## Pass the Ball

## Concepts

- Graph scatter plots
- Graph linear functions
- Analyze proportional relations
- Interpret, predict, and analyze data and graphs


## Materials

- TI-84 Plus


#### Abstract

\section*{Overview}

Many things in every day life follow patterns. We can use mathematics to examine the patterns that occur in a specific scenario and then predict future events for that scenario. This activity includes two examples of such patterns. The data collected is the time it takes to complete a task. The data will be used to make predictions about how long it will take to repeat that same task a certain number of times. The timed task is passing a ball. The basic concept has relevance to many areas of math, science, and every day life.


- Stopwatch
- Small bouncing ball (tennis/racquet balls work well)


## DATA COLLECTION

1. For this activity, begin by having four students stand in a circle.

- The starter will bounce, catch, and pass the ball to the next person.
- Continue bouncing, catching, and passing the ball until it is back to the one who started.
- Explain to students the importance of working at a consistent pace rather than working too quickly.

2. Start and stop with the same person.
3. Time how long it takes the ball to get all the way around the circle.

- Have the students record three trials of this process on their worksheet, and calculate the average of those three times (Figure 1).
- Have them record the average in the chart on their worksheet.

4. Complete the table for five different sets of students.

- There should be a different number of students for each set.
- To maximize participation, do not allow the same student to be in more than one group.

| Activity Sample Data |  |
| :---: | :---: |
| Number <br> of People | Average Time <br> in Seconds |
| 4 | 6 |
| 6 | 9 |
| 9 | 16 |
| 7 | 13 |
| 15 | 28 |

Figure 1
5. For the last row in each table, have everyone in the room participate.

- Before timing this trial, have students predict how long they think it will take and write it on their worksheet.
- Ask them to share their predictions.
- Then time the trial for the entire class.

6. Press STAT ENTER to access the Stat List Editor window.

- Have all students enter the data in L1 and L2.
- Do NOT include the data for the trial that included the entire class.
- Have one student enter the data on the overhead calculator (Figure 2).


## Data Analysis

1. Create a scatter plot.

- Press 2nd YY to access the [STAT PLOT] menu.
- 1: Plot 1 will be highlighted.
- Press ENTER.
- Use the arrow keys to highlight the On choice, and press ENTER to select On.
- Continue using the arrow keys to navigate through this screen and adjust it as shown in the screen shot (Figure 3).

2. Press ZOOM, and select 9:ZoomStat by pressing 9 or by scrolling down until it is highlighted and pressing ENTER.

- This feature automatically adjusts the size of the graph window to include all the data points.
- When the scatter plot is displayed, press TRACE, and scroll right and left to see the coordinates of each point displayed below the graph.
- Notice the P1 in the upper left corner.
- This tells you it is tracing the points from Plot1, and displays the lists that are the coordinates of the plot (Figure 4).


Figure 2


Figure 3


Figure 4
3. Have students fit the data with a function rule. There are several ways you can do this depending on your students' understanding of linear functions and features of the calculator. Below are three suggestions. Choose the one most appropriate for your students. Linear Regression and Manual-Fit Features are taught in subsequent lessons.

- Use the Linear Regression feature of the calculator.
- Use Manual-Fit feature of the calculator.
- Use paper and pencil to sketch a line of best fit, and find its equation.

The directions here will be for this last option.
a. It may be helpful to take control of the window settings rather than staying with the 9:ZoomStat setting.

- You can assure yourself of "friendly" numbers (without a lot of places after the decimal point) when you scroll if the spread on the X -values is a multiple of 47 as shown Figure 5.
- Expand the Y-values to include the approximation of the time you expect the entire class will take to pass the ball.
b. Have students sketch their points in the screen provided on their worksheet.
- Next, have them sketch their approximation of the line of best fit (Figure 6).
c. After sketching the line on their worksheet, direct students to find an equation and enter it in Y1 of the $Y=$ window.
- To do this, have them identify two points and use their X - and Y -values to determine the slope of the line (Figure 7).
d. Discuss with your students why the point $(0,0)$ would be a point that should be included with this data.
- How many minutes did it take for no students to pass the ball?
- Add the $(0,0)$ point to your data lists.


Figure 5


Figure 6


Figure 7

- Since it is likely that the points do not lie in a straight line, have students work in groups and have each group choose two points from the lists.
- Have each group choose a different set of points. One possible answer is $(28-6) /(15-4)=2$.
e. Using this slope, have students write the equation of the line in Y1 and compare their lines with each other.
- Have them graph the line to determine if their lines fit the plotted points.

4. Have students use either the trace or table feature to predict how long they would expect it to take the entire class to complete the activity.

- Discuss how close their predictions were to the actual time it took the whole class and possible reasons for any discrepancy.

5. Another option for predicting the time it took the entire class to pass the ball is to have the calculator find the $\mathbf{Y}$-value for a given $\mathbf{X}$-value.

- From the graph screen, press 2nd TRACE to access the [CALC] menu.
- 1:Value is highlighted already, so just press ENTER.
- An $X=$ is displayed in the bottom left-hand corner.
- Enter the value for the total number of students in the class, and press ENTER.
- The corresponding $\mathbf{Y}$-value will be displayed, and a cursor will mark the ordered pair on the graph (Figure 8).
NOTE: The value you enter must be included in the domain that was set in the WINDOW screen.


## Extension Activity

1. Repeat the process, but this time pass the ball without first bouncing it.

- A list of sample data is shown in Figure 9.


