American Diploma Project

Algebra II
End-of-Course Exam

Released Items
March 2008

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Achieve, Inc.

Created by the nation's governors and business leaders, Achieve, Inc., is a bipartisan non-profit organization that helps states raise academic standards, improve assessments and strengthen accountability to prepare all young people for postsecondary education and training, careers, and citizenship.

PEARSON

Pearson was awarded the contract to develop, deliver, score, and report the ADP Algebra II End-of-Course Exam. Through its Educational Measurement group, Pearson is the largest comprehensive provider of educational assessment products, services and solutions. As a pioneer in educational measurement, Pearson has been a trusted partner in district, state and national assessments for more than 50 years. Pearson helps educators and parents use testing and assessment to promote learning and academic achievement.

Background

The American Diploma Project (ADP) Network includes 32 states dedicated to making sure every high school graduate is prepared for college and work. Together, Network members are responsible for educating nearly 75% of all U.S. public school students. In each state, governors, state superintendents of education, business executives, and college and university leaders are working to restore value to the high school diploma by raising the rigor of high school standards, assessments, and curriculum, and better aligning these expectations with the demands of postsecondary education and work.

In May 2005, leaders from several of the ADP Network states began to explore the possibility of working together, with support from Achieve, to develop a common end-of-course exam for Algebra II.

In the fall of 2005, nine states—Arkansas, Indiana, Kentucky, Maryland, Massachusetts, New Jersey, Ohio, Pennsylvania and Rhode Island—came together to develop specifications for a common end-of-course exam in Algebra II. These states were planning to require or strongly encourage students to take Algebra II, or its equivalent, in order to better prepare them for college and careers, as Algebra II, or its equivalent, is a gateway course for higher education and teaches quantitative reasoning skills important for the workplace. State leaders recognized that using an end-of-course test would help ensure a consistent level of content and rigor in classes within and across their respective states. They also understood the value of working collaboratively on a common test: the potential to create a higher quality test faster and at lower cost to each state, and to compare their performance and progress with one another.

In recent months, five additional states—Arizona, Hawaii, Minnesota, North Carolina, and Washington—have joined the partnership to create the American Diploma Project (ADP) Algebra II End-of-Course Exam, bringing the total number of participating states to fourteen.

Pearson conducted field testing of the ADP Algebra II End-of-Course Exam in October 2007 and February 2008. This spring the exam will be administered to over 100,000 students across several of the consortium states, for the first time as an operational assessment.

The ADP Algebra II End-of-Course Exam serves three main purposes:

1. ***To improve curriculum and instruction.*** The test will help classroom teachers focus on the most important concepts and skills in Algebra II and identify areas where the curriculum needs to be strengthened.

2. ***To help colleges determine if students are ready to do credit-bearing work.*** Because the test is aligned with the ADP mathematics benchmarks, it will measure skills students need to enter and succeed in first-year, credit-bearing mathematics courses. Postsecondary institutions will be able to use the results of the test to tell high school students whether they are ready for college-level work, or if they have content and skill gaps that need to be filled before they enroll in college. This information should help high schools better prepare their students for college, and reduce the need for colleges to provide costly remediation courses.

3. ***To compare performance and progress among the participating states.*** Having agreed on the core content expectations of Algebra II, states are interested in tracking student performance over time. Achieve will issue a report each year comparing performance and progress among the participating states. This report will help state education leaders, educators and the public assess performance, identify areas for improvement, and evaluate the impact of state strategies for improving secondary math achievement.

Algebra II level curriculum: Function modeling and problem solving is the heart of the curriculum at the Algebra II level. Mathematical modeling consists of recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem. Students must be able to solve practical problems, representing and analyzing the situation using symbols, graphs, tables, or diagrams. They must effectively distinguish relevant from irrelevant information, identify missing information, acquire needed information, and decide whether an exact or approximate answer is called for, with attention paid to the appropriate level of precision. After solving a problem and interpreting the solution in terms of the context of the problem, they must check the reasonableness of the results and devise independent ways of verifying the results.

Algebra II level classroom practices: Effective communication using the language of mathematics is essential in a class engaged in Algebra II level content. Correct use of mathematical definitions, notation, terminology, syntax, and logic should be required in all work at the Algebra II level. Students should be able to translate among and use multiple representations of functions fluidly and fluently. They should be able to report and justify their work and results effectively. To the degree possible, these elements of effective classroom practice are reflected in the *ADP Algebra II End-of-Course Exam Content Standards* (*Exam Standards*).

The *Exam Standards* consist of a common set of core standards and seven optional modules (posted at: http://www.achieve.org/AlgebraIITestOverview). The Spring 2008 exam will be based upon only the common core standards content.

The Core Algebra II End-of-Course Exam: The core Algebra II End-of-Course Exam covers a range of algebraic topics. Successful students will:

- demonstrate conceptual understanding of the properties and operations of real and complex numbers
- be able to make generalizations through the use of variables resulting in facility with algebraic expressions
- solve single and systems of linear equations and inequalities and be able to use them to represent contextual situations
- be able to demonstrate facility with estimating and verifying solutions of various non-linear equations, making use of technology where appropriate to do so
- demonstrate knowledge of functions and their properties—distinguishing among quadratic, higher degree polynomial, exponential, and piecewise-defined functions—and recognize and solve problems that can be modeled by these functions
- be required to analyze the models, both symbolically and graphically, and determine and effectively represent their solution(s)

There are a variety of types of test items developed that will assess this content, including multiple-choice, short-answer, and extended-response items. Test items, in particular extended-response items, may address more than one content objective and benchmark within a standard, requiring students to make connections and, where appropriate, solve rich contextual problems.

There are fifty-seven questions on the core Algebra II End-of-Course Exam including forty-six multiple choice (1 point each), seven short answer (2 points each), and four extended response (4 points each). At least one-third of the student's score will be based on the short-answer and extended-response items. The Exam is comprised of two 45-60 minute sessions, with only the second session allowing calculator usage. However, some students may require—and should be allowed—additional time to complete the test. Each standard within the exam is assigned a priority, indicating the approximate percentage of points allocated to that standard on the test.

- Operations and Expressions 15%
- Equations and Inequalities 20%
- Polynomial and Rational Functions 30%
- Exponential Functions 20%
- Functional Operations and Inverses 15%

Algebra II End-of-Course Exam calculator use: The appropriate and effective use of technology is an essential practice in the Algebra II classroom. At the same time, students should learn to work mathematically without the use of technology. Computing mentally or with paper and pencil is required on the Algebra II End-of-Course Exam and should be expected in classrooms where students are working at the Algebra II level. It is therefore important that the Algebra II End-of-Course Exam reflect both practices. For purposes of the Algebra II End-of-Course Exam, students are expected to have access to a calculator for one of the two testing sessions, and use of a graphing calculator is strongly recommended. Scientific or four-function calculators are permitted but not recommended because they do not have graphing capabilities. Students should use the calculator they are accustomed to and use every day in their classroom work. It should be noted that not all items found on the calculator portion of the exam require the use of a calculator. It is important that students learn to assess for themselves whether or not a calculator would be helpful. For more information about technology use on the Algebra II End-of-Course Exam, see the calculator policy at the end of this document in Appendix D.

Released Items

The following released items are intended to offer insight into the format and expectations of this exam. All of the items being released in this document have been field-tested by students and reviewed by both a content review committee and a data review committee of professional educators. However, these particular items will never appear in a live test form. Over time, additional items will be released to further illustrate items, both in content and format, that students will see on the Algebra II End-of-Course Exam.

For this item release document, items are categorized as to whether they fall on the calculator or non-calculator portion of the exam. Within each category, items are then grouped by item type—multiple choice (MC), short answer (SA), or extended response (ER). On the actual Algebra II End-of-Course Exam, the sessions are separated into calculator and non-calculator sessions and items of different types will be intermingled throughout each session. Be aware that even though the released short-answer items are calculator items and the extended-response item is a non-calculator item, both item types will be represented in both the calculator and non-calculator sessions on the actual exam. On the exam, the student will know whether an item is a short answer (two-point item) or extended response (four-point item) by the amount of work space provided in the answer document. The work space provided in the answer document for a short answer item is about one-half page, while work space for an extended response item is about one full page. At the end of the "Released Items" section of this document, examples of the work space allocated for each type of constructed-response item are provided.

