

Math Objectives

- Students will determine that a right triangle exists when the sum of the areas of the squares built on the short sides is equal to the area of the square built on the longest side.
- Students will construct viable arguments and critique the reasoning of others (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

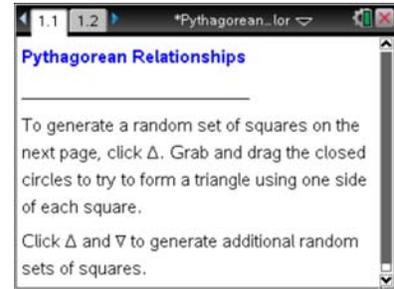
- Pythagorean Theorem
- converse
- acute triangle
- right triangle
- obtuse triangle

About the Lesson

- This activity allows students to experiment with three squares to see if they can make a triangle using one side of each square. They are then asked to classify the triangles and conjecture about the relationships between the areas of the three squares that produced acute, right, and obtuse triangles.
- This activity is a geometric visualization of the Pythagorean relationship: if the sum of the areas of the two small squares is equal to the area of the large square, then the triangle formed by one side of each square will be right.
- Note that there is point in Question 3 where students will need to share data in groups of three or four or as a class lead by the teacher.

TI-Nspire™ Navigator™ System

- Send the .tns file to students.
- Use Screen Capture to examine the side lengths that make a triangle and those lengths that do not make a triangle.
- Use Quick Poll questions to adjust the pace of the lesson according to student understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing **ctrl** **G**.

Lesson Materials:

Student Activity

- Pythagorean_Relationships_Student.pdf
- Pythagorean_Relationships_Student.doc

TI-Nspire document

- Pythagorean_Relationships.tns

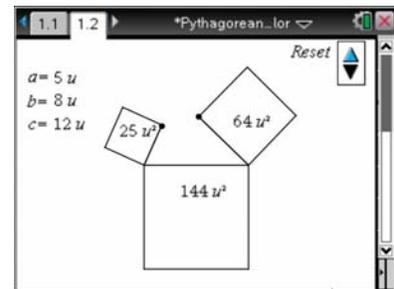
Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging the point, check to make sure that they have moved the arrow until it becomes a hand (☞) getting ready to grab the point, not a hand pointing at the point (☜). Press   to grab the point and close the hand (☞).

Move to page 1.2.

- Click on the up or down arrows on the screen. Describe the numbers and figures that appear and the connections among them.



Answer: The three numbers that are generated are the lengths of the sides of the three squares, and the numbers in the squares are the areas of the squares.

Tech Tip: The up and down arrows are selecting, at random, one of 16 groups of three numbers.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 1 at the end of this lesson.

- Drag the closed points to create a triangle so that one side of each square is a side of the triangle.
 - Fill in the table for Trial 1. Record the side lengths and areas and classify the triangle. Enter the smaller side measures in rows A and B and the largest side measure in row C.

Possible Answer: See the table below.

		Trial 1			Trial 2		
		Side Length	Area	Classify Δ by angles	Side Length	Area	Classify Δ by angles
Small	A	3	9	right	4	16	acute
Medium	B	4	16		6	36	
Large	C	5	25		7	49	



		Trial 3			Trial 4		
		Side Length	Area	Classify \triangle by angles	Side Length	Area	Classify \triangle by angles
Small	A	5	25	obtuse	3	9	no triangle
Medium	B	8	64		5	25	
Large	C	12	144		11	121	

Tech Tip: Note that some of the combinations will not form a triangle. If students question this, tell them to record *no triangle*.

TI-Nspire Navigator Opportunity: Quick Poll
See Note 2 at the end of this lesson.

- b. Click on the up or down arrows to do three more trials with a different set of numbers for each trial, and record the results in the table.

Answer: See table above.