Figure 8

| Extension Sample Data |  |
| :---: | :---: |
| Number <br> of People | Average Time <br> in Seconds |
| 5 | 3.98 |
| 7 | 4.91 |
| 9 | 6.42 |
| 11 |  |
| 15 |  |

Figure 9
2. After finding the regression equation and entering it into $\mathbf{Y} 1$, solve a problem that requires a prediction for values that are beyond the values in the recorded data.

- For example, "If it took 50 seconds, how many people were there?"

3. The equation of the line for the sample question would be $50=\mathrm{mX}$. If the slope was found to be 0.787 , the equation becomes $50=0.787 \mathrm{X}$ and X is approximately 64 people.

## Discussion Notes

- Guide your students toward an understanding of the concept that a change in the $\mathbf{X}$-values will cause a change in the $\mathbf{Y}$-values.
- Point out that if they were able to all pass the ball at exactly the same pace, they would be simulating a constant rate of change in the $\mathbf{X}$-values that would bring about a constant and predictable change in the Y -values.
- The fact that the estimation from the line of best fit is not an exact match to the actual time measured for the entire class is easily explained by human inconsistencies.
- The approximation from the line of best fit was not exact, but it should have been close enough to allow students to understand how this type of model building and reasoning could be used to predict future events with some reasonable degree of accuracy.


## WORKSHEET ANSWERS

The answers to the questions on the worksheet will vary depending on the data collected.

## Pass the Ball <br> Student Worksheet

Name

## Class

$\qquad$

## Math Objectives:

- Graph scatter plots
- Graph linear functions
- Analyze proportional relations
- Interpret, predict, and analyze data and graphs

Materials:

- TI-83/TI-84 Plus Family
- Stopwatch
- $\quad$ Small bouncing ball (tennis/racquet balls work well)

| Pass the Ball Data |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> People | Trial 1 | Trial 2 | Trial 3 | Average Time in <br> Seconds |  |
| 4 |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

1. Fill in the table above for the Pass the Ball activity.
2. Estimate how long you think it will take to do this activity with everyone in the room participating. $\qquad$
3. Enter the number of people in the group in $\mathbf{L} 1$ and the time for that group to pass the ball in L2.

- Turn on the [STAT PLOT] for $\mathbf{L 1}$ and $\mathbf{L 2}$ using the $\mathbf{9 : Z o o m S t a t}$ feature from the ZOOM menu to scale your window.
- Make a sketch of these data points in the graph in Figure 1.


Figure 1
4. Draw what you consider to be the line of best fit.
5. Select two points through which your line passes and use them to find the slope of your line.

- Write the equation of your line here $\qquad$ , enter it into the calculator beside $\mathbf{Y} 1=$, and then press GRAPH.
- Examine how closely the line you drew matches the line on your calculator.

6. It may be helpful to take control of the window settings rather than staying with the 9:ZoomStat setting.

- Press WINDOW.
- Adjust the spread on the $\mathbf{X}$-values to be a multiple of 47 as shown here to assure yourself of "friendly" numbers when you scroll (Figure 2).


Figure 2

- Next, adjust the Y-values to include the approximation of the time you expect the entire class will take to pass the ball.

7. Press TRACE, and use the down arrow key to scroll along the line entered in Y1, not the data points.

- Scroll to the right until your $\mathbf{X}$-value matches the total number of students in your class.
- Record your $\mathbf{Y}$-value of that point here and explain what it represents.
$\mathbf{Y}=$ $\qquad$

8. Turn off Plot 1, and clear Y1 before beginning the Extension activity.

## EXTENSION ACTIVITY

1. Fill in the table for the Extension Activity.

| Extension Activity Data |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> People | Trial 1 | Trial 2 | Trial 3 | Average Time in <br> Seconds |  |
| 4 |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

2. Estimate how long you think it will take to do this activity with everyone in the room participating. $\qquad$
3. Enter the number of people in $\mathbf{L} \mathbf{3}$ and the time for passing the ball in $\mathbf{L 4}$.

- Turn on the [STAT PLOT] for $\mathbf{L 3}$ and $\mathbf{L 4}$.
- Use the 9:ZoomStat feature from the ZOOM menu to scale your window.
- Make a sketch of these data points in the graph in Figure 3.



## Figure 3

4. Draw what you consider to be the line of best fit.
5. Select two points through which your line passes and use them to find the slope of your line.

- Write the equation of your line here $\qquad$ , enter it into the calculator beside $\mathbf{Y} \mathbf{1}=$, and then press GRAPH.
- Examine how closely the line you drew matches the line on your calculator.

6. It may be helpful to take control of the window settings rather than staying with the 9:ZoomStat setting.

- Press WINDOW.
- Adjust the spread on the $\mathbf{X}$-values to be a multiple of 47 as shown here to assure yourself of "friendly" numbers when you scroll (Figure 4).


Figure 4

- Next, adjust the $\mathbf{Y}$-values to include the approximation of the time you expect the entire class will take to pass the ball.

7. Press TRACE, and use the down arrow key to scroll along the line entered in Y1, not the data points.

- Scroll to the right until your $\mathbf{X}$-value matches the total number of students in your class.
- Record your $\mathbf{Y}$-value of that point here and explain what it represents.

$$
\mathbf{Y}=
$$

$\qquad$
8. Was your approximation for the time it took the entire class an exact match to the time you recorded from actually doing the activity?

- What could be the cause of any discrepancy? $\qquad$

