

Taxicab Geometry – ID: 8643

By Michele Patrick

Time required
40 minutes

Activity Overview

In this activity, students begin a study of taxicab geometry by discovering the taxicab distance formula. They then use the definition of radius to draw a taxicab circle and make comparisons between a circle in Euclidean geometry and a circle in taxicab geometry. Lastly, they construct taxicab perpendicular bisectors and discover that only certain pairs of points have taxicab perpendicular bisectors.

Concepts

- Taxicab Distances
- Taxicab Circles
- Taxicab Perpendicular Bisectors

Teacher Preparation

This activity is designed to be used in a high-school geometry classroom.

- Taxicab geometry is a non-Euclidean geometry. For the sake of comparisons, students are best off if they know, before starting this activity, how to find the distance between two points, as well as the definitions and characteristics of circles and perpendicular bisectors in Euclidean geometry.
- The screenshots on pages 2–5 demonstrate expected student results. Refer to the screenshots on page 6 for a preview of the student TI-Nspire document (.tns file).
- **To download the student .tns file and student worksheet, go to education.ti.com/exchange and enter “8643” in the quick search box.**

Classroom Management

- This activity is intended to be mainly **teacher-led**, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.
- The student worksheet *GeoAct19_Taxicab_worksheet_EN* helps guide students through the activity and provides a place for students to record their answers.
- The TI-Nspire solution document (.tns file) *GeoAct19_Taxicab_Soln_EN* shows the expected results of working through the activity.

TI-Nspire[™] Applications*Calculator, Graphs & Geometry, Notes*

Begin the activity with a brief introduction to taxicab geometry. Taxicab geometry is a form of non-Euclidean geometry—meaning it has different rules and characteristics than the geometry your students are probably used to studying. There are several types of non-Euclidean geometries. Much as it sounds, taxicab geometry considers what it would be like to drive a taxi through a city in which the streets are set up on a grid of parallel and perpendicular lines. Taxicab drivers can not drive through parks and buildings; therefore, they cannot travel from one intersection to another diagonally. Likewise, no diagonal movements are allowed in taxicab geometry.

Problem 1 – Taxicab distances

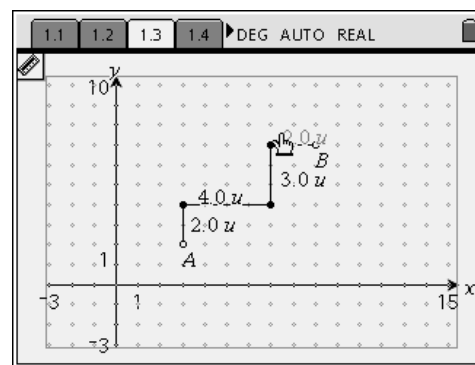
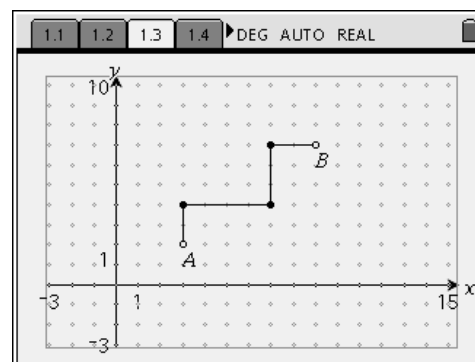
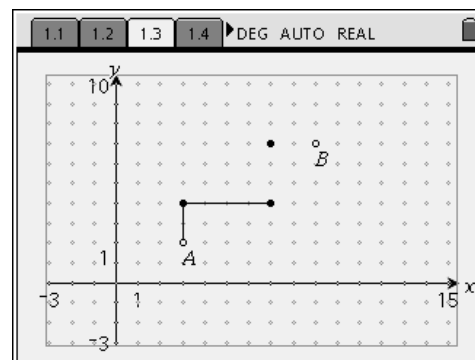
Points A and B are plotted on page 1.3. Tell students to pretend they are taxicab drivers and need to “drive” from point A to point B by drawing segments between the points. Reiterate that no diagonal movements are allowed. Tell them they can make as many, or as few, turns as they wish.

As it is difficult to construct the endpoints of segments **directly** on the grid points when using the handheld, have students make segments by first plotting the endpoints (**MENU > Points & Lines > Point On**), and then drawing the segments connecting the endpoints (**MENU > Points & Lines > Segment**).

Note: If students happen to be working with software instead of handhelds, they can easily make segments whose endpoints are the points on the grid by using just the **Segment** tool.

Once students have traveled from A to B , they should measure the distance they traveled. They can do this by either counting the number of “blocks,” or by using the **Length** tool from the Measurement menu. Students often make errors in counting the distances because they tend to count points instead of the number of spaces between the points.

Determine the greatest and shortest distances traveled within your classroom.



