$ is the mirror of $f(x)$ ("reflected about the y-axis")

Additional Transformations:

There are additional transformations called dilations and compressions, but it is a bit harder to see these changes. If you compare the graphs of $2x^2$, x^2, and ½ x^2, you'll see that the graph doesn't change its position as much as its size. The parabola for $2x^2$ "grows" twice as fast as x^2, so its graph is tall and narrow, which is "stretching" it away from the x-axis (dilation). The parabola for the function ½ x^2 "grows" only half as fast, so its graph is short and wide, which "compresses" it toward the x-axis. So, multiplying the function by a number in front changes the distance of the graph from the x-axis, by stretching it or compressing the picture.



Now compare the graphs of $(2x)^2$, x^2, and $(½\,x)^2$ The parabola for $(2x)^2$ is "compressed" toward the y-axis, and "stretched" away from the y-axis for $(½\,x)^2$. So, multiplying the x by a number "inside" the function (or parentheses, in this case) changes the distance of the graph from the y -axis, by stretching it or compressing the picture.

Dilations / Compressions

- $a \cdot f(x)$ is a vertical transformation of the graph of $f(x)$, with a "stretch" or "dilation" if $a > 1$, and a "compression" if $a < 1$.)
- $f(a \cdot x)$ is a horizontal transformation of the graph of $f(x)$, with a "stretch" or "dilation" if $a < 1$, and a "compression" if $a > 1$.

Formula for Putting it All Together

let *g(x)* be a transformation of some function *f(x)*

horizontal transformation ↓

$$g(x) = a(bx - h) + k$$

↑ ***vertical transformation* & reflect over x** ↑ ***horizontal shift*** ↑ ***vertical shift***

Practice: name the parent function, and describe the changes:

$f(x) = -3(x-2)^2 + 5$

$f(x) = |x - 3| - 2$

$g(x) = \dfrac{1}{x} + 1$

$g(x) = -2^x + 1$