Computer Algebra Systems: A basic tool for teaching Mathematics in Engineering

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The main goal of this paper is to reflect on the habitual uses that nowadays are given to CAS (Computer Algebra Systems) in the teaching of Mathematics at the degrees of Engineering. The traditional use of computer in mathematical teaching for Engineers is normally reduced to the resolution of some kind of theoretical-practical exercises where the students use a specific mathematical software. The strong tendency that is taking place with the purpose of changing the traditional didactic uses of these tools will be emphasized. We will describe our experiences when using CAS in Engineering. In this use of CAS the students have an active role in the sense they should elaborate themselves utility files to solve the typical problems for the different subjects. This fact implies that the students need to deal with programming, understand the subject and know how to solve typical problems. We will finish with the conclusions obtained after using CAS in this sense in the last years.

Keywords Mathematics; Computer Algebra System; Education; Engineering

1. Computer Technology and Mathematics Education

Mathematics teaching should keep in mind that students must be taught in conditions that favour their integration into larger society. Such an acknowledgement leads one to consider the characteristics of today's society and in particular the characteristics involving recent technological advances, which continue at such a frenetic pace that it is difficult to predict what kind of knowledge mankind will need tomorrow. As a consequence, it is essential for mathematics education to be as complete as possible: it must help students "learn how to learn", keep an open-minded attitude and acquire a certain level of confidence in their own thinking capabilities. Such adaptation to social changes can only be accomplished by teaching students to skilfully use information technologies.

2. CAS as didactic resources in Mathematics teaching

Given the instrumental role that Math courses play in technical degrees and considering the fact that computers and calculation programs are common tools in an engineer's professional activities, computers and CAS in particular have also started to enjoy widespread use in Math courses for engineers. Additionally, using computers makes it enormously easier to do exercises and apply mathematical subject matter to engineering problems, a situation that makes the use of computers especially appropriate in education.

However, as stated in [1], even nowadays there are still many professors who hesitate to use such technology due to technical, personal or even political reasons. An explanation for such behaviour can be found in the fact that most teachers were not taught to use CAS when they were studying to be teachers ([2]). It therefore seems that without such previous training, teachers are not sufficiently willing or prepared to make appropriate activities, and moreover, they are not able to recognize the potential risks involved in such use.

We believe that it is necessary to use CAS when teaching Mathematics for Engineering for several reasons. In fact, there exists a significant body of publications on different tasks and concepts that can be

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carried out in such courses using CAS, making it easier for teachers to teach students how to use them, and above all, helping to improve students' learning. Among other recorded benefits, CAS help make especially difficult abstract concepts more accessible and understandable to students ([3]). In addition, they help to increase student motivation and improve students' attitudes towards Mathematics ([4]).

On the other side of things, using CAS helps to bring about certain changes in the way classes are held: when CAS are not used, the professor tends to be the sole centre of attention; when they are used, there is an observable increase in student participation, autonomous activity and interaction among students ([5]), thereby making the process of acquiring and constructing mathematical knowledge more student-centred ([6]). Lastly, due to the potential interactivity of these tools, students are able to attain a higher level of abstraction in mathematical problem-solving ([7]), something which clearly represents a significant didactic accomplishment.

The above-mentioned qualitative improvements are complemented by significant quantitative improvements in students' academic performance when using CAS ([8]). These improvements are especially significant in cases where there exist deficiencies or a lack of prior mathematical knowledge and skills ([9]). When one considers the fact that, in addition to the foregoing, using CAS also makes it possible to hold classes with a variable pace, it is clear that CAS represent extremely useful tools for providing an appropriate attention to diversity in the classroom ([10]).

Based on these arguments, some Universities have fully integrated CAS into Mathematics teaching for several different university degrees, to the extent that their use is no longer considered to be novel or innovative, but rather something commonplace in such courses ([11]).

But one must also keep in mind the limitations and risks involved. For example, using CAS can potentially prevent students from making the proper connections between the techniques used and their mental approach to Mathematics ([12]); on the other hand, using CAS could theoretically lead to undesirable modifications due to the significant changes made to the teaching process itself ([13]). It is also necessary to adopt a series of appropriate measures to help students know what to do when a CAS fails to give them an answer to a problem ([14]) and to help them recognize when CAS are useful for solving a given problem and when they are not.

Regardless, a common characteristic has been detected regarding the effects of working with CAS: despite the improvements in attained practical results, most students are not aware of the improvements in their knowledge and skills, nor are they aware of the improvements in their assimilation of the contents presented in class ([15]).

However, in most cases, the use of CAS is reduced to using computers as powerful high-performance calculators. While such use already represents a step forward in comparison to traditional teaching methodologies, it nonetheless fails to take full advantage of the potential of CAS resources. It is therefore necessary to change the way people think about information technologies in order to optimize the opportunities they offer and try to encourage mathematical creativity among students ([16]).

In recent years a strong movement has been taking place among math teachers that use CAS to change the traditional uses given to these tools. Specifically, as stated in [17], it is a mistake to use CAS in teaching as simple problem-solving machines, while [18] argues that their use should not be reduced to high-performance calculators. On the contrary, they should be used in ways that maximize the opportunities that these technologies offer ([19]), focusing on positively affecting student learning ([20]), significantly increasing opportunities for experimentation ([21]) and allowing students to construct their own mathematical knowledge under the guidance of their teachers ([22]). This is the direction we have taken in our work.