It should be noted that not all items found on the calculator portion of the exam require the use of a calculator. It is important that students learn to assess for themselves whether or not a calculator would be helpful.

Non-Calculator Items

Multiple Choice

1.

Consider the equation below.

$$8^x = 32$$

Solve for x.

A. $\frac{3}{5}$

B. $\frac{5}{3}$

C. 4

D. 5

2.

Write $\frac{1-3i}{-2-4i}$ in the form $a+bi$.

A. $\frac{1}{2}+\frac{1}{2}i$

B. $\frac{1}{20}-\frac{3}{20}i$

C. $-\frac{1}{2}+\frac{3}{4}i$

D. $-\frac{5}{6}-\frac{5}{6}i$

3.

The graph of the equation $y=3^{x}$ is reflected over the y-axis. What is the equation of the image?

A. $y=-3^{x}$

B. $y=-\left(\frac{1}{3}\right)^{x}$

C. $y=\left(\frac{1}{3}\right)^{x}$

D. $y=3^{x}$

Extended Response

4.

Let $f(x) = x^2 + x + c$.

Part A For what values of c are the roots of $f(x)$ not real? Show or explain your work.

Part B Using one of the values for c that you found in Part A, determine the roots of $f(x)$. Show or explain your work.

Calculator Items

Multiple Choice

5.

What is the solution set of $|x-1| = 2x+3$?

A. $\{-4\}$

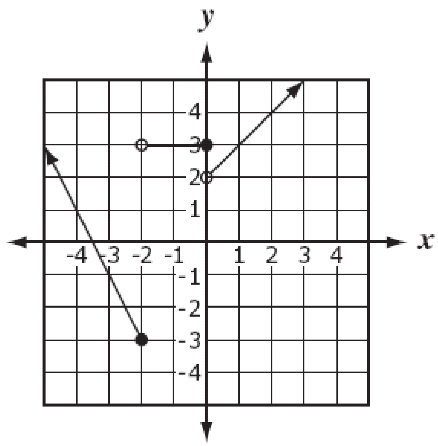
B. $\left\{-\frac{2}{3}\right\}$

C. $\left\{-4, -\frac{2}{3}\right\}$

D. $\varnothing$

6.

Consider the graph below.



Which function is best represented by this graph?

A. $y = \begin{cases} -2x - 3\frac{1}{2}, x \le -2 \\ 3, -2 < x \le 0 \\ x + 2, x > 0 \end{cases}$

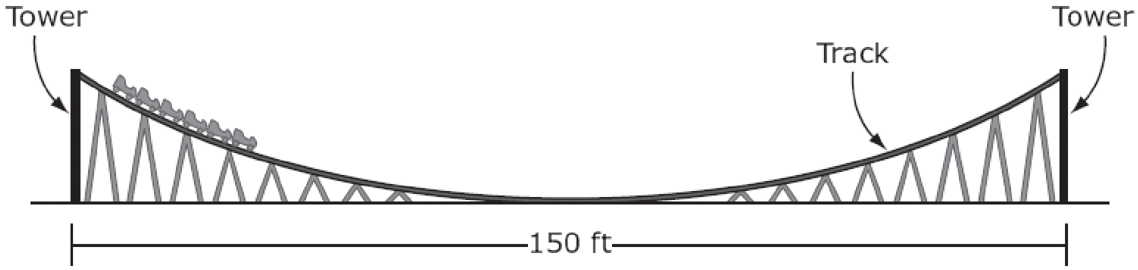
B. $y = \begin{cases} -2x - 7, x \le -2 \\ 3, -2 < x \le 0 \\ 2x, x > 0 \end{cases}$

C. $y = \begin{cases} -2x - 7, x \le -2 \\ 3, -2 < x \le 0 \\ x + 2, x > 0 \end{cases}$

D. $y = \begin{cases} -2x - 7, x \le -2 \\ 3x, -2 < x \le 0 \\ x + 2, x > 0 \end{cases}$

7.

Consider the diagram below.



The function $h(x) = \frac{1}{98}x^2$ describes $h(x)$, the height of part of a rollercoaster track, where x is the horizontal distance in feet from the center of this section of the track. The towers that support this part of the track are the same height and are 150 feet apart. Which is the best estimate of the height of the towers?

A. 57.4 feet
B. 85.7 feet
C. 121.2 feet
D. 229.6 feet

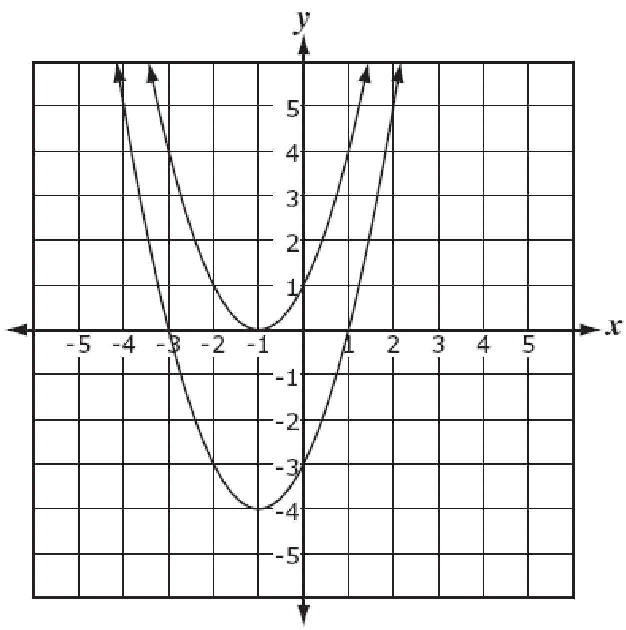
Short Response

8.

Solve $5e^{6x} + 4 = 13$ for x. Show or explain your work.

9.

The two quadratic functions graphed in the figure below are vertical translations of each other.



The equation for one of the functions is $y = x^2 + 2x - 3$. Write an equation that will describe the graph of the other function. Explain your reasoning.

10.

A car has an original value of $20,000. The value decreases at a rate of 18% each year.

Part A Write a function where $f(x)$ represents the value of the car in dollars and x represents years.

Part B After how many years will the car be worth less than $\frac{1}{2}$ of the original value? Show or explain your work.

Student Work Space

Short Answer
(paper-and-pencil exam)

Extended Response

(paper-and-pencil exam)

Released Item Commentaries

As stated earlier, these released items are intended to offer insight into the format and expectations of this exam. Of course, no set of ten items could possibly exemplify all of the available information surrounding an exam such as this one. These sample solutions and commentaries are provided as guidance to teachers and students that will be participating in the ADP Algebra II End-of-Course Exam. Please note that the format of and style of text in the solutions and commentaries does not reflect exactly how items appear in the test books. Some of the released items have been selected because they illustrate a particular type of notation with which students should become familiar; some of the released items attempt to show the intended assessment limit of a particular standard; still others demonstrate the use of diagrams, graphs, or contextual problem situations. Sample solutions and commentary are provided as guidance and should not be considered exhaustive. On the actual Algebra II End-of-Course Exam, the student's choice of solution method is limited by the placement of the items in the calculator or non-calculator session of the exam. The solutions presented in this document are aligned with the session in which an item appears. Students should be able to solve test problems in multiple ways, with and without a calculator.

For each item, this commentary will include the following:

- item type
- item calculator usage
- benchmark
- item
- the correct answer (multiple choice)
- distractor analyses (multiple choice) to indicate common errors that students could make
- implications for instruction
- sample solution method(s)

Item 1

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: E2.a Solve single-variable quadratic, exponential, rational, radical, and factorable higher-order polynomial equations over the set of real numbers, including quadratic equations involving absolute value.

Item:

Consider the equation below.

$$8^x = 32$$

Solve for x.

A. $\frac{3}{5}$

B. $\frac{5}{3}$

C. 4

D. 5

Correct Answer: B

Explanation of Distractors:

A	Determines the equivalent equation is $2^{3x} = 2^5$ but solves $3x = 5$ incorrectly
C	Solves the equation as $8x = 32$ (Students need to be careful to interpret the location of the x correctly.)
D	Solves the equation as $2^x = 2^5$, ignoring the difference in the two bases

Instructional Implications: This is an example of an exponential equation that can be solved without the use of a calculator. To find the solution, students must be familiar with the properties of exponents, in particular the property

$$\left(b^m\right)^n = b^{mn} \text{ for } b > 0.$$

Also fundamental to the solution of this equation is an understanding that

$$\text{if } b^m = b^n \text{, then } m = n.$$

This result relies on the fact that any exponential function is one-to-one—sometimes a difficult concept for students to comprehend, but important to this solution.

