Chapter 111. Texas Essential Knowledge and Skills for Mathematics

Subchapter C. High School

Statutory Authority: The provisions of this Subchapter C issued under the Texas Education Code, §28.002, unless otherwise noted.

§111.31. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades 9-12.

The provisions of this subchapter shall be implemented beginning with the 2006-2007 school year. This implementation date shall supersede any other implementation dates found in this subchapter.

Source: The provisions of this §111.31 adopted to be effective September 1, 1996, 21 TexReg 7371; amended to be effective August 1, 2006, 30 TexReg 4479.

§111.32. Algebra I (One Credit).

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students will continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra; symbols provide powerful ways to represent mathematical situations and to express generalizations. Students use symbols in a variety of ways to study relationships among quantities.

(3) Function concepts. A function is a fundamental mathematical concept; it expresses a special kind of relationship between two quantities. Students use functions to determine one quantity from another, to represent and model problem situations, and to analyze and interpret relationships.

(4) Relationship between equations and functions. Equations and inequalities arise as a way of asking and answering questions involving functional relationships. Students work in many situations to set up equations and inequalities and use a variety of methods to solve them.

(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(b) Knowledge and skills.

(A.1) **Foundations for functions.** The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(A) describe independent and dependent quantities in functional relationships;

(B) gather and record data and use data sets to determine functional relationships between quantities;

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

(A.2) **Foundations for functions.** The student uses the properties and attributes of functions.

The student is expected to:

(A) identify and sketch the general forms of linear ($y = x$) and quadratic ($y = x^2$) parent functions;

(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

(C) interpret situations in terms of given graphs or creates situations that fit given graphs; and

(D) collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.

(A.3) **Foundations for functions.** The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

(B) look for patterns and represent generalizations algebraically.

(A.4) **Foundations for functions.** The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

(B) use the commutative, associative, and distributive properties to simplify algebraic expressions; and

(C) connect equation notation with function notation, such as $y = x + 1$ and $f(x) = x + 1$.

(A.5) **Linear functions.** The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(A) determine whether or not given situations can be represented by linear functions;

(B) determine the domain and range for linear functions in given situations; and

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

(A.6) **Linear functions.** The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations;

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(C) investigate, describe, and predict the effects of changes in m and b on the graph of $y = mx + b$;

(D) graph and write equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept;

(E) determine the intercepts of the graphs of linear functions and zeros of linear functions from graphs, tables, and algebraic representations;

(F) interpret and predict the effects of changing slope and y-intercept in applied situations; and

(G) relate direct variation to linear functions and solve problems involving proportional change.

(A.7) **Linear functions.** The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;

(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and

(C) interpret and determine the reasonableness of solutions to linear equations and inequalities.

(A.8) **Linear functions.** The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations and formulate systems of linear equations in two unknowns to solve problems;

(B) solve systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) interpret and determine the reasonableness of solutions to systems of linear equations.

(A.9) **Quadratic and other nonlinear functions.** The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(A) determine the domain and range for quadratic functions in given situations;

(B) investigate, describe, and predict the effects of changes in a on the graph of $y = ax^2 + c$;

(C) investigate, describe, and predict the effects of changes in c on the graph of $y = ax^2 + c$; and

(D) analyze graphs of quadratic functions and draw conclusions.

(A.10) **Quadratic and other nonlinear functions.** The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (x-intercepts) of the graph of the function.

(A.11)	**Quadratic and other nonlinear functions.** The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.	The student is expected to: (A) use patterns to generate the laws of exponents and apply them in problem-solving situations; (B) analyze data and represent situations involving inverse variation using concrete models, tables, graphs, or algebraic methods; and (C) analyze data and represent situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods.

Source: The provisions of this §111.32 adopted to be effective September 1, 1996, 21 TexReg 7371; amended to be effective August 1, 2006, 30 TexReg 1931.

§111.33. Algebra II (One-Half to One Credit).

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra; symbols provide powerful ways to represent mathematical situations and to express generalizations. Students study algebraic concepts and the relationships among them to better understand the structure of algebra.

(3) Functions, equations, and their relationship. The study of functions, equations, and their relationship is central to all of mathematics. Students perceive functions and equations as means for analyzing and understanding a broad variety of relationships and as a useful tool for expressing generalizations.

