



Science Objectives

- Students will measure and describe one- and two-dimensional position, displacement, speed, velocity, and acceleration over time.
- Students will graphically calculate the resultant of two or more vectors.
- Students will use the Pythagorean theorem and the tangent function to calculate the resultant of two or more vectors.

Vocabulary

- scalar
- vector
- x- and y-components of a vector
- magnitude of a vector

About the Lesson

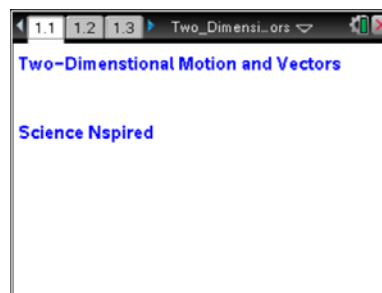
- This lesson visually shows how a vector can be expressed as two component vectors, and an animated demonstration of the addition of two perpendicular vectors.
- As a result, students will:
 - Investigate two-dimensional vectors.
 - Explore the resultant vectors produced when two perpendicular vectors are added.
 - Use these relationships to solve problems involving the motion of a boat crossing a river.

TI-Nspire™ Navigator™

- Send out the *Two_Dimensional_Motion_and_Vectors.tns* file.
- Monitor student progress using Screen Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- *Two_Dimensional_Motion_and_Vectors.tns* document
- TI-Nspire™ Technology
- pen or pencil
- blank sheet of paper



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag point

Tech Tips:

Make sure that students understand how to play ►, pause ||, and reset ◀ a simulation.

Lesson Materials:

Student Activity

- *Two_Dimensional_Motion_and_Vectors.doc*
- *Two_Dimensional_Motion_and_Vectors.pdf*

TI-Nspire document

- *Two_Dimensional_Motion_and_Vectors.tns*



Discussion Points and Possible Answers

TI-Nspire Navigator Opportunities

If you are using TI-Nspire Navigator, students can answer questions throughout the activity in the .tns document. Then, you can retrieve the file to assess understanding.

Before carrying out this activity, review the differences between scalars and vectors. Also review the Pythagorean theorem and the sine and cosine relationships in right triangles.

The screenshots on pages 2–6 demonstrate expected student results. Refer to the screenshots on pages 7–8 for a preview of the student TI-Nspire document (.tns file).

This activity is **teacher-led** with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented **only** in this document, so you should make sure to cover all the material necessary for students to comprehend the concepts.

Students may answer the questions posed in the .tns file, using the Notes application or on blank paper. In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.

The following questions will guide student exploration in this activity:

- What is the relationship between the length of a vector and the lengths of its x - and y -components?
- What is the relationship between the magnitude and angle of a vector and the lengths of its x - and y -components?
- What is the overall motion of an object subjected to forces in two directions?
- How do the components of a velocity vector affect time and distance?

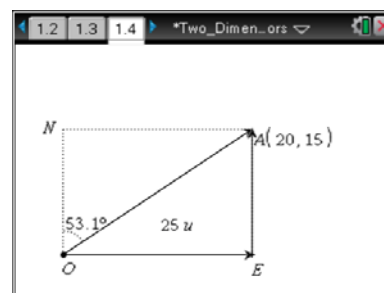
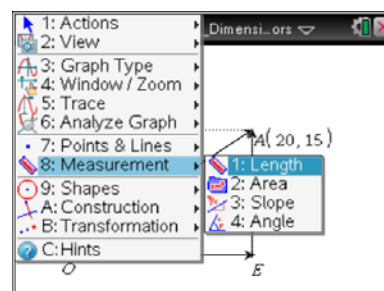
Students will manipulate vectors and observe changes in magnitude, angle, x -component, and y -component. Then they will explore the motion of an object subjected to forces in two directions.



Move to pages 1.2–1.4.

1. Students should open the file **Two_Dimensional_Motion_and_Vectors.tns** and read the first three pages.
2. Page 1.4 shows the displacement vector formed by moving 20 m east and 15 m north. Students should use the **Measurement** tool (**Menu > Measurement > Length**) to determine the length (magnitude) of the vector.

Press **ctrl** **▶** and **ctrl** **◀** to navigate through the lesson.



Move to pages 1.5–1.7.

Have students answer the questions on either the handheld, on the activity sheet, or both.

Q1. What is the length of vector OA in meters?

Answer: 25 m

Q2. What is the length of vector EA in meters?

Answer: 15 m

Q3. What is the length of vector OE in meters?