But to accomplish these goals one must first set out a series of appropriate activities. Therefore, as stated in [23], students should be given problems that are extremely hard to solve without using CAS, given the fact that "one cannot solve the same problems that were solved when CAS did not exist" ([24]). [25] makes a similar argument by stating that students should tackle much more realistic exercises and problems in order to take full advantage of computers. On the other side of things, given the fact that CAS often present results in unusual or unexpected ways, [26] proposes using such unexpected results to reinforce the learning of concepts and encourage a spirit of critical thinking among students. Even the "errors" that CAS make can be used to help in the teaching and learning process ([27]).

To address other pertinent matters, we agree with teachers who argue that the use of CAS in Mathematics has not yet reached optimum conditions ([28]). In fact, the most commonly used CAS are *blackbox* (showing the result in one step without teaching students how to get there), while [29] argues that in order to take full advantage of CAS, they should be *whitebox* (showing intermediate steps). Another new development which can be seen to a lesser extent (not because it is less important but rather due to the novelty of the idea), involves an increase in the use of computer programming languages in Mathematics courses ([30]). In such cases, when students program, they must read, construct and refine strategies, modify previously written programs and lastly, use the programs to solve problems. This makes them the protagonists of their own learning ([31]). Furthermore, the use of programming helps teachers find appropriate tasks that directly correspond to the mental constructions of mathematical concepts ([32]). To provide an example, the programming language ISETL was used by [33] to teach math to university students because its syntax favoured the assimilation of mathematical content being taught.

Lastly, it is a good idea to combine programming and CAS by having students design specific commands or functions ([34]), something that makes it possible to significantly increase the standard library of functions offered by CAS. More specifically, we share the approach of [35], which shows the great potential of combining the power of a CAS (in this case DERIVE) with the flexibility of a programming language. We believe that the most appropriate approach involves using programming and CAS together to allow students to create the specific necessary functions that will allow them to solve the problems involved in the subject matter under study.

3. Our experience

Our experience over the past ten years has consisted mainly of continuously carrying out practical experiments in the computer laboratory as part of a series of courses assigned to teachers in the *Department of Applied Mathematics* at the *University of Malaga*, including Mathematics courses for the degrees of *Technical Industrial Engineering* and *Technical Telecommunications Engineering*.

In many of the experiences in the first half of the above-mentioned time period, we could clearly see, among other things, that computers were insufficiently used in Mathematics classrooms ([16]), a situation which led us to introduce, over the course of the past four years, a series of new activities aimed at taking full advantage of computers and decreasing the exclusive use of CAS as calculation machines used to solve problems mechanically. These new activities aimed mainly at encouraging active participation among students in the teaching and learning process and giving greater attention to allowing students to construct mathematical knowledge under the guidance of teachers.

As a consequence of the experience we accumulated over the years, we decided that in addition to solving typical problems that were standard for the math course in question, the students themselves would have to construct (program) commands (specific functions) with DERIVE to solve such problems in their lab sessions ([16]). This new element, in which students had to create the commands themselves, represents the main innovative aspect of our experiments and requires an exhaustive knowledge of the subject matter on the part of the student.

As an example, in order to make a command that will check if a differential form is an exact differential form, students have to know what condition makes the differential form exact; on the other hand, in order to make commands that calculate triple integrals, students must keep the following elements in mind: the function to be integrated, the coordinate system and the three variables of integration with their corresponding integration limits. Additionally, since the order of integration is significant, students must take it into account as well when making the commands. Obviously, the fact that the students themselves are making the commands and including all of their arguments has a positive influence on their ability to subsequently apply them to solve specific exercises.

Through this type of practical lab sessions it becomes possible to accomplish the goal of not reducing computers to their most classical and typical use as a way of making calculations on a high-power calculator, but rather as tools that encourage active learning, greater comprehension of subject matter and mathematical creativity among students.

Lastly, it is worth mentioning that our experiences were channelled through the following projects, carried out at the *University of Malaga*, all of which involved having students make commands with DERIVE as a fundamental aspect:

- The Virtual Teaching Project *Computers in Math Courses in Technical Degrees: Much More Than a Powerful Calculator*.
- The Educational Innovation Project *Mathematics for Engineering. Computers as a Tool for Mathematical Creativity*.
- The Educational Innovation Project *Command-Making with DERIVE. An Experience in Degrees in Technical Telecommunications Engineering*.

4. Final conclusions

From the reflections contained in this paper, the following conclusions should be highlighted:

- Our accumulated experience reveals that CAS are computer tools which are easy to use and useful in Mathematics courses for Engineering.
- The traditional uses given to CAS in the teaching of Mathematics for Engineering must be changed to maximize the opportunities offered by CAS technologies. Optimal use should be aimed at improving student motivation, autonomy and achieving participatory and student-centred learning.
- One powerful idea involves combining CAS resources with the flexibility of a programming language.
- There exists reasonable evidence to show that making commands with DERIVE facilitates learning and improves student motivation.
- Although it would be desirable to do so, it is not necessary to substantially modify the traditional program of studies of Math courses for Engineering to introduce the innovation of having students make their own commands with DERIVE.

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