Sample Solution:
Students should be familiar enough with the integral powers of several numbers—at least 2, 3, 5 and 10—and should notice that both 8 and 32 can be expressed as a power of 2. The solution of the equation relies then on re-expressing 8 and 32 as powers of 2 and applying the exponential properties referenced above.

$$\begin{aligned} 8^x &= 32 \\ \left(2^3\right)^x &= 2^5 \\ 2^{3x} &= 2^5 \\ 3x &= 5 \\ x &= \frac{5}{3} \end{aligned}$$

Item 2

Item Type: Multiple Choice

Calculator: NOT permitted

Benchmark: O2.b Perform operations on the set of complex numbers.

Item:

Write $\frac{1-3i}{-2-4i}$ in the form $a+bi$.

A. $\frac{1}{2}+\frac{1}{2}i$

B. $\frac{1}{20}-\frac{3}{20}i$

C. $-\frac{1}{2}+\frac{3}{4}i$

D. $-\frac{5}{6}-\frac{5}{6}i$

Correct Answer: A

Explanation of Distractors:

B	Multiplies only the denominator by the conjugate of $-2-4i$
C	Breaks up the numbers in the denominator so that -2 becomes the denominator under 1 and -4 becomes the denominator under $-3i$
D	Multiplies the numerator and denominator by the conjugate of $-2-4i$ but incorrectly evaluates $4-16i^2$ as -12

Instructional Implications: In this item, students have to understand that division of complex numbers is indicated by the fraction bar, or *vinculum*. Students must also understand that division of complex numbers is carried out by *rationalizing* the denominator of the fraction, that is, multiply by something that will make the denominator of the fraction real instead of complex.

Sample Solutions:
To rationalize the denominator, the student must identify the *conjugate* of the complex number in the denominator. The conjugate of the complex number $a+bi$ is $a-bi$. *The numerator and denominator are then both multiplied by the conjugate of the denominator. The product of two complex conjugates is a real number.*

Before rationalizing the denominator, 2 or -2 could be factored out of the denominator. The three columns below illustrate the rationalization process for three possible expressions of the denominator.

The conjugate of the denominator is $-2+4i$	The conjugate of the complex factor in the denominator is $-1+2i$	The conjugate of the complex factor in the denominator is $1-2i$
$\frac{1-3i}{-2-4i}$	$\frac{1-3i}{-2-4i}$	$\frac{1-3i}{-2-4i}$
$\frac{(1-3i)(-2+4i)}{(-2-4i)(-2+4i)}$	$\frac{(1-3i)}{2(-1-2i)}$	$\frac{(1-3i)}{-2(1+2i)}$
$\frac{-2+4i+6i-12i^2}{4-16i^2}$	$\frac{(1-3i)(-1+2i)}{2(-1-2i)(-1+2i)}$	$\frac{(1-3i)(1-2i)}{-2(1+2i)(1-2i)}$
$\frac{10+10i}{20}$	$\frac{-1+2i+3i-6i^2}{2(1-4i^2)}$	$\frac{1-2i-3i+6i^2}{-2(1-4i^2)}$
$\frac{1+i}{2}$	$\frac{5+5i}{2(5)}$	$\frac{-5-5i}{-2(5)}$
$\frac{1}{2}+\frac{1}{2}i$	$\frac{1+i}{2}$	$\frac{-1-i}{-2}$
	$\frac{1}{2}+\frac{1}{2}i$	$\frac{1}{2}+\frac{1}{2}i$

Item 3

Item Type: Multiple Choice

Calculator: NOT permitted

Benchmark: X1.c Describe the effect that changes in a parameter of an exponential function have on the shape and position of its graph.

Item:

The graph of the equation $y = 3^x$ is reflected over the y-axis. What is the equation of the image?

A. $y = -3^x$

B. $y = -\left(\frac{1}{3}\right)^x$

C. $y = \left(\frac{1}{3}\right)^x$

D. $y = 3^x$

Correct Answer: C

Explanation of Distractors:

A	Reflects the graph of the equation over the *x*-axis instead of the *y*-axis.
B	Reflects the graph of the equation over the *x*-axis and the *y*-axis.
D	Uses the given equation to represent the reflection of the graph over the *y*-axis.

Instructional Implication: Reflection of a graph involves negating—or changing the sign of—one of the variables, either the x or y in this problem. Reflection over the x-axis involves changing the sign on the y-variable, $-y = 3^x$ or $y = -3^x$, while reflection over the y-axis involves changing the sign on the x-variable, $y = 3^{-x}$. Students will need practice to recognize that 3^{-x} is equivalent to $\left(\frac{1}{3}\right)^x$ in order to identify the correct answer.

Sample Solution:
There is relatively little for students to do to solve this problem. The key is for them to recognize that changing the signs on all x's—which translates into $y = 3^{-x}$ for the given equation—is how a reflection of a graph over the y-axis is represented in the symbolic form of the function.

While the solution $y = 3^{-x}$ would be acceptable if this question was presented in short answer format, here students must make an additional adjustment because that form of the answer is not one of the choices in the multiple choice format. By applying the definition of a negative exponent, answer choice C, $y = \left(\frac{1}{3}\right)^x$, can be seen to be equivalent to $y = 3^{-x}$.

Item 4

Item Type: Extended Response

Calculator: NOT Permitted

Benchmarks: Extended-response items can be written to address multiple aspects of the standard. This particular item was written to the E standard, *Equations and Inequalities*, and addresses the following benchmarks within the standard.

E2.a Solve single-variable quadratic, exponential, rational, radical, and factorable higher-order polynomial equations over the set of real numbers, including quadratic equations involving absolute value.

E2.c Use the discriminant, $D = b^2 - 4ac$, to determine the nature of the solutions of the equation $ax^2 + bx + c = 0$.

Item:

Let $f(x) = x^2 + x + c$.

Part A For what values of c are the roots of $f(x)$ not real? Show or explain your work.

Part B Using one of the values for c that you found in Part A, determine the roots of $f(x)$. Show or explain your work.

Correct Answer:

Part A: $c > \frac{1}{4}$

Part B: Answers will vary depending on student's choice for the c value. For example, if $c = 1$, then the roots are $\frac{-1 \pm i\sqrt{3}}{2}$ or equivalent.

To earn full credit, student solutions should correctly address all parts of the item, including showing sufficient work to justify the answer and/or providing a complete explanation, where necessary.

Instructional Implications:

Extended response items are intended to test the student's ability to synthesize and apply the concepts that cross an entire standard. In this problem, students need to know the conditions under which the solution of a quadratic equation will be non-real—this can be taught using the discriminant. It is important that students understand why having a negative discriminant guarantees that the roots will be non-real (complex). The restrictions on c will be found as an inequality. Students must then know that they need to select *a single number* that meets these restrictions and use it to determine the roots of the resulting function.

Students should have experience solving multi-step problems where they must make decisions—such as choosing a value for c that satisfies the restrictions found.

Sample Solution:

Part A

The roots of a quadratic equation of the form $ax^2 + bx + c = 0$ are not real when the discriminant, $b^2 - 4ac$ is negative.

For this problem, the function is given as $f(x) = x^2 + x + c$. The roots of the equation $x^2 + x + c = 0$ will yield the zeros, or roots, of this function. If the function is to have non-real roots, the discriminant of the equation $x^2 + x + c = 0$ must be less than zero.

The discriminant of $x^2 + x + c = 0$ is $(1)^2 - 4(1)(c)$, or $1 - 4c$.

$$\begin{aligned} 1 - 4c &< 0 \\ 1 &< 4c \\ c &> \frac{1}{4} \end{aligned}$$

Part B

Students must select one possible value for *c* and use it to determine the zeros for the resulting function.

For example, suppose that $c = 1$. Because $1 > \frac{1}{4}$, it fits the restrictions on *c* that guarantee that the zeros will not be real.

The resulting function is $f(x) = x^2 + x + 1$.