3. Work with the others in your group to organize your results according to the type of triangle by angle.

Acute Triangles			Obtuse Triangles			Right Triangles			No Triangles		
a^2	b^2	c^2	a^2	b^2	c^2	a^2	b^2	c^2	a^2	b^2	c^2
16	16	16	9	25	36	9	16	25	16	36	100
25	25	49	16	16	49	36	64	100	9	25	121
16	36	49	9	36	64	81	144	225	9	9	64
49	81	121	25	64	144	25	144	169	9	49	169

Tech Tip: Some groups may not have all four examples in each category. These represent all the possibilities from the .tns file. It is important that each group have sufficient examples in a category to make a conjecture. The compilation of the data can be done in groups of three or four as a whole class.



Describe the relationship between a^2 , b^2 , and c^2 that seems to be true for each class of triangles.

a. acute

Answer: The sum of the areas of the two smaller squares is greater than the area of the largest square.

b. obtuse

Answer: The sum of the areas of the two smaller squares is less than the area of the largest square.

c. right

Answer: The sum of the areas of the two smaller squares is equal to the area of the largest square.

d. no triangle was formed

Answer: The sum of the areas of the two smaller squares is less than the area of the larger square, and the sum of the side lengths of the two smaller squares is less than the length of the side of the larger square.

Tech Tip: Students may not see the relationships between the linear measures in the *no triangle* case, but for this activity the focus is really on the trials where right triangles are formed. If they do see this relationship, the triangle inequality theorem can be discussed.

4. Brianna has five squares with areas 1 in^2 , 2 in^2 , 3 in^2 , 4 in^2 , and 5 in^2 . Which sets of three squares will fit on the sides of a right triangle? Explain your reasoning.

Answer: 1 in^2 , 2 in^2 , 3 in^2 ; 1 in^2 , 3 in^2 , 4 in^2 ; 1 in^2 , 4 in^2 , 5 in^2 ; 2 in^2 , 3 in^2 , 5 in^2 ; The two smaller areas add to the larger area.

5. If a , b , and c are the lengths of the three sides of a triangle with c the longest side, which of the following will be **never** true, **sometimes** true, and **always** true? Use the work you have done with squares and triangles above to justify your answers.

a. $a^2 + b^2 = c^2$

Answer: Sometimes true; when the triangle is a right triangle, then the sum of the areas of the two smaller squares will equal the area of the largest square.



b. $a + b = c$

Answer: Never true; in this case a triangle cannot be formed.

c. $a + b > c$

Answer: Always true; in this case a triangle can be formed.

d. $c^2 - a^2 = b^2$

Answer: Sometimes true, because this says that the difference between the area of the largest square and the area of one of the others is equal to the area of the third. This is the same as saying the sum of the areas of the two smallest squares is equal to the area of the largest square, which only happens if it is a right triangle.

Tech Tip: Answers to parts b and c will tell you whether students have grasped the triangle inequality. These statements are included because students often miss the area relationship in the Pythagorean Theorem and try to infer a relationship between the linear measures. The statement in part a allows for a discussion about when the Pythagorean relationship holds. Some students may think this true for all triangles. Statement d allows you to see if students recognize an equivalent form of the Pythagorean relationship.



Wrap Up

Upon completion of the discussion, the teacher should ensure that students:

- Can state the relationship between the areas of the three squares that form a right triangle using one side of each square as a side of the triangle.
- Can find the area of one of the three squares that were used to make a right triangle using one side of each square if they know the areas of the other two squares.

TI-Nspire Navigator

Note 1

Question 2, *Live Presenter*

Before beginning Question 2 on their own, use the Teacher Software or Live Presenter to demonstrate to students how to move the point to attempt to make a triangle out of the three squares. Explain to students how they should record the results in the table on their worksheet.

Note 2

Question 2, *Quick Poll*

After students complete the table in Question 2, send an open response Quick Poll asking them to give the side lengths of one triangle they found to be acute. Repeat for obtuse and right triangles. Discuss several of the submissions with the class.