Mathematics (0060)

Test Guide Available
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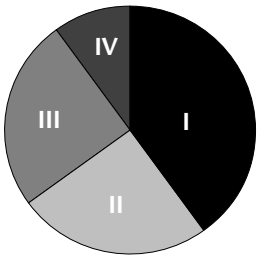
Test at a Glance

Test Name	**Mathematics**
Test Code	**0060**
Time	**2 hours**
Number of Questions	**110**
Format	**Multiple-choice questions, graphing, scientific, or four-function calculators allowed**



	Content Categories	Approximate Number of Questions	Approximate Percentage of Examination
I.	Basic (Precollege) Mathematics	42	38%
II.	Pre-Calculus Mathematics	30	27%
III.	Advanced College-Level Mathematics	27	25%
IV.	Professional Understanding and Pedagogy	11	10%

About this test

The Mathematics test is designed to assess the mathematical knowledge and abilities expected of examinees who intend to teach mathematics at the secondary school level. Examinees have typically completed a bachelor's degree program in mathematics and mathematics education. The test measures the examinees' knowledge and skills, understanding of concepts, and ability to solve problems involving proofs and applications. The 110 multiple-choice test questions cover four content areas: basic mathematics, pre-calculus mathematics, advanced college-level mathematics, and professional understanding and pedagogy.

In addition, the test is designed to measure the following abilities: interpreting symbols and terms; demonstrating skills; applying concepts; extending concepts to unfamiliar situations; justifying statements and constructing proofs; relating essential knowledge and concepts to the learning situation; selecting appropriate teaching methods and principles; and identifying and interpreting trends and curriculum development in the light of the history of mathematics education.

Graphing calculators without QWERTY keyboards are allowed on this test, although no question on the test will require the use of a graphing calculator.

Topics Covered

Representative descriptions of topics covered in each category are provided below.

I. Basic Mathematics

- Number concepts and elementary number theory topics include measurement in both the English and metric systems, area and volume, estimation, factors and multiples, ratio and proportion, percent
- Elementary and intermediate algebra topics include the fundamental operations with monomials and polynomials; algebraic fractions; linear, quadratic, and higher degree equations and inequalities; systems of equations and inequalities; radicals and exponents; translation; arithmetic and geometric progressions
- High school geometry topics include visualization in 2-space and 3-space; reflections, rotations, and translations; graphing lines and planes that are parallel or perpendicular to a given line or plane

II. Pre-Calculus Mathematics

- College algebra, trigonometry, and analytic geometry: absolute value, fractional and negative exponents, scientific notation, quadratic inequalities, irrational numbers, complex numbers, logarithms, recognition and application of trigonometric identities
- Functions: algebraic, trigonometric, logarithmic, exponential, and absolute value functions; domain and range of functions; composite and inverse functions; one-to-one mappings; graphical properties of functions; functions that are defined recursively

III. Advanced College-Level Mathematics

- Calculus: properties of real numbers; limits; maxima and minima; least upper bound property; points of inflection; asymptotes; polar coordinates; evaluation and application of derivatives and integrals; continuity; mappings into or onto a set; convergence of series. Only single variable calculus is covered.
- Linear and abstract algebra: the properties of groups, rings, and fields; matrices and determinants; vectors and vector spaces; linear transformations
- Computer science and discrete mathematics: symbolic logic; simple computer programs; congruence for integers; more advanced topics in number theory
- Probability and statistics: permutations and combinations; finite and continuous probability; conditional probability; mean, median, and mode as measures of central tendency; range; standard deviation; simple distributions, particularly the normal distribution

IV. Professional Understanding and Pedagogy

- Important trends in mathematics education
- Important developments in the history of mathematics
- Knowledge of professional journals and resources
- Pedagogical issues, such as teaching methods, curriculum, and analysis of student errors

The sample questions that follow illustrate the kinds of questions in the test. They are not, however, representative of the entire scope of the test in either content or difficulty. Answers with explanations follow the questions.

Directions: Each of the questions or statements below is followed by five suggested answers or completions. Select the one that is best in each case.

Basic (Precollege) Mathematics

1. If x, $x + 1$, and $x + 2$ are consecutive positive integers, which of the following must be true?

   I. The average of the three integers is divisible by 2.
   II. The sum of the three integers is divisible by the middle integer.
   III. $(x + 1)(x + 2) - x(x + 1)$ is divisible by 2.

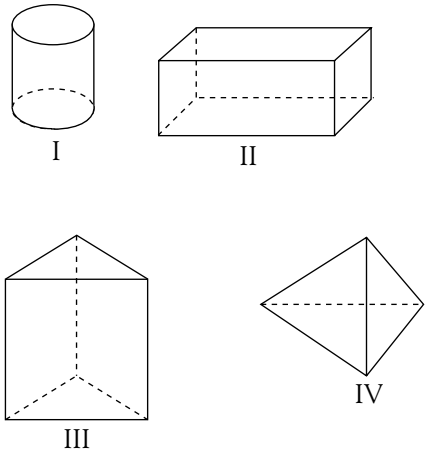
   (A) I only
   (B) II only
   (C) III only
   (D) II and III only
   (E) I, II, and III

2. If a student states that

$$-\frac{1}{2}\left(-\frac{2}{3}x + \frac{1}{2}\right) = \frac{1}{3}x + \frac{1}{2},$$

   it is most likely that the mistake results from a misunderstanding of which of the following?