Students should repeat the trip from A to B , but this time choosing a different path. Once done, have them find the distance of this second trip.

Tip: To avoid a cluttered screen, students can first either “undo” every move they have made ($\text{ctrl} + \text{Z}$) or “hide” the segments and lengths of their first trip (**MENU > Tools > Hide/Show**).

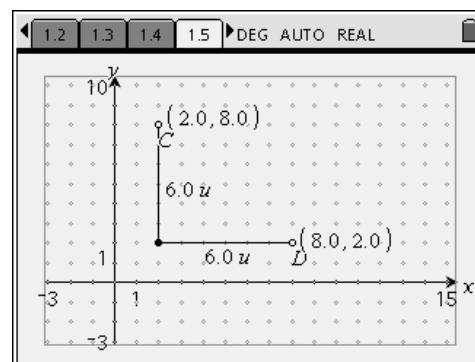
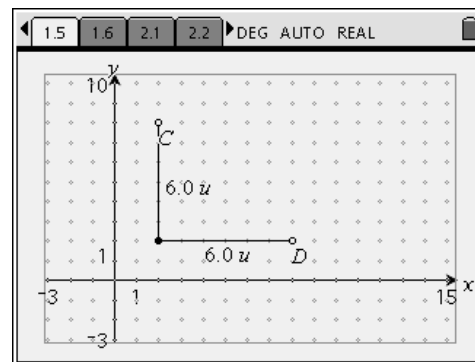
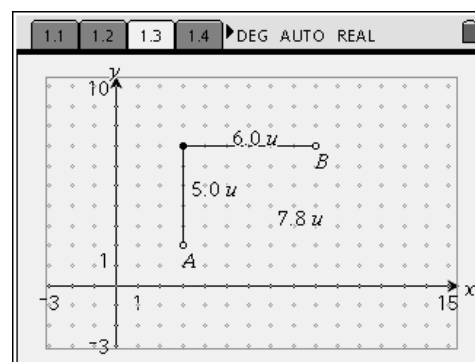
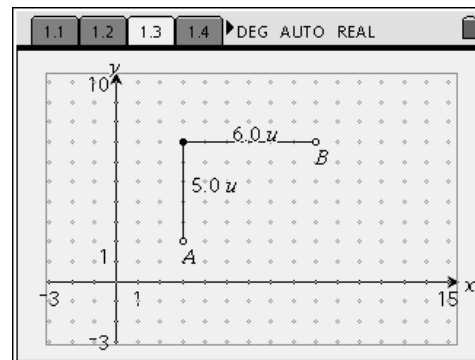
Determine the shortest distance that any student made from A to B . The shortest possible distance is 11. Discuss with students how this can be achieved with just one turn. (See figure at right.)

Ask students how they would normally determine the distance between two points (outside of taxicab geometry). They might discuss the distance formula or the Pythagorean Theorem. For the sake of comparison, have students measure the distance from A to B directly and then answer the first question on their worksheets.

Note: Students can use the **Length** tool without drawing a segment from A to B .

On page 1.5, students will see points C and D . Have them try drawing a path from C to D such that it is the shortest possible path, and that only one turn is made. (There are two possible ways: down then to the right, or to the right and then down.) Use the **Length** tool to find the lengths of the two segments.

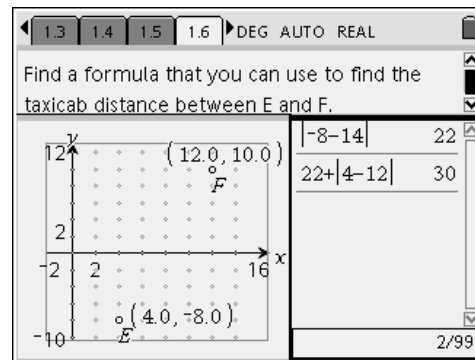
The coordinates of the points should be displayed (**MENU > Tools > Coordinates and Equations**) to allow students to brainstorm a way they could find the lengths of the two segments they drew if given only these coordinates. This is the second question on the worksheet. (Subtract the x -values, take the absolute value, subtract the y -values, take the absolute value, and then add the absolute values.)



On page 1.6, students will first find the coordinates of points E and F . Then they should move to the *Calculator* application and use the coordinates to find the taxicab distance between E and F .

Depending on the ability level of your class, either display the following formula or have students write it themselves, based on what they already know. This is the third question on the worksheet.