The quadratic formula can be used to determine the roots of $x^2 + x + 1 = 0$ and the zeros for the function.

$$\begin{aligned} x &= \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} \\ &= \frac{-1 \pm \sqrt{1 - 4}}{2} \\ &= \frac{-1 \pm \sqrt{-3}}{2} \\ &= \frac{-1 \pm i\sqrt{3}}{2} \text{ or } -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i \end{aligned}$$

So the pair of non-real zeros for the function $f(x) = x^2 + x + 1$ is $\frac{-1 \pm i\sqrt{3}}{2}$, or $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$.

Item 5

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: E1.a Solve equations and inequalities involving the absolute value of a linear expression.

Item:
What is the solution set of $|x-1| = 2x+3$?

A. $\{-4\}$

B. $\left\{-\frac{2}{3}\right\}$

C. $\left\{-4, -\frac{2}{3}\right\}$

D. $\varnothing$

Correct Answer: B

Explanation of Distractors:

In each case, the incorrect response indicates eliminating possible solutions incorrectly.

A	Only solves $x-1=2x+3$ and does not check the solution in the original absolute value equation.
C	Solves $x-1=2x+3$ and $x-1=-(2x+3)$ correctly but does not check either answer in the original absolute value equation.
D	Solves $x-1=2x+3$ and $x-1=-(2x+3)$ correctly and checks both solutions in the original absolute value equation. Incorrectly determines that neither solution satisfies the original equation.

Instructional Implications: This item contains the set notation that may be found on the exam. In addition, the problem reminds teachers and students of the need to check for extraneous solutions, that is, for potential solutions that do not fit all of the constraints of the problem.

Regardless of how the problem is solved, it is important for students to understand the mathematical notation that will be used on the exam to express the solution. For this equation, $x=-\frac{2}{3}$ is the only solution—another way to say this is that $-\frac{2}{3}$ is the only *element* of the *solution set* for the equation. The symbol $\{\ \}$, sometimes called *braces* or *set brackets*, is often used to denote a set. The solution set for this equation is given by $\left\{-\frac{2}{3}\right\}$. The symbol $\varnothing$ will be used to represent the *empty set*—or a set having no elements—and would be selected in a case where there were no solutions.

Sample Solutions:
This item is found on the calculator-active portion of the exam. If a graphing calculator is available to students—one with which they are familiar—a graphical solution method provides an alternative to the more traditional algebraic methods.

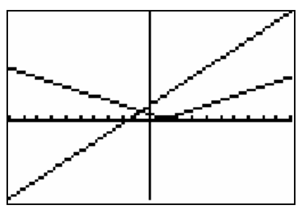
The only solution to this absolute value equation is $-\frac{2}{3}$. While -4 is a potential solution, it does not satisfy the equation when it is checked.

Sample Graphical Method:
While the problem is presented as a single-variable equation, it is often possible to gain insight into its solutions by introducing a second variable.

To solve the single-variable equation $|x-1| = 2x+3$
Consider the system of equations

$$y = |x-1|$$
$$y = 2x+3$$

Use the graphing calculator to graph these two equations in *an appropriate window*.



For this problem, the *ZoomFit* capability on one type of calculator yields a window where $-1 < x < 3$ and $0 < y < 9$. The single intersection point for these two graphs is visible in this window.

The standard default window, $-10 < x < 10$ and $-10 < y < 10$ will also show the critical features of this system of equations.

Other options are possible as long as the window covers the intersection point $\left(-\frac{2}{3},\frac{5}{3}\right)$.

It is important for students to understand the nature of the graphs of both a linear function and an absolute value function in order to be confident that there is *no additional point of intersection* for this system that lies outside the chosen window.

This method relies on student understanding that the solution of a system of equations in two variables occurs where the graphs of the equations making up the system intersect. Students using the calculator to determine the point of intersection of these two graphs should have repeated experiences finding solutions to systems on the calculator they will be using for the exam. Often the coordinates of the point will be approximated by decimals such as $(-0.6666667,\ 1.6666667)$. It is important for students to realize that this is the way many calculators express the fractional solution $\left(-\frac{2}{3},\frac{5}{3}\right)$.

Because a calculator may be providing only an approximate set of coordinates, it is important that students verify the solution by substitution. Translating back to the original problem, the solution only calls for the *x*-value that makes the original equation true. Substituting $-\frac{2}{3}$ for *x*:

$$\left|-\frac{2}{3}-1\right| = 2\left(-\frac{2}{3}\right)+3$$
$$\left|-\frac{5}{3}\right| = -\frac{4}{3}+\frac{9}{3}$$
$$\frac{5}{3} = \frac{5}{3}$$

The solution to the equation is $x = -\frac{2}{3}$.

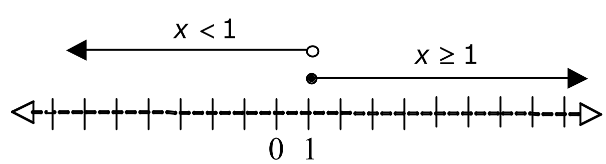
Sample Algebraic Methods

Algebraic approaches to this problem rely on the definition of absolute value:

$$\begin{cases} |x| = x \text{ for } x \geq 0 \\ |x| = -x \text{ for } x < 0 \end{cases}$$

Sample Algebraic Method: Number Line approach

Students should recognize that two critical regions of the number line come into play. If $|x-1| \geq 0$, then $x \geq 1$ defines a closed ray beginning at 1 and pointed to the right. If $|x-1| < 0$, then $x < 1$ defines an open ray beginning at 1 and pointed to the left.



If a point is selected from the ray defined by $x \geq 1$, then $|x-1| = x-1$ and the equation can be solved as shown below:

$$x - 1 = 2x + 3$$
$$-x - 1 = 3$$
$$-x = 4$$
$$x = -4$$

The solution to this equation, $x = -4$, IS NOT on the ray $x \geq 1$, so it cannot be a solution for this problem and must be discarded.

If a point is selected from the ray defined by $x < 1$, then $|x - 1| = -(x - 1)$ and the equation can be solved as shown below:

$$\begin{aligned} -(x - 1) &= 2x + 3 \\ -x + 1 &= 2x + 3 \\ -3x &= 2 \\ x &= -\frac{2}{3} \end{aligned}$$

The solution to this equation, $x = -\frac{2}{3}$, IS on the ray $x < 1$, so it is a solution for the problem.

All of the solutions that make the original absolute value equation true are called the *solution set* of the equation. In this case, the only solution for the equation is $x = -\frac{2}{3}$.

Sample Algebraic Method: Case Method approach

Case 1: If $|x - 1| \geq 0$, then $x \geq 1$ and $|x - 1| = x - 1$.
The equation can be solved as shown below:

$$\begin{aligned} x - 1 &= 2x + 3 \\ -x - 1 &= 3 \\ -x &= 4 \\ x &= -4 \end{aligned}$$

The inequality $x \geq 1$ and the equation $x = -4$ have no solutions in common so do not yield a solution for the problem.

Case 2: If $|x - 1| < 0$, $x < 1$, then $x < 1$ and $|x - 1| = -(x - 1)$.
The equation can be solved as shown below:

$$\begin{aligned} -(x - 1) &= 2x + 3 \\ -x + 1 &= 2x + 3 \\ -3x &= 2 \\ x &= -\frac{2}{3} \end{aligned}$$

The inequality $x < 1$ and the equation $x = -\frac{2}{3}$ have a single solution in common so a $x = -\frac{2}{3}$ is a solution for the problem.

Regardless of the method of solution used, it is important that students recognize that $\left\{-\frac{2}{3}\right\}$ is a correct representation for the answer.

Item 6

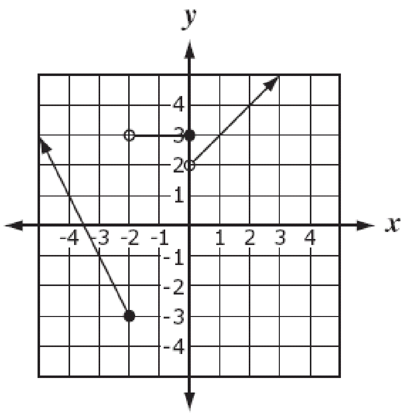
Item Type: Multiple Choice

Calculator: Permitted

Benchmark: F3.b Represent piecewise-defined functions using tables, graphs, verbal statements, and equations. Translate among these representations.

