A "Mathematics Background Check"

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Early in my career someone else reported that the best indicator of success in calculus-based physics (CBP) at our school was whether students had taken mathematics in a certain region of New Brunswick. I sat down with a very longtime mathematics teacher and asked him what he thought students should know in mathematics after high school to succeed in college. He quickly gave me five areas that every student should know and pointedly indicated that it was best to give them the questions and watch how they attacked the problems. A solution was not even important; all one had to note was the immediate steps that the student took.

I did not have that kind of time with more than 100 students in the first week of class, so I made up a multiple-choice test.[1] The questions varied, but were always in the same format and of the same type. I used this "Mathematics Background Check" (MBC) for more than 30 years to advise students what their success might be in CBP. For the first 10 years I dealt only with engineering/science majors, but for the next 22 years I had the luxury of allowing students to sign up for algebra-trig or calculus-based physics and used the results of the MBC to admit or suggest an alternative course.

Students who scored *20 or above* (out of 25) were allowed into CBP, *15 and above* into algebra-trig, and all others into a two-semester Conceptual Physics course. The success rate in the CBP was very high.

Meanwhile, here is an idealized conversation of the one that I had with a mathematics teacher of many years, after which I designed the "Mathematics Background Check."

Q. *Suppose a student comes to you seeking immediate advice and you do not have the time to administer a test. What do you do?*

A. I have found that five questions are usually sufficient to yield an idea of the student's mathematical level.

1. Write out a randomly generated quadratic with the coefficient of the squared term not equal to one and with at least one of the coefficients negative. For example:

   $$5x^2 - 3x + 2 = 0.$$

   Then ask the student what the values of x are that satisfy the equation. If he begins by writing "(5x ... " then he is probably not as well-prepared as the student who begins with

   "$x = -(-3) \pm \sqrt{(-3)^2 - 4(5)(2)}$."

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2. Set up two simultaneous equations with coefficients other than one and ask the student to solve for the two unknowns. If she immediately tries substituting by solving for one variable in terms of the other from one of the equations, that is good, but then give her three simultaneous equations. If instead she attempts to multiply each equation through by suitable constants to eliminate one of the variables or she sets up determinants, then she is probably well-prepared.

3. Ask a question that requires the use of Pythagoras' theorem. If the student immediately writes down the correct expression, you can be certain that he is probably well-acquainted with the theorem. It would probably be best to ask for one of the legs of the triangle rather than the hypotenuse.

4. Ask a problem that requires the use of proportionalities. For example, one dealing with similar triangles or of the form "If the surface area of a such-and-such doubles, what happens to the volume?"

5. Lastly, see what the student can do with a problem dealing with binomial expansion. Even checking to see whether he is familiar with the notation will give some insight as to his capabilities.

None of the above questions require that the student actually complete the solution. If you find that you still have time, you might ask some questions that check whether the student tends to be careless. For example:

a) Add two fractions with different denominators.

b) A simple ratio problem. If 2.5 cm represents 6 m/s, what speed is represented by 7.3 cm?

c) Calculate an area or a volume of a fairly common geometric figure.

d) Take the square root of a product that contains constants and variables raised to various powers.

e) Solve a problem like the following where there is a negative sign in front of the parentheses containing an expression with a negative sign in at least one of the terms:

What is the value of t that satisfies the following equation?

$$12t - (4t - 6) = 3t - (9t - 27)$$

f) Solve for the unknown in an inverse proportionality problem when three variables are given. E.g., given that $[(P^*V)/T] = k$, solve for V.

Reference

1. The complete multiple-choice test can be found online at http://ftp.aip.org/cgi-bin/epaps?ID=E-PHTEAH-47-014905. For more information on EPAPS, visit http://www.aip.org/pubservs/epaps.html.

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