   (A) Multiplication of fractions
   (B) Arithmetic of negative numbers
   (C) Commutative property of multiplication
   (D) Distributive property
   (E) Equivalence relation



3. The volume(s) of which of the solids shown above can be found by applying the formula $V = Bh$, where V represents the volume, B represents the area of a base, and h represents the altitude to the base that has area B?

   (A) II only
   (B) I and II only
   (C) I and III only
   (D) I, II, and III only
   (E) I, II, III, and IV

Pre-Calculus Mathematics

4. Each of the following defines a function such that the interval [0, 1] is mapped into the interval [0, 1] EXCEPT

   (A) $f(x) = \sqrt{x}$
   (B) $f(x) = |x|$
   (C) $f(x) = \frac{1}{x}$
   (D) $f(x) = x^2$
   (E) $f(x) = x^3$

5. From a point 1,000 meters from the base of an airport tower, the angle of elevation to the top of the tower is 15 degrees. If h is the height, in meters, of the tower, then $h =$

   (A) $1{,}000 \tan 15°$

   (B) $\dfrac{1{,}000}{\tan 15°}$

   (C) $1{,}000 \sin 15°$

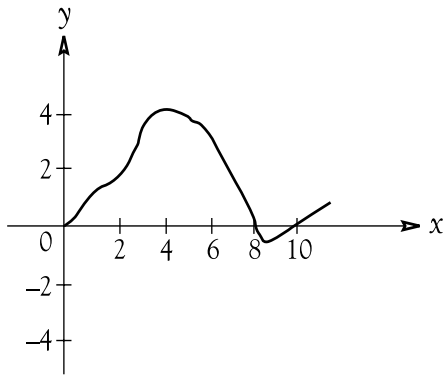
   (D) $\dfrac{1{,}000}{\sin 15°}$

   (E) $1{,}000 \cos 15°$

Advanced College-Level Mathematics

6. Let G be a group with operation $*$. If a, b, and c are elements in G, then which of the following statements is NOT necessarily true?

   (A) $a * b$ is in G.
   (B) $a * (b * c) = (a * b) * c$
   (C) $a * b = b * a$
   (D) There exists an element e in G such that $a * e = e * a = a$ for each a in G.
   (E) For each a in G there exists an element a^{-1} in G such that $a * a^{-1} = a^{-1} * a = e$, the identity element.



7. For all x in the interval $[0, 10]$, F is the function defined by $F(x) = \int_0^x f$, where f is the function that has the graph shown above. At what value of x does F attain its maximum value?

   (A) 0
   (B) 4
   (C) 6
   (D) 8
   (E) 10

8. The scores of 500 students on an examination constitute a random sample from a normal distribution that has a mean of 70 and a standard deviation of 10. Letter grades were assigned as follows:

   Above 86: A
   81-86: B
   60-80: C
   53-59: D
   Below 53: F

   If the distribution of the sample mirrors the properties of the normal distribution, approximately how many students received a grade higher than C?

   (A) 50
   (B) 80
   (C) 100
   (D) 120
   (E) 160

Go on to the next page.

Professional Understanding and Pedagogy

9. Which of the following sources would be most likely to have some suggestions on classroom-tested techniques for motivating students in a high school geometry class?

   (A) *Mathematical Reviews*
   (B) *Mathematics Teacher*
   (C) *The American Mathematical Monthly*
   (D) *Arithmetic Teacher*
   (E) *Transactions of the American Mathematical Society*

10. Of the following mathematicians, which one is credited with the creation of coordinate geometry?

   (A) Leonhard Euler
   (B) Leonardo Fibonacci
   (C) Evariste Galois
   (D) René Descartes
   (E) Carl Gauss

Answers

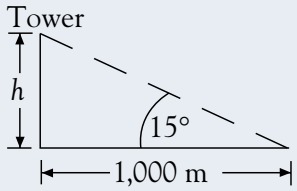
1. In statement I, the average of the three positive integers is $\frac{x + (x + 1) + (x + 2)}{3}$ or $\frac{3x + 3}{3} = x + 1$, which is not necessarily divisible by 2. Therefore, statement I is not always true. In statement II, the sum of the three positive integers is $3x + 3$, which is a product of 3 and the middle integer, $x + 1$. Therefore, II must be true. In statement III, $(x + 1)(x + 2) - x(x + 1) = (x + 1)(x + 2 - x) = 2(x + 1)$, which is divisible by 2. Therefore, statements II and III must be true. The correct answer is D.