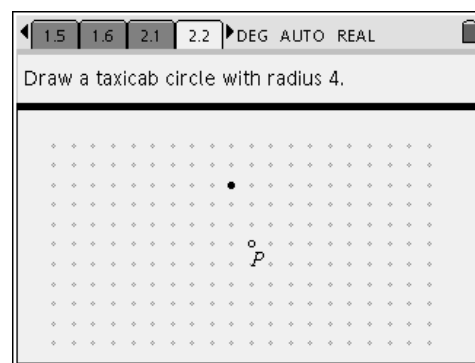
$$\text{taxicab distance} = |y_2 - y_1| + |x_2 - x_1|$$



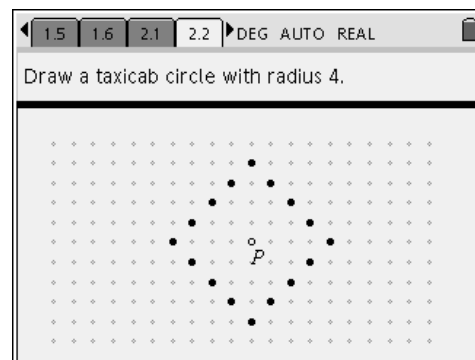
Problem 2 – Taxicab circles

On page 2.2, students create a taxicab “circle” with point P as the center and 4 as the radius. To do this, students will need to plot every point that is a total of 4 units from P . For example, one such point is 3 units up and 1 unit left.

From the Points & Lines menu, remind them to use the **Point On** tool, rather than the **Point** tool.



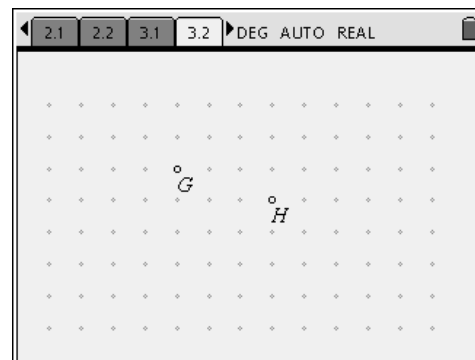
Ask students how many points make up the circle. (16) Lead a class discussion about the differences between a circle in Euclidean geometry and a circle in taxicab geometry. This is the fourth question on the worksheet. (shape, infinite v. finite number of points)



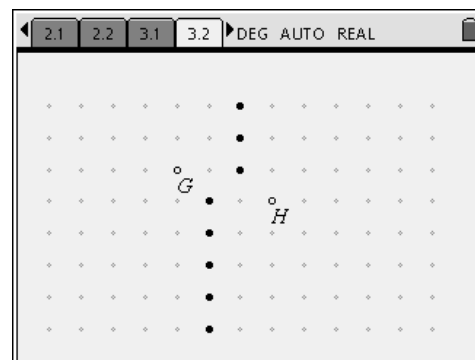
If time permits, you can have students insert a new page by pressing $\text{ctrl} + \text{I}$ and draw one taxicab circle with a radius of 2 units and another with a radius of 3 units to make a conjecture about the relationship between the radius and number of points in the circle. (number of points increase as radius increases; the number of points is equal to 4 times the radius)

Problem 3 – Taxicab perpendicular bisectors

On page 3.2, have students try to plot every point that is equidistant from points G and H .

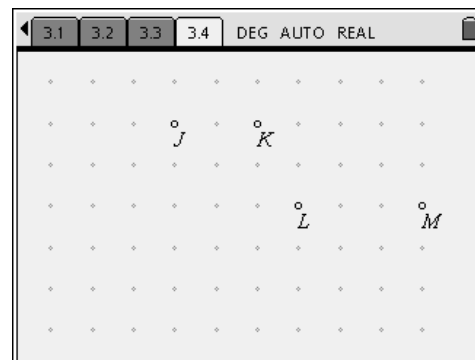


Once they have finished, explain that they have just created a taxicab perpendicular bisector. Discuss similarities and differences between perpendicular bisectors in Euclidean and taxicab geometry. This is the fifth question on the worksheet. (equidistance, infinite v. finite)

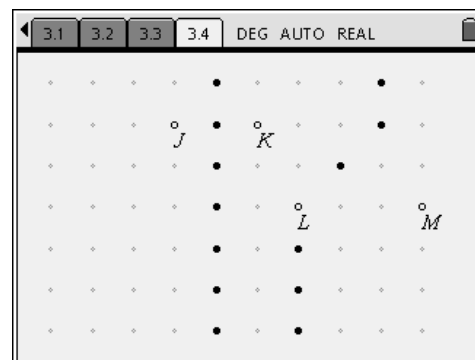


On page 3.4, students try to create perpendicular bisectors for points J and K and then for points K and L .

Students will see that it is impossible to create equidistant points for K and L .



Ask students if they can create perpendicular bisectors for K and M (yes) and J and L (no). See if students can identify the condition required for two points to have a perpendicular bisector. This is the sixth question on the worksheet. (the minimum distance between the two points must be an even number of units)



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(Student)TI-Nspire File: *GeoAct19_Taxicab_EN.tns*

