Name ______________________________

Date ______________________________

EXPLORATIONS

ACTIVITY 13

Good Vibrations

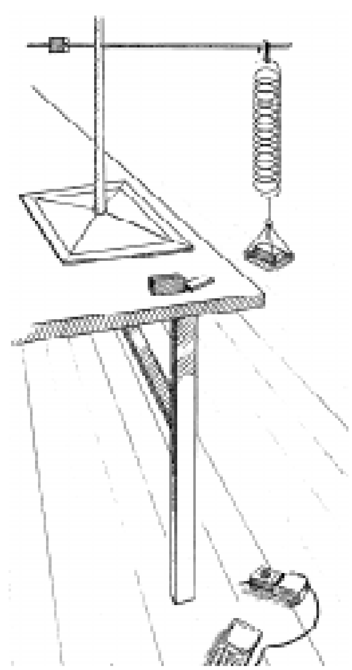
Simple harmonic motion is a periodic motion in which a restoring force is directly proportional to the position and in the opposite direction. The force equation for an object in simple harmonic motion is in the form $F = -k x$, where F is the force, k is a constant that depends on the stiffness of the spring, and x is the position of the object from the at-rest position. The negative means that if an object is stretched in one direction, the restoring force is pulling it back to its natural position, thus the name *restoring force*. A mass on a spring demonstrates simple harmonic motion. As the spring is stretched from its natural position, the spring force pulls it back to its natural position.

One of the basic equations of physics is $F = m a$, where F is the force on an object, m is the mass of the object, and a is the acceleration of the object. Setting the two force equations equal to each other, we have $m a = -k x$. If you solve this equation for a, i.e. you find that $a = -(k/m) x$.

In this activity, you will investigate the motion of a mass moving up and down on a spring. You will collect data for the position, velocity, and acceleration of the mass's motion and examine the relationship between them.

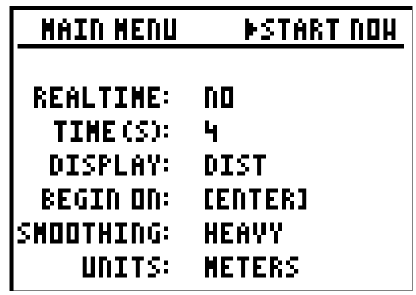
You'll Need

- 1 CBR unit
- 1 TI-83 or TI-82 Graphing Calculator
- Spring with a hanging mass (50 or 100 grams)
- Support stand to suspend the spring and mass



Instructions

1. Set up the support stand and suspend the spring so that it is at least 1 meter from the floor. Test your setup by pulling slightly on the mass so that it will oscillate up and down. Be sure it is secure and that the mass and spring will not fall on the floor. Place the CBR on the floor so that it is aimed at the hanging mass. Test your apparatus by pulling the mass down and allowing it to oscillate. *Be sure that the mass is securely attached so that it will not fall on the CBR, which will be placed underneath the spring and mass.*
2. Run the **RANGER** program on your graphing calculator.
3. Enter the setup instructions.
   a. From the **MAIN MENU** select **1:SETUP/SAMPLE** to access the setup menu.
   b. Press [ENTER] until the **REALTIME** option reads **no**.
   c. Press [▼] (the down arrow) to select the next line **TIME (S)** and press [ENTER] [4] [ENTER] to change the time to **4** seconds.
   d. Press [▼] to select the next line. Correct or verify the settings and press [ENTER]. Repeat until the options for each line read as shown at right.
   e. Press [▼] to move the cursor to the **START NOW** command. Do *not* press [ENTER].



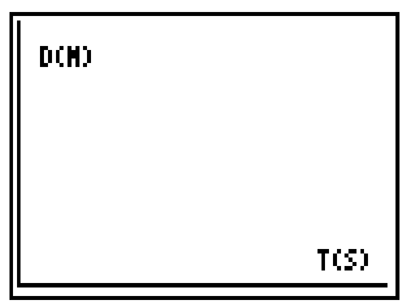
4. Pull the mass from its resting position so that it is oscillating up and down. Press [ENTER] to begin collecting data once the mass is moving. Your data should look like a sine or cosine graph with at least 4 complete cycles of the motion. If you are not happy with your results, press [ENTER] and select **5:REPEAT SAMPLE**. You may need to adjust the time if 4 seconds is too little or too much time for your spring. When you are happy with your plot, press [ENTER] and select **7:QUIT.**

Questions

As you see on the calculator screen, the data has been stored in the calculator as follows:

Time data is stored in **L1**, Distance data is stored in **L2**, Velocity data is stored in **L3**, Acceleration data is stored in **L4**

1. Press [GRAPH] and sketch the spring's Distance-Time graph in the space provided. Label your axes with numbers.



2. The mass oscillates about the position it would have at rest. In physics, this would be called its *equilibrium position*. In mathematical terms, we call this the vertical shift of the cosine graph. The equilibrium position should be the average of the Y values since the mass oscillates about this position. Using the following key strokes, find the mean value for the heights that are stored in **L2**. Press [2nd] **[LIST]** and use the right arrow to access the MATH menu. To find the mean value for the distances, press [3] [2nd] **[L2]** [ENTER]. Record the value below including the units.

Equilibrium position ______________________________

You will now use the following keystrokes to subtract that value from all values in **L2**. Press [STAT] [ENTER]. Move the cursor to **L2** so that it is highlighted. Type **L2 – [Your equilibrium position value]** and press [ENTER]. How does this transform the data? Predict what the new Distance-Time plot will look like. Explain your thinking.