Item:

Consider the graph below.



Which function is best represented by this graph?

A. $y = \begin{cases} -2x - 3\frac{1}{2}, \ x \le -2 \\ 3, \ -2 < x \le 0 \\ x + 2, \ x > 0 \end{cases}$

B. $y = \begin{cases} -2x - 7, \ x \le -2 \\ 3, \ -2 < x \le 0 \\ 2x, \ x > 0 \end{cases}$

C. $y = \begin{cases} -2x - 7, \ x \le -2 \\ 3, \ -2 < x \le 0 \\ x + 2, \ x > 0 \end{cases}$

D. $y = \begin{cases} -2x - 7, \ x \le -2 \\ 3x, \ -2 < x \le 0 \\ x + 2, \ x > 0 \end{cases}$

Correct Answer: C

Explanation of Distractors:

A	Uses the *x*-intercept of the graph of the first piece of the function as the *y*-intercept in its equation.
B	Uses the *y*-intercept of the graph of the third piece of the function as the slope in its equation.
D	Uses the *y*-value of the graph of the second piece of the function as the slope in its equation.

Instructional Implication: Students should be familiar with the symbolic notation for a step function—being certain to pay special attention to the domain associated with each piece of the function.

Sample Solution:
Students should begin by identifying the *x*-values associated with the two rays and one segment that make up the graph. Special attention should be paid to whether an open circle or a dot, which can also be referred to as a point, is used to mark the beginning/end of each piece of the graph.

The ray with endpoint at $(-2, -3)$ covers all *x*-values less than or equal to -2 since the endpoint is indicated by a dot.

$$x \le -2$$

The segment in the middle covers all *x*-values strictly greater than -2, since the left endpoint is indicated by an open circle, but less than or equal to 0, since the right endpoint is indicated by a dot.

$$-2 < x \le 0$$

Finally, the ray with endpoint $(0,2)$ covers all *x*-values strictly greater than 0 since the endpoint is indicated by an open circle.

$$x > 0$$

In this problem, the domains are the same for all possible answers, but similar questions might offer distractors having different piecewise domains so students should be aware of how they are determined. However, if a similar problem appeared in a constructed-response format, students might be required to express piecewise functions on their own, including determining the appropriate domain.

Once the piecewise domains have been determined, students may simply wish to write the equations of the lines that *carry* each ray and segment using one of the given points and a counted slope.

For the ray on the left: $(-2, -3)$ is a point on the line. It appears that the ray also passes through the point $(-4,1)$ so the slope is $-\frac{4}{2} = -2$.

$$y - (-3) = -2(x - (-2))$$
$$y + 3 = -2x - 4$$
$$y = -2x - 7$$

This effectively eliminates choice A.

The segment is part of a horizontal line that passes through the point $(0,3)$.

$$y = 3$$

This effectively eliminates choice D where the equation of the line carrying the segment is given as $y = 3x$.

Finally, the ray on the right has a *y*-intercept of 2 and a slope of 1, so its line has the following equation.

$$y = x + 2$$

This final equation leads to identifying C as the correct answer.

Whether students simply reason through the solution or actually write the equation of the related lines, students need to be familiar with the notation for step function—being certain to pay special attention to the domain associated with each piece of the function.

Item 7

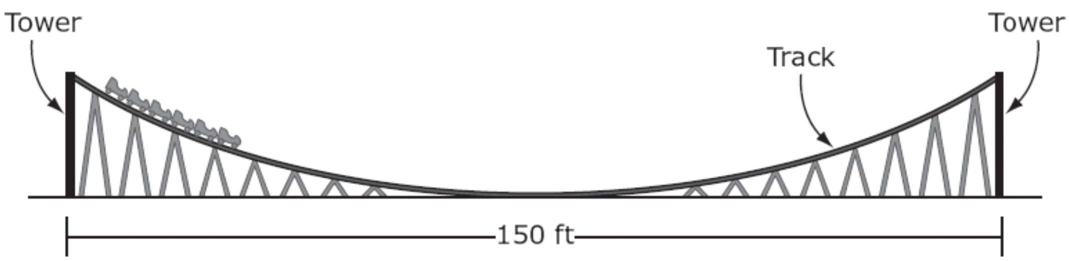
Item Type: Multiple Choice

Calculator: Permitted

Benchmark: P1.d Recognize, express, and solve problems that can be modeled using quadratic functions. Interpret their solutions in terms of the context.

Item:

Consider the diagram below.



The function $h(x) = \frac{1}{98}x^2$ describes $h(x)$, the height of part of a rollercoaster track, where x is the horizontal distance in feet from the center of this section of the track. The towers that support this part of the track are the same height and are 150 feet apart. Which is the best estimate of the height of the towers?

A. 57.4 feet
B. 85.7 feet
C. 121.2 feet
D. 229.6 feet

Correct Answer: A

Explanation of Distractors:

B	Substitutes 75 for $h(x)$ instead of for x, resulting in the equation $75 = \frac{1}{98}x^2$, and solves for x.
C	Substitutes 150 for $h(x)$, resulting in the equation $150 = \frac{1}{98}x^2$, and solves for x.
D	Substitutes 150 for x and evaluates $h(150) = \frac{1}{98}(150)^2$.

Instructional Implications: This released item is an example of a quadratic application that is not related to the speed or motion of a falling body. Its inclusion is intended to provide insight for teachers and students about how a non-graphical diagram can communicate graphical information.

Critical to solving this problem is the ability to understand how to impose a coordinate system onto the existing diagram. Students must choose a position for the *x*- and *y*-axes; a judicious decision clarifies the problem and simplifies its solution. This understanding will develop over time as students are presented with contextual problems that can be solved using a coordinate plane and given the opportunity to explore the result of different axis placements.

Sample Solution:
For this problem, the equation of the quadratic function that describes the path of the rollercoaster is given as $h(x) = \frac{1}{98}x^2$. The graph of that quadratic function goes through the origin since it is of the general form $h(x) = ax^2$. For that reason, the horizontal axis (*x*-axis) must be located to include the lowest point the rollercoaster ever reaches—at *ground level*. Locating the vertical axis (*y*-axis) at the point where the rollercoaster hits its lowest point—the midpoint of the ride as it is shown, guarantees that the graph of the equation will have its vertex at (0, 0) and will be of the correct general form.

Once the axes are located, the origin falls at (0, 0), that is, at the midpoint of the ride where the height above the ground is 0 feet. Each of the two towers is 75 feet away from the midpoint. By substituting 75 or -75 for *x* in the function $h(x) = \frac{1}{98}x^2$ and evaluating, the height of the towers can be determined.

$$\begin{aligned} h(x) &= \frac{1}{98}x^2 \\ h(75) &= \frac{1}{98}(75)^2 \\ &= \frac{5625}{98} \\ &\approx 57.397959 \\ &\approx 57.4 \end{aligned}$$

$$\begin{aligned} h(x) &= \frac{1}{98}x^2 \\ h(-75) &= \frac{1}{98}(-75)^2 \\ &= \frac{5625}{98} \\ &\approx 57.397959 \\ &\approx 57.4 \end{aligned}$$

Students should notice that it is NOT necessary to evaluate the height for both towers. Because of the squared *x* and because of the symmetry of a quadratic graph, both answers will be the same.

Item 8

Item Type: Short Answer

Calculator: Permitted

Benchmark: E2.a Solve single-variable quadratic, exponential, rational, radical, and factorable higher-order polynomial equations over the set of real numbers, including quadratic equations involving absolute value.

Item:

Solve $5e^{6x} + 4 = 13$ for x. Show or explain your work.

Correct Answer: $x = \frac{\ln(1.8)}{6}$ **or** $x = \frac{\log(1.8)}{6}$ **or** $x \approx 0.098$

To earn full credit, student solutions should correctly address all parts of the item, including showing sufficient work to justify the answer and/or providing a complete explanation, where necessary.

Instructional Implications: Some exponential growth or decay situations are *continuous* in nature—for example, continuous compounding of interest or the size of a bacterial culture that is considered to be constantly increasing. Such situations are best represented using *e* for the base of the exponent. In this problem, students are asked to solve an equation that could arise from such a situation.