(4) Relationship between algebra and geometry. Equations and functions are algebraic tools that can be used to represent geometric curves and figures; similarly, geometric figures can illustrate algebraic relationships. Students perceive the connections between algebra and geometry and use the tools of one to help solve problems in the other.

(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(b) Knowledge and skills.

(2A.1) **Foundations for functions.** The student uses properties and attributes of functions and applies functions to problem situations.

The student is expected to:

(A) identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations; and

(B) collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.

(2A.2) **Foundations for functions.** The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations; and

(B) use complex numbers to describe the solutions of quadratic equations.

(2A.3) **Foundations for functions.** The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations.

The student is expected to:

(A) analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems;

(B) use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities; and

(C) interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.

(2A.4) **Algebra and geometry.** The student connects algebraic and geometric representations of functions.

The student is expected to:

(A) identify and sketch graphs of parent functions, including linear ($f(x) = x$), quadratic ($f(x) = x^2$), exponential ($f(x) = a^x$), and logarithmic ($f(x) = \log_a x$) functions, absolute value of x ($f(x) = |x|$), square root of x ($f(x) = \sqrt{x}$), and reciprocal of x ($f(x) = 1/x$);

(B) extend parent functions with parameters such as a in $f(x) = a/x$ and describe the effects of the parameter changes on the graph of parent functions; and

(C) describe and analyze the relationship between a function and its inverse.

(2A.5) **Algebra and geometry.** The student knows the relationship between the geometric and algebraic descriptions of conic sections.

The student is expected to:

(A) describe a conic section as the intersection of a plane and a cone;

(B) sketch graphs of conic sections to relate simple parameter changes in the equation to corresponding changes in the graph;

(C) identify symmetries from graphs of conic sections;

(D) identify the conic section from a given equation; and

(E) use the method of completing the square.

(2A.6) **Quadratic and square root functions.** The student understands that quadratic functions can be represented in different ways and translates among their various representations.

The student is expected to:

(A) determine the reasonable domain and range values of quadratic functions, as well as interpret and determine the reasonableness of solutions to quadratic equations and inequalities;

(B) relate representations of quadratic functions, such as algebraic, tabular, graphical, and verbal descriptions; and

(C) determine a quadratic function from its roots or a graph.

(2A.7) **Quadratic and square root functions.** The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations.

The student is expected to:

(A) use characteristics of the quadratic parent function to sketch the related graphs and connect between the $y = ax^2 + bx + c$ and the $y = a(x - h)^2 + k$ symbolic representations of quadratic functions; and

(B) use the parent function to investigate, describe, and predict the effects of changes in a, h, and k on the graphs of $y = a(x - h)^2 + k$ form of a function in applied and purely mathematical situations.

(2A.8) **Quadratic and square root functions.** The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving quadratic functions and formulate quadratic equations or inequalities to solve problems;

(B) analyze and interpret the solutions of quadratic equations using discriminants and solve quadratic equations using the quadratic formula;

(C) compare and translate between algebraic and graphical solutions of quadratic equations; and

(D) solve quadratic equations and inequalities using graphs, tables, and algebraic methods.

(2A.9) **Quadratic and square root functions.** The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) use the parent function to investigate, describe, and predict the effects of parameter changes on the graphs of square root functions and describe limitations on the domains and ranges;

(B) relate representations of square root functions, such as algebraic, tabular, graphical, and verbal descriptions;

(C) determine the reasonable domain and range values of square root functions, as well as interpret and determine the reasonableness of solutions to square root equations and inequalities;

(D) determine solutions of square root equations using graphs, tables, and algebraic methods;

(E) determine solutions of square root inequalities using graphs and tables;

(F) analyze situations modeled by square root functions, formulate equations or inequalities, select a method, and solve problems; and

(G) connect inverses of square root functions with quadratic functions.

(2A.10) **Rational functions.** The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) use quotients of polynomials to describe the graphs of rational functions, predict the effects of parameter changes, describe limitations on the domains and ranges, and examine asymptotic behavior;

(B) analyze various representations of rational functions with respect to problem situations;

(C) determine the reasonable domain and range values of rational functions, as well as interpret and determine the reasonableness of solutions to rational equations and inequalities;

(D) determine the solutions of rational equations using graphs, tables, and algebraic methods;

(E) determine solutions of rational inequalities using graphs and tables;

(F) analyze a situation modeled by a rational function, formulate an equation or inequality composed of a linear or quadratic function, and solve the problem; and

(G) use functions to model and make predictions in problem situations involving direct and inverse variation.