Press [GRAPH]. Since your entire graph may not be visible, press [WINDOW] and adjust the **Ymin** and **Ymax** values so that the entire plot can be seen. Sketch the new graph, labeling your axes.

How does this compare with the prediction that you made above?

3. Since this data is sinusoidal, you will find a cosine equation to describe the motion of the mass on the spring. On your graphing calculator, press [Y=] and move the cursor to **Y1**. Enter the equation **Y = A cos(B(X – C))** and then turn it off by moving the cursor over the equal sign and pressing [ENTER]. You will now find the values of A, B, and C that will produce a model that fits your data.

4. The variable A represents the *amplitude* or the vertical stretch of the sinusoidal curve that models this data set. Since the curve is now centered about the x-axis, the amplitude is equal to the maximum value on the graph. To find this value, press [TRACE]. Use the arrow keys to move the cursor to one of the apparent maximum values of the data set. Store the y-value of this point as A by pressing [ALPHA] **[Y]** [STO▶] [ALPHA] **[A]** [ENTER]. Record the value of A below.

A = ______________________

What is the physical representation of A in terms of the spring's motion?

5. The *period* of the motion is the time interval for one complete vibration. In other words, it is the time between any two adjacent maximums or minimums. Press TRACE. Use the arrow keys to trace to the first maximum of the graph.

> **For the TI-83:** Press X,T,Θ,*n* ENTER to place the x-value of this point on the home screen. Press TRACE and use the arrow keys to trace to the next maximum of the graph. Press X,T,Θ,*n* − 2nd **[ANS]** ENTER.
>
> **For the TI-82:** Press X,T,Θ,*n* ENTER to place the x-value of this point on the home screen. Press TRACE and use the arrow keys to trace to the next maximum of the graph. Press X,T,Θ,*n* − 2nd **[ANS]** ENTER.

This difference in time values represents the period. Record your value for the period below.

Period = ______________________________

6. In your model, B represents the number of cycles that the mass moves over the course of the natural period of the function. The natural period for cosine is 2π.
This means

$$Period = \frac{2\pi}{B} \text{ and therefore, } B = \frac{2\pi}{Period}.$$

Calculate the value of B and store it in your calculator, by entering the value followed by STO▶ ALPHA **[B]** ENTER. Record the value of B below.

B = ______________________________

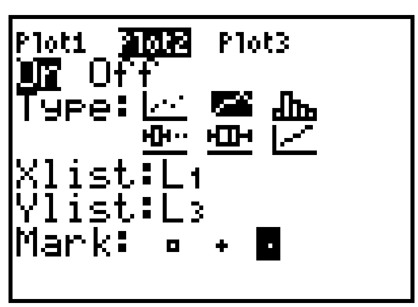
7. The value of C represents the horizontal shift of the data. When no shift is involved ($C = 0$), the graph of cosine begins with a maximum value at time zero. To find the horizontal shift for your data, press TRACE and use the arrow keys to move the cursor to any maximum value. Store the x-value as C by pressing X,T,Θ,*n* (or X,T,Θ,*n* if you are using the TI-82) STO▶ ALPHA **[C]** ENTER. Record the value of C below.

C = ______________________________

8. Press the MODE key and adjust the mode to **Radian**. Press Y= and move the cursor back over the equal sign of your equation. Press ENTER to turn the equation back on. Press GRAPH to view the equation with the data plot. If the equation does not match the data, make adjustments to the values of A, B, or C until your equation does match the data. If you are not sure which variable to change, review the procedures listed above to see how each value, A, B, and C, is obtained from your graph. Record your final equation below.

Y = ______________________________

9. The velocity values for the mass are stored in **L3**. Press 2nd **[STAT PLOT]** 2 to access **Plot2**. Set up the plot as shown.



Press GRAPH to view the Velocity-Time plot along with the Distance-Time plot. If the maximum and minimum values of the velocity are out of range, press WINDOW and adjust the **Ymin** and **Ymax** values so that the entire plot can be seen. Sketch the resulting plot. Be sure to label your axes.

How does the period of the velocity compare with the period of the distance plot?

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Look at the maximum values for the distance. Describe the velocity at these times.

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Explain why this makes sense for the mass oscillating at the end of the spring.

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Describe the position of the mass when it is moving the fastest.

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10. The acceleration values for the mass are stored in **L4.** Press 2nd **[STAT PLOT]** 2 to access **Plot2**. Change the **Ylist** to **L4**. Press GRAPH to view the Acceleration-Time plot along with the Distance-Time plot. If the maximum and minimum values of the acceleration are out of range, press WINDOW and adjust the **Ymin** and **Ymax** values so that the entire plot can be seen. It may be difficult to see both the position and the entire acceleration plot at the same time. Adjust the window so that you can see the essence of both graphs. Sketch the resulting plots. Label your axes.

How does the period of the acceleration compare with the period of the distance plot?

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Look at the maximum values for the distance. Describe the acceleration at these times.

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Describe the position of the mass when its acceleration is greatest.

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11. Rewrite the relationship between acceleration and distance for an object moving in simple harmonic motion that was given at the start of this activity.

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Do your graphs for distance and acceleration verify this relationship? Explain your answer.

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Extension

1. Are there any other values for *C* that will work in the equation for distance? If so, how many are there?

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   How would you find one without going back to the graph for more information? Try it. Test your new value.

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2. What variables above would change if you modeled this data with an equation for sine instead of cosine? Try making the change. Does the equation fit the data?

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3. Use your graphs of distance and acceleration to find the value for (k/m).

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