Because it has been determined that problems involving *e* are permitted on the exam, students need to be able to interpret that symbol and its relationship to the natural logarithm, $\ln(x) = \log_e(x)$.

If the curriculum has introduced students to the concept of a logarithm, use of the *natural logarithm* is appropriate here. The solution may be expressed in logarithmic form or, if a calculator with a logarithm function is available, expressed as a numeric approximation. If logarithms have not been studied, calculator technology—used to graph and solve or used to explore a numerical approach to a solution—provides an alternative solution method.

Regardless of the solution method used, student experience with notation and any technology that is to be used is important. In particular, students will need instruction on how to enter *e* on the calculator they will be using on the exam.

Sample Solutions:

Sample algebraic approach:
A good algebraic strategy will demonstrate student understanding that the solution of the given equation begins by isolating e^{6x} on one side of the equation.

$$5e^{6x} + 4 = 13$$
$$5e^{6x} = 9$$
$$e^{6x} = \frac{9}{5} \text{ or } 1.8$$

From this point, if the student understands logarithms, several methods are possible to solve for x.

Students can apply the definition of a logarithm to rewrite this equation:

$$\ln(1.8) = 6x$$

and solve this equation

$$x = \frac{1}{6}\ln(1.8)$$
$$\approx 0.0979644442$$

Students can take the natural (base e) or common (base 10) logarithm of both sides of the equation:

$$\ln\left(e^{6x}\right) = \ln(1.8)$$
$$6x\ln(e) \approx 0.5877866649$$
$$6x \approx 0.5877866649$$
$$x \approx 0.0979644442$$

or

$$\log\left(e^{6x}\right) = \log(1.8)$$
$$6x\log(e) \approx 0.2552725051$$
$$6x(0.4342944819) \approx 0.2552725051$$
$$x \approx 0.0979644442$$

The answer may be expressed using the logarithm or approximated—both will receive full credit if expressed correctly. No level of precision is specified for this problem so approximations may be given to any degree of precision. For this problem, $x \approx 0.098$ seems to express the answer with reasonable precision. Students should be reminded to look for indications of precision, either expressed explicitly or implied by the context of a problem, whenever answers are expressed as decimal approximations.

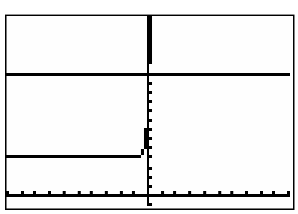
Sample graphical approach:
This item is found on the calculator-active portion of the exam. If a graphing calculator is available to students—one with which they are familiar—a graphical solution method provides an alternative to the more traditional algebraic methods.

While the problem is presented as a single-variable equation, it is often possible to gain insight into its solutions by introducing a second variable.

To solve the single-variable equation $5e^{6x} + 4 = 13$
Consider the system of equations

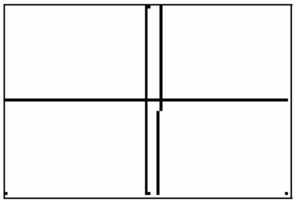
$$y = 5e^{6x} + 4$$
$$y = 13$$

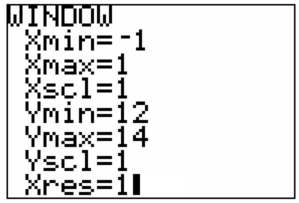
Use the graphing calculator to graph these two equations in *an appropriate window*.

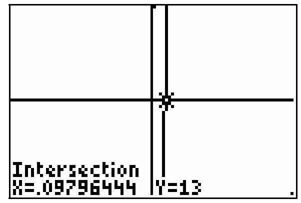


```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-1
 Ymax=15
 Yscl=1
 Xres=1
```

Identification of an appropriate window is a skill that students will need to practice. It seems reasonable to select a window where $12 < y < 14$ since we know that one equation is $y = 13$. Students may need to experiment to determine bounds for *x* for which both functions are visible. Any reasonably small boundaries on *x* that contain 0.98 will work.







It is important for students to understand the nature of the graphs of both a linear function and an exponential function in order to be confident that there is *no additional point of intersection* for this system that lies outside the chosen window.

This graphical method relies on student understanding that the solution of a system of equations in two variables occurs where the graphs of the equations making up the system intersect and that the *x*-value of solution of that system is the solution to the original single-variable equation.

To determine the solution of the system, most graphing calculators have a *calculate* capability that will approximate the intersection of the two graphs. Alternatively, students may *trace* one of the graphs to identify the intersection—if this method is used students may need to zoom in on the intersection so that smaller increments are used when tracing.

Students who will be using a calculator to determine the point of intersection of these two graphs should have repeated experiences finding solutions to system of equations on the model of calculator they will be using for the exam.

While students should generally be encouraged to check solutions by substitution, when the decimal answer is an approximation, substitution is also likely to be only an approximation.

Sample numerical approach:
If a calculator is available that creates a table of values for a given expression, this feature can be used to generate a table that will contain the answer to the solution—graphing calculators do this by entering equations and then looking at the table of values.

For this method, students will need to set the beginning entry of the table and the increment between entries of the table. For this problem, an appropriate starting point is -1 or 0 and an appropriate increment would be 0.001 to produce a solution to three-decimal precision. A portion of the resulting table might look like the following:

x	e^{6x}	1.8
.093	1.7472	1.8
.094	1.7577	1.8
.095	1.7683	1.8
.096	1.7789	1.8
.097	1.7896	1.8
.098	1.8004	1.8

The value in the second column should approximate 1.8 half way between the last two entries. The solution can be estimated to be $x \approx .0975$ which could be rounded if desired.

Item 9

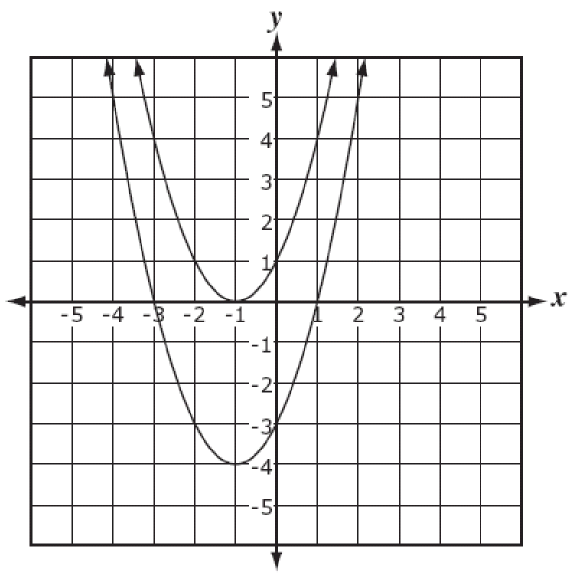
Item Type: Short Answer

Calculator: Permitted

Benchmark: P1.c Describe the effect that changes in the parameters of a quadratic function have on the shape and position of its graph.

Item:

The two quadratic functions graphed in the figure below are vertical translations of each other.



The equation for one of the functions is $y = x^2 + 2x - 3$. Write an equation that will describe the graph of the other function. Explain your reasoning.

Correct Answer: $y = x^2 + 2x + 1$ or equivalent

To earn full credit, student solutions should correctly address all parts of the item, including showing sufficient work to justify the answer and/or providing a complete explanation, where necessary.

Instructional Implications: Students should have practice writing the symbolic representation of a function from its graph. They should experience doing this when two or more graphs are shown on a single coordinate system as well as when only one graph is given. In addition they should be able to compare two graphs presented on the same coordinate system and note how they differ.

Students will also need to be familiar with the manner in which translations of the graph of a function are reflected in its symbolic representation—in particular, for this problem recognizing that a positive vertical translation of 4 units results in adding 4 to the symbolic expression that defines the function.

<u>Sample Solutions</u>:

Identification of the graph of the equation $y = x^2 + 2x - 3$ is the first step in solving this problem. Several methods of identifying the correct graph are possible.

Determining the vertex:

The two function graphs have different vertices. Students can use the technique of *completing the square* to change the given equation into *vertex form* and identify its vertex.

$$\begin{aligned} y &= x^2 + 2x - 3 \\ &= \left(x^2 + 2x + \underline{?}\right) - 3 \\ &= \left(x^2 + 2x + \underline{1}\right) - 3 - 1 \\ &= (x + 1)^2 - 4 \end{aligned}$$

The vertex of the given equation is $(-1, -4)$. This identifies the lower of the two graphs as representing the equation $y = x^2 + 2x - 3$.

