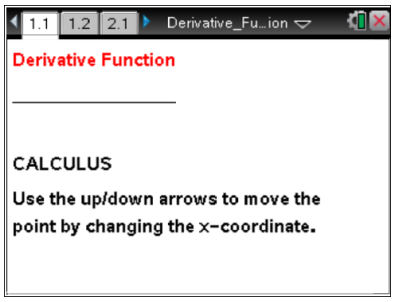
Derivative Function
Student Activity

Name _____
Class _____

Open the TI-Nspace document *Derivative_Function.tns.*



If a function **f** is differentiable at $x = a$, then its graph will appear to become linear as you zoom in on the point $(a, \mathbf{f}(a))$.

The derivative $\mathbf{f}'(a)$ is the slope of the tangent line to the graph of $y = \mathbf{f}(x)$ at the point $(a, \mathbf{f}(a))$.

In this activity, you will define a new function, $\mathbf{f}'(x)$, for the derivative at every value of x.

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Press ctrl ▶ and ctrl ◀ to navigate through the lesson.

1. The graph shown on the left is $y = \mathbf{f}(x) = x^2$ with one point $(a, \mathbf{f}(a))$ boxed in. A magnified "zoomed-in" view of the box is shown on the right with the slope $\mathbf{f}'(a)$ of the tangent line to the graph at that point. In fact, the graph becomes indistinguishable from the tangent line when you zoom in close. Increase or decrease the value of a by using the up/down arrows.
   a. What is $\mathbf{f}'(2)$?

   b. At what value(s) of a is the derivative $\mathbf{f}'(a) = -2$?

   c. Fill out the following table of values for a and $\mathbf{f}'(a)$.

***a* =**	–2	–1.3	–0.5	0	0.7	1.5	2.1
f′(*a*) =							

Move to page 2.1.

2. Grab the white point labeled x on the x-axis and move it to see the slope of the tangent line change as you move along the graph of $y = \mathbf{f}(x) = x^2$.
   a. Describe any pattern you see in the slopes of the tangent lines.

   b. Describe the relationship between each value of x and the slope of the tangent line at $(x, \mathbf{f}(x))$.

Move to page 3.1.

3. If you plot the value of the derivative **f′**(*x*) as the *y*-coordinate for each value *x*, the ordered pairs (*x*, **f′**(*x*)) trace out the graph of a new function *y* = **f′**(*x*), the derivative function. Use the up arrow for *x* in the top window to see the graph of the derivative traced out.
   a. What can you say about the graph of *y* = **f**(*x*) = x^2 when **f′**(*x*) < 0?

   b. What can you say about the graph of *y* = **f**(*x*) = x^2 when **f′**(*x*) > 0?

   c. What can you say about the graph of *y* = **f**(*x*) = x^2 when **f′**(*x*) = 0?

   d. What is the equation of the graph of **f′**(*x*)? What is a general rule that gives a relationship between *x* and **f′**(*x*)? Explain.

Move to page 4.1.

4. The graph shown in the left window is of *y* = **f**(*x*) = sin(*x*) with one point (*a*, **f**(*a*)) boxed in. Again, a magnified "zoomed-in" view of the box is shown on the right along with the slope **f′**(*a*) of the tangent line to the graph at that point. Increase/decrease the value of *a* using the up/down arrows.
   a. What is **f′**(0)?

   b. At what values of *a* (in this window) is the derivative **f′**(*a*) = 0?

   c. Fill out the following table of values for *a* and **f′**(*a*).

a	$-\pi$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
f′(*a*) =								

Move to page 5.1.

5. Grab the white point labeled *x* on the *x*-axis and move it to see the slope of the tangent line change as you move along the graph of *y* = **f**(*x*) = sin(*x*).
   a. What can you say about the slope of the tangent line when the graph of **f**(*x*) = sin(*x*) is decreasing?

   b. What can you say about the slope of the tangent line when the graph of **f**(*x*) = sin(*x*) is increasing?

Move to page 6.1.

6. Use the up arrow for x in the top window to plot the graph of the derivative function $\mathbf{f}'(x)$.
   a. What can you say about the graph of $y = \mathbf{f}(x) = \sin(x)$ when $\mathbf{f}'(x) < 0$?

   b. What can you say about the graph of $y = \mathbf{f}(x) = \sin(x)$ when $\mathbf{f}'(x) > 0$?

   c. What can you say about the graph of $y = \mathbf{f}(x) = \sin(x)$ when $\mathbf{f}'(x) = 0$?

   d. Does the graph $y = \mathbf{f}'(x)$ look familiar? What is the equation of the graph of $\mathbf{f}'(x)$? What is a general rule that gives a relationship between x and $\mathbf{f}'(x)$? Explain.

Move to page 7.1.

7. Increase the value of a using the up/down arrows.
   a. What is $\mathbf{f}'(0)$?

   b. For how many values of a (in this window) is the derivative $\mathbf{f}'(a) = 0$?

Move to page 8.1.

8. Grab the white point labeled x on the x-axis and move it to see the slope of the tangent line change as you move along the graph of $y = \mathbf{f}(x)$.
   a. For approximately what values of a (in this window) is the slope of the graph negative?

   b. For approximately what values of a (in this window) is the slope of the graph positive?

Move to page 9.1.

9. Use the up arrow for x in the top window to plot the graph of the derivative function $\mathbf{f}'(x)$.
   a. What can you say about the graph of $y = \mathbf{f}(x)$ when $\mathbf{f}'(x) < 0$?

   b. What can you say about the graph of $y = \mathbf{f}(x)$ when $\mathbf{f}'(x) > 0$?

   c. What can you say about the graph of $y = \mathbf{f}(x)$ when $\mathbf{f}'(x) = 0$?