(2A.11) **Exponential and logarithmic functions.** The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) develop the definition of logarithms by exploring and describing the relationship between exponential functions and their inverses;

(B) use the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describe limitations on the domains and ranges, and examine asymptotic behavior;

(C) determine the reasonable domain and range values of exponential and logarithmic functions, as well as interpret and determine the reasonableness of solutions to exponential and logarithmic equations and inequalities;

(D) determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods;

(E) determine solutions of exponential and logarithmic inequalities using graphs and tables; and

(F) analyze a situation modeled by an exponential function, formulate an equation or inequality, and solve the problem.

Source: The provisions of this §111.33 adopted to be effective September 1, 1996, 21 TexReg 7371; amended to be effective August 1, 2006, 30 TexReg 1931.

§111.34. Geometry (One Credit).

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Geometric thinking and spatial reasoning. Spatial reasoning plays a critical role in geometry; geometric figures provide powerful ways to represent mathematical situations and to express generalizations about space and spatial relationships. Students use geometric thinking to understand mathematical concepts and the relationships among them.

(3) Geometric figures and their properties. Geometry consists of the study of geometric figures of zero, one, two, and three dimensions and the relationships among them. Students study properties and relationships having to do with size, shape, location, direction, and orientation of these figures.

(4) The relationship between geometry, other mathematics, and other disciplines. Geometry can be used to model and represent many mathematical and real-world situations. Students perceive the connection between geometry and the real and mathematical worlds and use geometric ideas, relationships, and properties to solve problems.

(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to solve meaningful problems by representing and transforming figures and analyzing relationships.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem solving contexts.

(b) Knowledge and skills.

(G.1) **Geometric structure.** The student understands the structure of, and relationships within, an axiomatic system.

The student is expected to:

(A) develop an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems;

(B) recognize the historical development of geometric systems and know mathematics is developed for a variety of purposes; and

(C) compare and contrast the structures and implications of Euclidean and non-Euclidean geometries.

(G.2) **Geometric structure.** The student analyzes geometric relationships in order to make and verify conjectures.

The student is expected to:

(A) use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships; and

(B) make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

(G.3) **Geometric structure.** The student applies logical reasoning to justify and prove mathematical statements.

The student is expected to:

(A) determine the validity of a conditional statement, its converse, inverse, and contrapositive;

(B) construct and justify statements about geometric figures and their properties;

(C) use logical reasoning to prove statements are true and find counter examples to disprove statements that are false;

(D) use inductive reasoning to formulate a conjecture; and

(E) use deductive reasoning to prove a statement.

(G.4) **Geometric structure.** The student uses a variety of representations to describe geometric relationships and solve problems.

The student is expected to select an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems.

(G.5) **Geometric patterns.** The student uses a variety of representations to describe geometric relationships and solve problems.

The student is expected to:

(A) use numeric and geometric patterns to develop algebraic expressions representing geometric properties;

(B) use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles;

(C) use properties of transformations and their compositions to make connections between mathematics and the real world, such as tessellations; and

(D) identify and apply patterns from right triangles to solve meaningful problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples.

(G.6) **Dimensionality and the geometry of location.** The student analyzes the relationship between three-dimensional geometric figures and related two-dimensional representations and uses these representations to solve problems.

The student is expected to:

(A) describe and draw the intersection of a given plane with various three-dimensional geometric figures;

(B) use nets to represent and construct three-dimensional geometric figures; and

(C) use orthographic and isometric views of three-dimensional geometric figures to represent and construct three-dimensional geometric figures and solve problems.

(G.7) **Dimensionality and the geometry of location.** The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly.

The student is expected to:

(A) use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures;

(B) use slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons; and

(C) derive and use formulas involving length, slope, and midpoint.

(G.8) **Congruence and the geometry of size.** The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations.

The student is expected to:

(A) find areas of regular polygons, circles, and composite figures;

(B) find areas of sectors and arc lengths of circles using proportional reasoning;

(C) derive, extend, and use the Pythagorean Theorem; and

(D) find surface areas and volumes of prisms, pyramids, spheres, cones, cylinders, and composites of these figures in problem situations.

(G.9) **Congruence and the geometry of size.** The student analyzes properties and describes relationships in geometric figures.