Answer: 20 m

Q4. Write and solve an equation relating the length of vector OA to the lengths of vectors OE and EA .

Answer:

$$OA^2 = OE^2 + EA^2$$

$$OA = \sqrt{OE^2 + EA^2}$$



Q5. Write an equation that relates vectors \vec{EA} , \vec{OE} , and \vec{OA} .

Answer: $\vec{OA} = \vec{OE} + \vec{EA}$

Q6. Another way to reach point A is to walk 25 m in the direction 53.1° east of north. Use θ to represent the angle between vector \vec{OA} and the vertical. Write equations showing the relationship between θ , the length of vector \vec{OA} , and the lengths of vectors \vec{EA} and \vec{OE} .

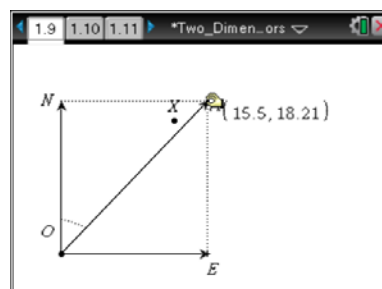
Answer: $\vec{EA} = \vec{OA} \cos \theta$, $\vec{OE} = \vec{OA} \sin \theta$

Teacher Tip: Students may have difficulty seeing these relationships at first. Help them understand the trigonometric relationships by pointing out that the vertical axis (ON) is parallel to vector \vec{EA} , which means that θ is equal to angle OAE .

Move to pages 1.8 and 1.9.

Have students answer the questions on either the handheld, on the activity sheet, or both.

3. Students should read page 1.8 for directions to page 1.9. On page 1.9 students will find a situation similar to page 1.4. Students should manipulate the vector \vec{OA} until point A coincides with point X .



Move to page 1.10.

Have students answer the questions on either the handheld, on the activity sheet, or both.

- Q7. How many meters north and how many meters east would you have to walk to get to point X from point O ?

Answer: I would have to walk 16 m north and 12 m east.

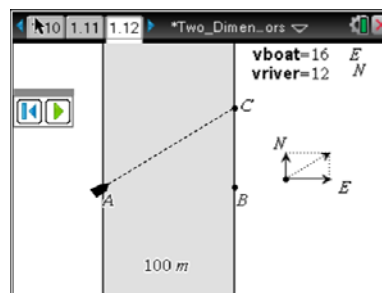
- Q8. Describe the displacement vector of a person who walks directly from point O to point X . Explain how you got your answer.

Answer: The person's displacement vector would be 20 m long and point in a direction 37° east of north. Students can use either of two ways to find the answer: 1. Use the **Angle** and **Length Measurement** tools to measure the length and direction of the vector; or 2. Calculate the length using the Pythagorean theorem and use trigonometric relationships to calculate the angle.



Move to pages 1.11 and 1.12.

4. Students should read page 1.11, which explains the animated simulation on page 1.12 of a boat crossing a river. Students can change the magnitudes of the velocities of the boat and the river by double-clicking on the variables **vboat** and **vriver**. The animation will then change to show the resulting path of the boat.

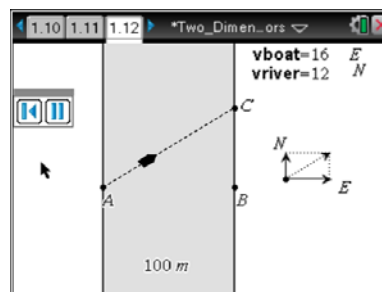


Teaching Tip: The direction that the “boat” is pointing represents the boat's final velocity vector, not its initial vector. This is why the boat does not point directly across the river.

Move to pages 1.13–1.18.

Have students answer the questions on either the handheld, on the activity sheet, or both.

5. Students should return to the animation on page 1.12 as needed as they answer Questions 9–15 on pages 1.13–1.18.



Teaching Tip: Students cannot use the **Measurement** tool to determine lengths and angles in this animation. The dimensions of the animation have been set so that they will give incorrect values if the **Measurement** tool is used. Students must use the Pythagorean theorem and trigonometric relationships to answer questions 9–15.

- Q9. How does changing the speed of the boat affect the boat's overall path?

Answer: As you increase the speed of the boat, point C moves closer to point B. The boat reaches the east bank of the river without going as far north. As you decrease the speed of the boat, point C moves farther from point B. The boat travels farther north before reaching the east bank.

- Q10. How does changing the speed of the river affect the boat's overall path?