Students will need to have practice with the technique of completing the square in order to be confident about the process and the interpretation of the resulting vertex form.

Determining the zeros:

The two function graphs have different zeros. A *zero* of a function is represented graphically as an *x*-intercept, that is, where the *y*-value of the function is 0. Students can solve the quadratic equation $x^2 + 2x - 3 = 0$ by any method—factoring, completing the square, or by using the quadratic formula—in order to identify the zeros of the given equation.

<u>Factoring</u>

$$\begin{aligned} x^2 + 2x - 3 &= 0 \\ (x - 1)(x + 3) &= 0 \\ x &= 1 \text{ or } -3 \end{aligned}$$

<u>Completing the Square</u>

$$\begin{aligned} x^2 + 2x - 3 &= 0 \\ x^2 + 2x &= 3 \\ x^2 + 2x + 1 &= 3 + 1 \\ (x + 1)^2 &= 4 \\ x + 1 &= \pm 2 \\ x + 1 = 2 &\text{ or } x + 1 = -2 \\ x &= 1 \text{ or } -3 \end{aligned}$$

<u>Using the Quadratic Formula</u>

For $x^2 + 2x - 3 = 0$,
$a = 1,\ b = 2,$ and $c = -3$.
Substituting into

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ yields

$$\begin{aligned} x &= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)} \\ &= \frac{-2 \pm \sqrt{4 + 12}}{2} \\ &= \frac{-2 \pm \sqrt{16}}{2} \\ &= \frac{-2 \pm 4}{2} \\ &= \frac{-2 + 4}{2} \text{ or } \frac{-2 - 4}{2} \\ &= 1 \text{ or } -3 \end{aligned}$$

In any case, the graph of the given function crosses the x-axis at $(1,0)$ and $(-3,0)$ identifying its graph as the lower graph.

Once the correct graph for the function $y = x^2 + 2x - 3$ is identified, there are several methods that can be used to determine the equation of the other graph.

Graphical approach:
Because this item permits the use of a calculator, if a graphing calculator is available to the student, he or she could graph the equation $y = x^2 + 2x - 3$ using the calculator and notice that its vertex falls below the x-axis at about $(-1,-4)$.

After the graph of the given function is determined, students should notice that the graph of the second function—the upper graph—is 4 units above the graph of the given function. Since a vertical translation of the graph 4 units up is reflected in the symbolic representation by adding 4 to the original function. The upper function is represented by the symbolic form $y = x^2 + 2x - 3 + 4$ or $y = x^2 + 2x + 1$.

Direct analytic approach:
The vertex of the upper parabola is $(1,0)$. That parabola passes through the point $(0,1)$ so its equation can be obtained by substituting the value of 0 for x and 1 for y in the equation $y = a(x+1)^2$ and solving for a.

$$\begin{aligned} y &= a(x+1)^2 \\ 1 &= a(0+1)^2 \\ 1 &= a(1) \\ a &= 1 \end{aligned}$$

The equation for the upper parabola will be

$$y = (x+1)^2 \text{ or } y = x^2 + 2x + 1.$$

Regardless of the method used to find the equation, students should fully explain their method and/or logic used to reach the final conclusion.

Item 10

Item Type: Short Answer

Calculator: Permitted

Benchmark: X1.d Recognize, express, and solve problems that can be modeled using exponential functions, including those where logarithms provide an efficient method of solution. Interpret their solutions in terms of the context.

Item:

A car has an original value of $20,000. The value decreases at a rate of 18% each year.

Part A Write a function where $f(x)$ represents the value of the car in dollars and x represents years.

Part B After how many years will the car be worth less than $\frac{1}{2}$ of the original value? Show or explain your work.

Correct Answer:

Part A: $f(x) = 20{,}000(0.82)^x$ or equivalent

Part B: The car will be worth less than $\frac{1}{2}$ its original value after approximately 3.493 years.

To earn full credit, student solutions should correctly address all parts of the item, including showing sufficient work to justify the answer and/or providing a complete explanation, where necessary.

Instructional Implications: Students should develop experience recognizing when a situation can be modeled by an exponential function. They also need to be able to identify the symbolic parameters that represent each key element in the problem.

This problem also points out the importance of distinguishing between situations that suggest exponential growth and those that suggest exponential decrease. This understanding needs to translate into the selection of an appropriate growth/decay factor. It may be helpful for students to participate in the development of the model (see Sample Solutions for Part A) in order to understand the need to add/subtract the growth/decay factor from one when creating the exponential function to model a given situation.

Sample Solution:

Part A

A student may simply know that a situation like this, one in which the percent decrease in value is applied to the remaining value year after year, can be modeled by a function of the form $f(x) = ab^x$ where *a* represents the original value, *b* represents the percentage increase or decrease and *x* indicates the number of time units the increase/decrease is applied. In this problem, the percentage decrease is $1 - .18 = .82$. The function

$$f(x) = 20{,}000(0.82)^x$$

represents the value of the car after *x* years.

If the exponential function model is not known, students may use a table to develop the model.

X	$f(x)$	
0	20,000	
1	$20,000 - 20,000(0.18)$	$20,000(1-0.18) = 20,000(0.82)$
2	$20,000(0.82) - 20,000(0.82)(0.18)$	$20,000(0.82)(1-0.18) = 20,000(0.82)^2$
3	$20,000(0.82)^2 - 20,000(0.82)^2(0.18)$	$20,000(0.82)^2(1-0.18) = 20,000(0.82)^3$
4	$20,000(0.82)^3 - 20,000(0.82)^3(0.18)$	$20,000(0.82)^3(1-0.18) = 20,000(0.82)^4$

It appears as if a good model for this situation would be $f(x) = 20,000(0.82)^x$ where x is the number of years after the establishment of the original value.

Part B

Since the original value of the car is $20,000, half its value would be $10,000. Solving the equation

$$10,000 = 20,000(0.82)^x$$

will determine the number of years that it will take for the value of the car to be reduced to $10,000. Each of the following solution methods is an example of a correct strategy.

Using logarithms

$$10,000 = 20,000(0.82)^x$$
$$0.5 = (0.82)^x$$
$$\log(0.5) = \log(0.82)^x$$
$$\log(0.5) = x\log(0.82)$$
$$x = \frac{\log(0.5)}{\log(0.82)}$$
$$\approx 3.492788621$$

Natural logarithms could also be used to solve the equation.

Using a graphing calculator

Simplify the equation to

$$0.5 = (0.82)^x$$

Graph the system of equations

$$y = 0.5$$
$$y = (0.82)^x$$

Use the calculate function or trace to determine the coordinates of the point of intersection of these equations, approximately $(3.493, 0.5)$.

The solution to the original equation is the x-value of this ordered pair.

Use technology to generate a table of values

To determine a starting value students might evaluate $(0.82)^x$ for $x = 1, 2, 3, 4, \ldots$ until a value less than 0.5 is attained. Doing these calculations will identify that the desired x lies between 3 and 4 so a starting value of 3 would be appropriate. To determine the solution to three-decimal precision, the increment should be set at 0.001.

Part of the table is

3.491	0.50018
3.492	0.50008
3.493	0.49998

The solution is closer to 3.493.

Having determined that the solution to the equation is approximately 3.493 years, the answer to the problem is that the value of the car will be *less than* half its original value *after approximately 3.493 years.* An acceptable answer for this problem would be 4 years. It would be best if students would accompany the answer 4 with an explanation indicating that they were assuming that the depreciation occurred on an *annual* basis since rounding up makes sense under this assumption.

ADP Algebra II End-of-Course Exam Notation Information

The information below is meant to inform teachers, schools, districts, and states about the notation **that students will see on the ADP Algebra II End-of-Course Exam.** It is not meant to exclude acceptable notation from being used in the classroom, but to let teachers know what students should expect to see on the exam. We expect students to be exposed to and use multiple forms of correct notation in their classrooms. In addition, students may answer items using any acceptable form of notation on the exam.