The student is expected to:

(A) formulate and test conjectures about the properties of parallel and perpendicular lines based on explorations and concrete models;

(B) formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models;

(C) formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models; and

(D) analyze the characteristics of polyhedra and other three-dimensional figures and their component parts based on explorations and concrete models.

(G.10) **Congruence and the geometry of size.** The student applies the concept of congruence to justify properties of figures and solve problems.

The student is expected to:

(A) use congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane; and

(B) justify and apply triangle congruence relationships.

(G.11) **Similarity and the geometry of shape.** The student applies the concepts of similarity to justify properties of figures and solve problems.

The student is expected to:

(A) use and extend similarity properties and transformations to explore and justify conjectures about geometric figures;

(B) use ratios to solve problems involving similar figures;

(C) develop, apply, and justify triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods; and

(D) describe the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and apply this idea in solving problems.

Source: The provisions of this §111.34 adopted to be effective September 1, 1996, 21 TexReg 7371; amended to be effective August 1, 2006, 30 TexReg 1931.

§111.35. Precalculus (One-Half to One Credit).

(a) General requirements. The provisions of this section shall be implemented beginning September 1, 1998, and at that time shall supersede §75.63(bb) of this title (relating to Mathematics). Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisites: Algebra II, Geometry.

(b) Introduction.

(1) In Precalculus, students continue to build on the K-8, Algebra I, Algebra II, and Geometry foundations as they expand their understanding through other mathematical experiences. Students use symbolic reasoning and analytical methods to represent mathematical situations, to express generalizations, and to study mathematical concepts and the relationships among them. Students use functions, equations, and limits as useful tools for expressing generalizations and as means for analyzing and understanding a broad variety of mathematical relationships. Students also use functions as well as symbolic reasoning to represent and connect ideas in geometry, probability, statistics, trigonometry, and calculus and to model physical situations. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model functions and equations and solve real-life problems.

(2) As students do mathematics, they continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(c) Knowledge and skills.

(P.1) The student defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power (including radical), exponential, logarithmic, trigonometric, and piecewise-defined functions.

The student is expected to:

(A) describe parent functions symbolically and graphically, including $f(x) = x^n$, $f(x) = 1n\ x$, $f(x) = \log_a x$, $f(x) = 1/x$, $f(x) = e^x$, $f(x) = |x|$, $f(x) = a^x$, $f(x) = \sin x$, $f(x) = \arcsin x$, etc.;

(B) determine the domain and range of functions using graphs, tables, and symbols;

(C) describe symmetry of graphs of even and odd functions;

(D) recognize and use connections among significant values of a function (zeros, maximum values, minimum values, etc.), points on the graph of a function, and the symbolic representation of a function; and

(E) investigate the concepts of continuity, end behavior, asymptotes, and limits and connect these characteristics to functions represented graphically and numerically.

(P.2) The student interprets the meaning of the symbolic representations of functions and operations on functions to solve meaningful problems.

The student is expected to:

(A) apply basic transformations, including $a \cdot f(x)$, $f(x) + d$, $f(x - c)$, $f(b \cdot x)$, and compositions with absolute value functions, including $|f(x)|$, and $f(|x|)$, to the parent functions;

(B) perform operations including composition on functions, find inverses, and describe these procedures and results verbally, numerically, symbolically, and graphically; and

(C) investigate identities graphically and verify them symbolically, including logarithmic properties, trigonometric identities, and exponential properties.

(P.3) The student uses functions and their properties, tools and technology, to model and solve meaningful problems.

The student is expected to:

(A) investigate properties of trigonometric and polynomial functions;

(B) use functions such as logarithmic, exponential, trigonometric, polynomial, etc. to model real-life data;

		(C) use regression to determine the appropriateness of a linear function to model real-life data (including using technology to determine the correlation coefficient);
		(D) use properties of functions to analyze and solve problems and make predictions; and
		(E) solve problems from physical situations using trigonometry, including the use of Law of Sines, Law of Cosines, and area formulas and incorporate radian measure where needed.
(P.4)	The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems.	The student is expected to:
		(A) represent patterns using arithmetic and geometric sequences and series;
		(B) use arithmetic, geometric, and other sequences and series to solve real-life problems;
		(C) describe limits of sequences and apply their properties to investigate convergent and divergent series; and
		(D) apply sequences and series to solve problems including sums and binomial expansion.
(P.5)	The student uses conic sections, their properties, and parametric representations, as well as tools and technology, to model physical situations.	The student is expected to:
		(A) use conic sections to model motion, such as the graph of velocity vs. position of a pendulum and motions of planets;
		(B) use properties of conic sections to describe physical phenomena such as the reflective properties of light and sound;
		(C) convert between parametric and rectangular forms of functions and equations to graph them; and
		(D) use parametric functions to simulate problems involving motion.

