Worksheet

Quadratics: vertex form

Introduction

Quadratic functions can be written in what is known as the "vertex form." The vertex form is: $\mathbf{y = a(x - h)^2 + k}$. This worksheet will help you understand how '**a**', '**h**', and '**k**' effect the graph of the quadratic function. Some applications of quadratic functions are also presented.

Let's explore quadratic functions in vertex form. Use your web browser to go to the "Quadratics: vertex form" activity:

http://www.exploremath.com/activities/Activity_page.cfm?ActivityID=14

Below are several questions designed to get you thinking about the activity. Answer them on a separate sheet of paper.

Activity

The meaning of 'a'
Grab the '**a**' slide bar and slide it left and right.

Question 1a. Which way does the graph open when '**a**' is positive?

Question 1b. Which way does the graph open when '**a**' is negative?

Question 1c. What type of graph is it when '**a**' is zero?

Question 1d. What conjectures can you make relating the sign of '**a**' to the direction the graphs opens?

Question 1e. What happens to the graph as '**a**' moves away from zero?

Question 1f. What happens to the graph as '**a**' moves close to zero?

Question 1g. What conjectures can you make relating the vertical scaling of the graph to '**a**'?

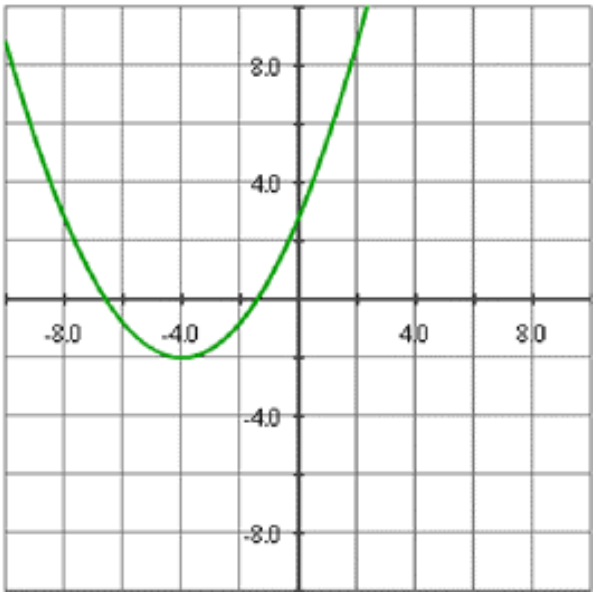
The meaning of 'h'
Select the "show vertex/intercept data" box, then grab the '**h**' slide bar and move it left and right.

Question 2a. What happens to the vertex of the graph as '**h**' moves to the right?

Question 2b. What happens to the vertex of the graph as '**h**' moves to the left?

Question 2c. In the equation $y = 3(x + 4)^2 - 8$, what is the value of '**h**'?

Question 2d. In the graph below, what is the approximate value of '**h**'?



Question 2e. What conjectures can you make relating '**h**' to the graph of a quadratic?

The meaning of 'k'

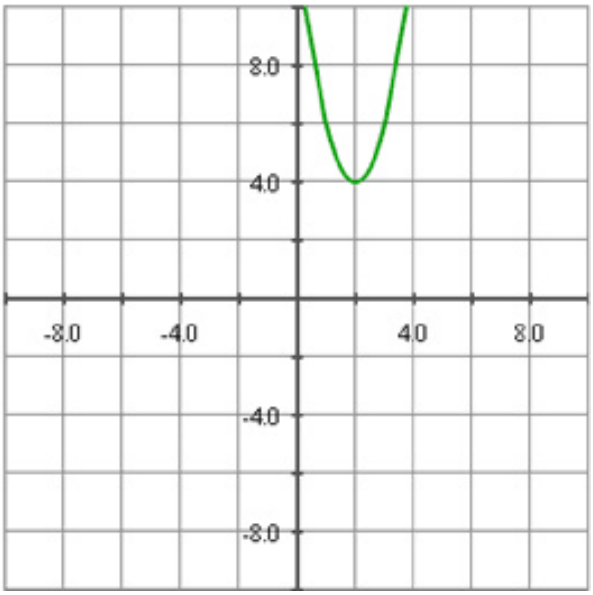
Grab the '**k**' slide bar and move it left and right.

Question 3a. What happens to the vertex of the graph as '**k**' moves to the right?

Question 3b. What happens to the vertex of the graph as '**k**' moves to the left?

Question 3c. In the equation $y = -3(x - 4)^2 + 8$, what is the value of '**k**'?

Question 3d. In the graph below, what is the approximate value of '**k**'?



Question 3e. When is does '**k**' represent the maximum value that the function attains?

Question 3f. What conjecture can you make relating '**k**' to the graph of a quadratic?

Putting it all together

Question 4a. What is the vertex of $y = -(x + 5)^2 - 7$?

Question 4b. Without graphing the function, how many roots (x-intercepts) does $y = -3(x - 2)^2 - 7$ have?

Question 4c. What is an equation for a quadratic function that has only one distinct real root?

Question 4d. Suppose a function has the equation $y = 5(x - 6)^2 + 1$. The vertex was then shifted to the left **3** units and down **4** units. What would the equation for this new function be?

Application

A ball is thrown vertically into the air from the ground. After **2** seconds it attains its max height of **96** feet. After **4** seconds the **4** seconds it hits the ground.

Question 5a. If $\mathbf{x}$ represents the number of seconds the ball was in the air, and $\mathbf{y}$ represents the height of the ball, the path of the ball can be modeled with a quadratic equation. What would be the value of '$\mathbf{h}$' and '$\mathbf{k}$' in this model?

Question 5b. What is the equation of this model?

Question 5c. At what point does the ball attain it max height?

Completing the square

Question 6a. What is the vertex form of the quadratic equation $\mathbf{y = 3x^2 + 6x + 8}$?

Question 6b. What is the vertex form of the quadratic equation $\mathbf{y = -2x^2 + 4x + 12}$?

Conclusion

Quadratic equations in the form $\mathbf{y = a(x - h)^2 + k}$ have a vertex at $\mathbf{(h, k)}$. If $\mathbf{a > 0}$ the graph opens upward, and when $\mathbf{a < 0}$ the graph opens downward. The magnitude of '$\mathbf{a}$' also determines the 'steepness' of the graph. The graphs of quadratic functions can have $\mathbf{0}$, $\mathbf{1}$, or $\mathbf{2}$ x-intercepts. Quadratics can be converted form polynomial form to vertex form by completing the square.