2. Since $\left(-\frac{1}{2}\right)\left(-\frac{2}{3}x\right) = \frac{1}{3}x$, the student has demonstrated the understanding of multiplication of fractions and the arithmetic of negative numbers. Therefore, the answer cannot be A or B; choices C and E are not involved in the example, so no conclusions can be drawn about these. However, since the student did not multiply the second term of the sum by $-\frac{1}{2}$, but apparently thinks that $-\frac{1}{2}\left(-\frac{2}{3}x + \frac{1}{2}\right) = \left(-\frac{1}{2}\right)\left(-\frac{2}{3}x\right) + \frac{1}{2}$, it is most likely that the student misunderstands the distributive property. The best answer is D.

3. The volume of a solid can be found by multiplying the area of its base, B, by its height, h, provided that (1) the solid has at least two congruent faces that lie in parallel planes, and (2) every cross section that is parallel to the planes of the congruent faces will be congruent to these faces. All prisms and cylinders have these two properties. Thus, the figures given that have these properties are I, II, and III. The correct answer is D.

4. Showing that a function f does not map [0, 1] into [0, 1] is the same as showing that for some x in [0, 1], $f(x)$ is not in [0, 1]; that is, for some x in [0, 1], either $f(x) < 0$ or $f(x) > 1$. For all x in [0, 1], $\sqrt{x}$, $|x|$, x^2, and x^3 are in [0, 1]. Thus, none of the choices A, B, D, and E is the correct answer. With respect to choice C, $f(0)$ is not defined and $f(x) > 1$ for every x in (0, 1). Therefore, $f(x) = \frac{1}{x}$ does *not* map [0, 1] into [0, 1], and C is the correct answer.

5. The following figure represents the information given in the problem.

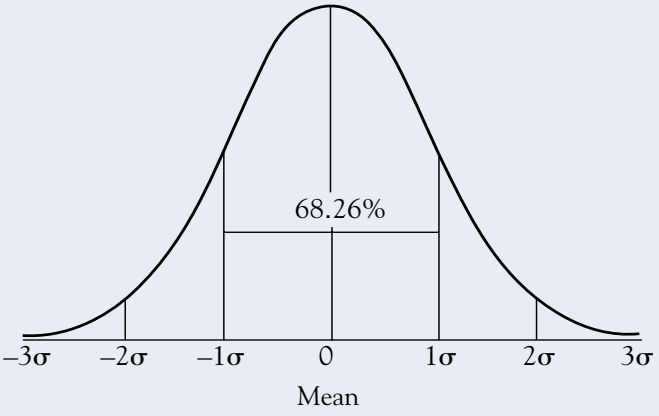


Now $\tan 15° = \frac{h}{1{,}000}$ and $h = 1{,}000 \tan 15°$. The correct answer is A.

6. A nonempty set of elements G together with operation * is said to form a group if those properties exemplified in choices A, B, D, and E are satisfied. These properties include closure, associativity, identity element, and an inverse for every element in G. With respect to choice C, a group need not be commutative. Since choice C is not necessarily true, the correct answer is C.

7. If a function $f(x)$ is non-negative on the interval $[a,b]$, then $\int_a^b f(x)\,dx$ represents the area of the region bounded by $y = f(x)$, the x-axis, and the lines $x = a$ and $x = b$. On the other hand, if $f(x)$ is negative on the interval $[a,b]$, then $\int_a^b f(x)\,dx$ represents the negative of the area of the region bounded by $y = f(x)$, the x-axis, and the lines $x = a$ and $x = b$. Therefore, the function $F(x)$ increases when $f(x) > 0$ and decreases when $f(x) < 0$. Since $f(x) > 0$ when $x < 8$ and $f(x) < 0$ when $x > 8$, it follows that $F(x)$ attains its maximum value when $x = 8$. The correct answer is D.

8. The figure below is a graph of a normal distribution with mean 0 and standard deviation σ. Note that the vertical line at the mean (and the median) splits the distribution in half. Furthermore, 34 percent of the cases (letter grades) fall between the mean (70, in this problem) and one standard deviation above the mean (70 + 10 = 80). The number of students who received a grade higher than C is precisely the number who scored more than one standard deviation above the mean (that is, more than 80). This proportion of the scores is 50% − 34% = 16%, and 16% of 500 students is 80. The best answer is B.



Standard Normal Distribution

9. Both *Mathematics Teacher* and *Arithmetic Teacher* are journals published by the National Council of Teachers of Mathematics and have suggestions about techniques for motivating and teaching students in mathematics. However, *Arithmetic Teacher* focuses on elementary school mathematics, and *Mathematics Teacher* focuses on secondary school mathematics. The other three journals listed are of greatest interest to university-level mathematics teachers and to nonteaching mathematicians. Therefore, *Mathematics Teacher* would be most likely to provide helpful suggestions for a high school geometry class. The best answer is B.

10. The rectangular coordinate system is often referred to as the Cartesian coordinate system in honor of René Descartes, who is credited with its creation. The correct answer is D.