(P.6)	The student uses vectors to model physical situations.	The student is expected to: (A) use the concept of vectors to model situations defined by magnitude and direction; and (B) analyze and solve vector problems generated by real-life situations.

Source: The provisions of this §111.35 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 1931.

§111.36. Mathematical Models with Applications (One-Half to One Credit).

(a) General requirements. The provisions of this section shall be implemented beginning September 1, 1998. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Algebra I.

(b) Introduction.

(1) In Mathematical Models with Applications, students continue to build on the K-8 and Algebra I foundations as they expand their understanding through other mathematical experiences. Students use algebraic, graphical, and geometric reasoning to recognize patterns and structure, to model information, and to solve problems from various disciplines. Students use mathematical methods to model and solve real-life applied problems involving money, data, chance, patterns, music, design, and science. Students use mathematical models from algebra, geometry, probability, and statistics and connections among these to solve problems from a wide variety of advanced applications in both mathematical and nonmathematical situations. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to link modeling techniques and purely mathematical concepts and to solve applied problems.

(2) As students do mathematics, they continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(c) Knowledge and skills.

(M.1)	The student uses a variety of strategies and approaches to solve both routine and non-routine problems.	The student is expected to: (A) compare and analyze various methods for solving a real-life problem; (B) use multiple approaches (algebraic, graphical, and geometric methods) to solve problems from a variety of disciplines; and (C) select a method to solve a problem, defend the method, and justify the reasonableness of the results.

(M.2) The student uses graphical and numerical techniques to study patterns and analyze data.

The student is expected to:

(A) interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatterplots, line plots, stem and leaf plots, and box and whisker plots to draw conclusions from the data;

(B) analyze numerical data using measures of central tendency, variability, and correlation in order to make inferences;

(C) analyze graphs from journals, newspapers, and other sources to determine the validity of stated arguments; and

(D) use regression methods available through technology to describe various models for data such as linear, quadratic, exponential, etc., select the most appropriate model, and use the model to interpret information.

(M.3) The student develops and implements a plan for collecting and analyzing data in order to make decisions.

The student is expected to:

(A) formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions;

(B) communicate methods used, analyses conducted, and conclusions drawn for a data-analysis project by written report, visual display, oral report, or multi-media presentation; and

(C) determine the appropriateness of a model for making predictions from a given set of data.

(M.4) The student uses probability models to describe everyday situations involving chance.

The student is expected to:

(A) compare theoretical and empirical probability; and

(B) use experiments to determine the reasonableness of a theoretical model such as binomial, geometric, etc.

(M.5)	The student uses functional relationships to solve problems related to personal income.	The student is expected to: (A) use rates, linear functions, and direct variation to solve problems involving personal finance and budgeting, including compensations and deductions; (B) solve problems involving personal taxes; and (C) analyze data to make decisions about banking.
(M.6)	The student uses algebraic formulas, graphs, and amortization models to solve problems involving credit.	The student is expected to: (A) analyze methods of payment available in retail purchasing and compare relative advantages and disadvantages of each option; (B) use amortization models to investigate home financing and compare buying and renting a home; and (C) use amortization models to investigate automobile financing and compare buying and leasing a vehicle.
(M.7)	The student uses algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning.	The student is expected to: (A) analyze types of savings options involving simple and compound interest and compare relative advantages of these options; (B) analyze and compare coverage options and rates in insurance; and (C) investigate and compare investment options including stocks, bonds, annuities, and retirement plans.
(M.8)	The student uses algebraic and geometric models to describe situations and solve problems.	The student is expected to: (A) use geometric models available through technology to model growth and decay in areas such as population, biology, and ecology; (B) use trigonometric ratios and functions available through technology to calculate distances and model periodic motion; and (C) use direct and inverse variation to describe physical laws such as Hook's, Newton's, and Boyle's laws.

(M.9)	The student uses algebraic and geometric models to represent patterns and structures.	The student is expected to: (A) use geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and architecture; and (B) use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music.

Source: The provisions of this §111.36 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 1931.