Core Test

Piecewise functions: $f(x) = \begin{cases} x - 2, & x \leq 0 \\ x + 2, & x > 0 \end{cases}$

Greatest Integer Function: $f(x) = [x]$

Absolute value functions: $f(x) = -3|x + 2| + 1$

Exponential functions of base e: $f(x) = 2e^{-0.024x}$

Domain restrictions: When a student is asked only to simplify a rational expression, all expressions are assumed to be defined. If a student is identifying an equivalent expression or solving equations, restrictions on the domain will be a part of the item, either in the question or the answer or both.

Composition of functions: $f(g(x))$

Set notation: $\{-1, 0, 4\}$ for solution sets

Negative fractions and rational expressions: $-\frac{2}{3}$ $\quad -\frac{x+1}{x}$

Variables with subscripts involving exponents: ${v_0}^2$ $\quad {R_1}^2$

Monomials involving roots and exponents: $x^2 y \sqrt[4]{z^3}$

(x squared multiplied by y multiplied by the fourth root of z cubed)

Modules

Correlation: r

Combinations/Permutations: ${}_5C_2$ ${}_5P_2$

Matrix elements: a_{32} for the element in row 3 column 2

Vectors: $(0, 1)$ $\mathbf{u}$ $\mathbf{AB}$ $\overrightarrow{\mathbf{AB}}$

Spreadsheet applications: A1 for column A row 1; * for multiplication; / for division; ^ for exponents

ADP Algebra II End-of-Course Exam
Expectations of Geometry Knowledge

The following topics are mathematical concepts with which students entering an Algebra II curriculum should be familiar from prior mathematics courses. The high school curriculum or course sequence that a student might follow that leads them to this exam varies by state, district, and sometimes even school. Some curriculum sequences include an Integrated series or geometry before or after Algebra II. Regardless of the course sequence followed, the mathematical concepts below are typically considered middle school concepts and taught before the Algebra II, or its equivalent, course(s).

Prior geometry knowledge/topics where formulas would not necessarily be provided:

Sum of the interior angles of a triangle equals 180°
Perimeter of polygons
Area of rectangles
Area of triangles (not requiring trigonometry)
Area and circumference of circles
Surface area of right prisms with rectangular or triangular bases
Volume of rectangular prisms
Pythagorean Theorem
Similar figures

Use of π:

When specified that an exact answer is required, answers should be expressed in terms of π. If not specified, answers may be expressed in terms of π, or 3.14 or 22/7 may be used as an approximation for π.

ADP Algebra II
End-of-Course Exam
Constructed-Response
General Guidelines

The following are general guidelines for constructed-response items on the ADP Algebra II End-of-Course Exam. These guidelines are to help students understand what is expected of their responses on these items.

1. RESPONSES

A student's response consists of the answer to the item, as well as any information requested of them. As such, when a student is asked to show or explain their work, justify their answer, explain their reasoning, etc., credit is given for a correct strategy or justification, in addition to credit for a correct answer. It is possible to receive credit for one without the other.

2. CALCULATOR USE

When an item requires students to show or explain a strategy, a student response of "I plugged it into my calculator" or an equivalent response does not earn credit for the strategy. The student must explain how the calculator was used, either by indicating key entries or describing the calculation process to earn credit for the strategy.

3. VERIFICATION STRATEGIES

When an item requires students to show or explain their work and the student uses a "guess and check" method, the student must show more than one trial to earn credit for the strategy.

4. UNITS OF MEASUREMENT

When an item, or part of an item, refers to only one particular unit of measure (e.g. inches, degrees Celsius, etc.) throughout the entire item or part, the response to that item or part does not need to restate the unit in the answer in order to earn credit for the answer. Also, if the question is phrased in terms of a particular unit, the answer will be considered to be in that measurement, unless otherwise indicated by the student.

However, in a case where multiple units of measure are referenced in the item or the unit of measurement would change in the answer (e.g., area: feet to feet squared), the student response must give the appropriate units (or change of units) to earn credit for the answer.

5. FOLLOW GIVEN DIRECTIONS

When an item gives specific instructions to a student about the format of the answer, the student must respond to the item in the format that is asked for to earn credit for the answer. For example:

- round to the correct decimal place, when specified;
- label axes and scales in graphs (including a scale of 1) when required; and
- if an equation is asked for, an equation, not an expression, must be given**.**

ADP Algebra II
End-of-Course Exam
Calculator Policy

The ADP Algebra II End-of-Course Exam is designed to take 90-120 minutes for most students to complete. The test will be administered in two sessions. The first session will last an estimated 45-60 minutes and will be completed by students without the use of a calculator. The second session will also last an estimated 45-60 minutes and will be completed by students using a calculator (preferably a graphing calculator). Some items in the calculator session assume access to a graphing calculator; therefore graphing calculators are recommended although not required. The following policy provides guidance on the use of calculators during the administration of the second session of the Algebra II End-of-Course Exam. The test will be designed so that some but not all questions on the calculator portion of the test require the use of a calculator.

The following types of calculators are permitted for use:

- graphing calculators (recommended)
- scientific calculators (not recommended)
- four-function calculators (not recommended)

The following types of calculators are not permitted for use on the ADP Algebra II End-of-Course Exam:

- **Texas Instruments: All model numbers that begin with TI-89 or TI-92; Voyage 200; N-Spire CAS**
- **Hewlett-Packard: hp 48GII and all model numbers that begin with hp 40G, hp 49G, or hp 50G**
- **Casio: Algebra fx 2.0, ClassPad 300, and all model numbers that begin with CFX-9970G**
- Calculators with built-in computer algebra systems
- Pocket organizers
- Handheld or laptop computers
- Calculators built into cellular phones or other electronic communication devices
- Calculators that have pen input/stylus-driven devices
- Calculators requiring access to an electrical outlet
- Calculators that make noises of any kind that cannot be noise disabled (except for students needing special accommodations)
- Calculators that use a QWERTY keyboard
- Calculators that use paper tape

Proctors are required to disallow the use of any of the above types of calculators. Substitute calculators may be provided in the event that a student's calculator is disallowed and the proctor has approved calculators available.

Students using a calculator with a raised display or a display one inch or larger will be seated at the discretion of the proctor.

Proctors are required to check calculators before the exam. Students should be thoroughly familiar with the operation of the calculator they plan to use on the exam. Calculators may not be shared, and communication between individual student calculators is prohibited during the exam. Students should use their calculator on a regular basis so that they become adept at using the calculator that they plan to use during the test.

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ADP Algebra II End-of-Course Exam Blueprint

Content Standards		Emphasis	Number of Multiple Choice Items	Number of Short Answer Items	Number of Extended Response Items
			1 point each	2 points each	4 points each
Operations and Expressions		**15% of Exam**	6-8	0-3	0-2
	O1. Real numbers				
	O2. Complex numbers				
	O3. Algebraic expressions				
Equations and Inequalities		**20% of Exam**	9-13	0-4	0-2
	E1. Linear equations and inequalities				
	E2. Nonlinear equations and inequalities				
Polynomial and Rational Functions		**30% of Exam**	13-17	1-3	1-2
	P1. Quadratic functions				
	P2. Higher-order polynomial and rational functions				
Exponential functions		**20% of Exam**	8-12	0-4	0-2
	X1. Exponential functions				
Functional Operations and Inverses		**15% of Exam**	5-9	0-4	0-2
	F1. Functional operations and composition				
	F2. Inverse functions				
	F3. Piecewise functions				
Totals per Exam*		**76 points**	**46 Multiple Choice Items**	**7 Short Answer Items**	**4 Extended Response Items**

Additional Notes Regarding the Test:

As described by Webb's Taxonomy:

Cognitive Level 1 items include the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula and will make up 20 - 30% of points on the exam.

Cognitive Level 2 items require students to make some decisions as to how to approach the problem or activity and will make up 40 - 60% of points on the exam.

Cognitive Level 3 items require reasoning, planning, using evidence, and a higher level of thinking (e.g., justifying or analyzing) and will make up 20 - 30% of points on the exam.

Short Answer (2 point) items will make up 14 points on every operational form.

Extended Response (4 point) items will make up 16 points on every operational form.

*Total per exam item counts above do not include embedded field test items that will consist of 6 multiple choice items and 1 or 2 constructed-response items. These items will not count towards a student's score.

Half of the items will appear in the session where a calculator is permitted and half will appear in the session where a calculator is not permitted.