Answer: Increasing the speed of the river takes the boat farther north. Decreasing the speed of the river allows the boat to reach the east bank of the river without traveling as far north.



- Q11. If the boat moves due east at 16 m/s and the river flows due north at 12 m/s, what is the speed of the boat relative to an observer at point A? What angle does the boat make with the west bank of the river? Show your work.

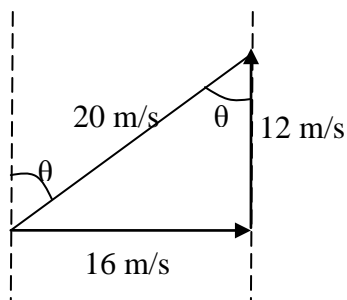
Answer: To calculate the speed of the boat, use the Pythagorean theorem, as shown below:

$$\text{speed} = \sqrt{v_{\text{river}}^2 + v_{\text{boat}}^2} = \sqrt{(12 \text{ m/s})^2 + 16 \text{ (m/s)}^2}$$

$$\text{speed} = \sqrt{144 \text{ m}^2/\text{s}^2 + 256 \text{ m}^2/\text{s}^2}$$

$$\text{speed} = \sqrt{400 \text{ m}^2/\text{s}^2} = 20 \text{ m/s}$$

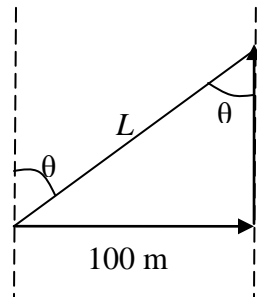
To calculate the angle between the boat's path and the west bank of the river, use the cosine function, as shown below:



$$\theta = \cos^{-1}\left(\frac{12}{20}\right) = \cos^{-1}(0.6) = 53.1^\circ$$

- Q12. If the boat moves due east at 16 m/s and the river flows due north at 12 m/s, how many seconds does it take for the boat to travel across the river? Show your work.

Answer: First, calculate the length of the boat's path using the sine function, as shown below:



$$L = \frac{100 \text{ m}}{\sin \theta} = \frac{100 \text{ m}}{0.8} = 125 \text{ m}$$

From Question 11, we know that the boat is traveling at 20 m/s. So, it will take the boat $125 \text{ m} \div 20 \text{ m/s} = 6.25 \text{ sec}$ to cross the river.



- Q13. If the boat moves due east at 16 m/s and the river flows due north at 6 m/s, how many seconds does it take for the boat to travel across the river? Based on your answer, draw a conclusion about the factors affecting the boat's crossing time.

Answer: Students should approach the problem as above. They will learn that the boat's overall velocity is 17.1 m/s at an angle 69.4° east of north. When they carry out the crossing-time calculation, they will find that the boat requires 6.25 sec to cross the river. From this observation, students should conclude that the northward velocity of the river does not affect the crossing time of the boat. Only the eastward component of the boat's velocity affects its crossing time.

- Q14. If the boat moves due east at 16 m/s and the river flows due north at 6 m/s, how many meters north is point *C* from point *B*? Show your work.

Answer: The distance between point *B* and point *C* is the distance north the boat travels while it is crossing the river. This is equal to the speed of the river multiplied by the time the boat is in the water. Thus, the distance between point *B* and point *C* is $6 \text{ m/s} \times 6.25 \text{ sec} = 37.5 \text{ m}$.

- Q15. If the boat moves due east at 10 m/s and the river flows due north at 6 m/s, how many meters north is point *C* from point *B*?

Answer: First, students must calculate the crossing time. From Question 13, we know that the crossing time depends only on the width of the river and the eastward speed of the boat. Therefore, the crossing time = $100 \text{ m} \div 10 \text{ m/s} = 10 \text{ sec}$. The distance between point *B* and point *C* is therefore $6 \text{ m/s} \times 10 \text{ sec} = 60 \text{ m}$.

TI-Nspire Navigator Opportunities

Use TI-Nspire Navigator to capture screen shots of student progress and to retrieve the file from each student at the end of the class period. The student questions can be electronically graded and added to the student portfolio.



Wrap Up

When students are finished with the activity, pull back the .tns file using TI-Nspire Navigator. Save grades to Portfolio. Discuss activity questions using Slide Show.

Assessment

- Formative assessment will consist of questions embedded in the .tns file. The questions will be graded when the .tns file is retrieved. The Slide Show will be utilized to give students immediate feedback on their assessment.
- Summative assessment will consist of questions/problems on the chapter test.