The College Board

AP® ADVANCED PLACEMENT PROGRAM®

Course Description

CALCULUS

Calculus AB, Calculus BC

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MAY 2004, MAY 2005

The College Board is a national nonprofit membership association whose mission is to prepare, inspire, and connect students to college and opportunity. Founded in 1900, the association is composed of more than 4,300 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT®, and the Advanced Placement Program® (AP®). The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.

For further information, visit www.collegeboard.com

The College Board and the Advanced Placement Program encourage teachers, AP Coordinators, and school administrators to make equitable access a guiding principle for their AP programs. The College Board is committed to the principle that all students deserve an opportunity to participate in rigorous and academically challenging courses and programs. All students who are willing to accept the challenge of a rigorous academic curriculum should be considered for admission to AP courses. The Board encourages the elimination of barriers that restrict access to AP courses for students from ethnic, racial, and socioeconomic groups that have been traditionally underrepresented in the AP Program. Schools should make every effort to ensure that their AP classes reflect the diversity of their student population.

For more information about equity and access in principle and practice, contact the National Office in New York.

For the College Board's online home for AP professionals, visit AP Central at apcentral.collegeboard.com.

Dear Colleagues:

In 2002, more than one million high school students benefited from the opportunity of participating in AP® courses, and nearly 940,000 of them then took the challenging AP Exams. These students felt the power of learning come alive in the classroom, and many earned college credit and placement while still in high school. Behind these students were talented, hardworking teachers who collectively are the heart and soul of the AP Program.

The College Board is committed to supporting the work of AP teachers. This AP Course Description outlines the content and goals of the course, while still allowing teachers the flexibility to develop their own lesson plans and syllabi, and to bring their individual creativity to the AP classroom. To support teacher efforts, a Teacher's Guide is available for each AP subject. Moreover, AP workshops and Summer Institutes held around the globe provide stimulating professional development for more than 60,000 teachers each year. The College Board Fellows stipends provide funds to support many teachers' attendance at these Institutes. Stipends are now also available to middle school and high school teachers using Pre-AP® strategies.

Teachers and administrators can also visit AP Central™, the College Board's online home for AP professionals at apcentral.collegeboard.com. Here, teachers have access to a growing set of resources, information, and tools, from textbook reviews and lesson plans to electronic discussion groups (EDGs) and the most up-to-date exam information. I invite all teachers, particularly those who are new to AP, to take advantage of these resources.

As we look to the future, the College Board's goal is to broaden access to AP while maintaining high academic standards. Reaching this goal will require a lot of hard work. We encourage you to connect students to college and opportunity by not only providing them with the challenges and rewards of rigorous academic programs like AP, but also by preparing them in the years leading up to AP.

Sincerely,

Gaston Caperton
President
The College Board

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Welcome to the AP® Program

The Advanced Placement Program® (AP®) is a collaborative effort between motivated students, dedicated teachers, and committed high schools, colleges, and universities. Since its inception in 1955, the Program has allowed millions of students to take college-level courses and exams, and to earn college credit or placement while still in high school.

Most colleges and universities in the U.S., as well as colleges and universities in 21 other countries, have an AP policy granting incoming students credit, placement, or both on the basis of their AP Exam grades. Many of these institutions grant up to a full year of college credit (sophomore standing) to students who earn a sufficient number of qualifying AP grades.

Each year, an increasing number of parents, students, teachers, high schools, and colleges and universities turn to AP as a model of educational excellence.

More information about the AP Program is available at the back of this Course Description and at AP Central™, the College Board's online home for AP professionals (apcentral.collegeboard.com). Students can find more information at the AP student site (www.collegeboard.com/apstudents).

AP Courses

Thirty-four AP courses in a wide variety of subject areas are currently available. Developed by a committee of college faculty and AP teachers, each AP course covers the breadth of information, skills, and assignments found in the corresponding college course. See page 2 for a list of the AP courses and exams that are currently offered.

AP Exams

Each AP course has a corresponding exam that participating schools worldwide administer in May. Except for Studio Art, which is a portfolio assessment, AP Exams contain multiple-choice questions and a free-response section (either essay or problem-solving).

AP Exams represent the culmination of AP courses, and are thus an integral part of the Program. As a result, many schools foster the expectation that students who enroll in an AP course will go on to take the corresponding AP Exam. Because the College Board is committed to providing homeschooled students and students whose schools do not offer AP access to the AP Exams, it does not require students to take an AP course prior to taking an AP Exam.

AP Courses and Exams

Art
Art History
Studio Art (Drawing Portfolio)
Studio Art (2-D Design Portfolio)
Studio Art (3-D Design Portfolio)

Biology

Calculus
Calculus AB
Calculus BC

Chemistry

Computer Science
Computer Science A
Computer Science AB

Economics
Macroeconomics
Microeconomics

English
English Language and Composition
English Literature and Composition

Environmental Science

French
French Language
French Literature

German Language

Government and Politics
Comparative Government and Politics
United States Government and Politics

History
European History
United States History
World History

Human Geography

Latin
Latin Literature
Latin: Vergil

Music Theory

Physics
Physics B
Physics C: Electricity and Magnetism
Physics C: Mechanics

Psychology

Spanish
Spanish Language
Spanish Literature

Statistics

Introduction to AP Calculus

Shaded text indicates important new information about this subject.

An AP course in calculus consists of a full high school academic year of work that is comparable to calculus courses in colleges and universities. It is expected that students who take an AP course in calculus will seek college credit, college placement, or both, from institutions of higher learning.

The AP Program includes specifications for two calculus courses and the examination for each course. The two courses and the two corresponding examinations are designated as Calculus AB and Calculus BC.

Calculus AB can be offered as an AP course by any school that can organize a curriculum for students with mathematical ability. This curriculum should include all the prerequisites for a year's course in calculus listed on page 6. Calculus AB is designed to be taught over a full high school academic year. It is possible to spend some time on elementary functions and still cover the Calculus AB curriculum within a year. However, if students are to be adequately prepared for the Calculus AB examination, most of the year must be devoted to the topics in differential and integral calculus described on pages 8 to 11. These topics are the focus of the AP Examination questions.

Calculus BC can be offered by schools that are able to complete all the prerequisites listed on page 6 before the course. Calculus BC is a full-year course in the calculus of functions of a single variable. It includes all topics covered in Calculus AB plus additional topics, but both courses are intended to be challenging and demanding; they require a similar depth of understanding of common topics. The topics for Calculus BC are described on pages 12 to 16. A Calculus AB subscore grade is reported based on performance on the portion of the exam devoted to Calculus AB topics.

Both courses described here represent college-level mathematics for which most colleges grant advanced placement and/or credit. Most colleges and universities offer a sequence of several courses in calculus, and entering students are placed within this sequence according to the extent of their preparation, as measured by the results of an AP Examination or other criteria. Appropriate credit and placement are granted by each institution in accordance with local policies. The content of Calculus BC is designed to qualify the student for placement and credit in a course that is one course beyond that granted for Calculus AB. Many colleges provide statements regarding their AP policies in their catalogs and on their Web sites.

Secondary schools have a choice of several possible actions regarding AP Calculus. The option that is most appropriate for a particular school depends on local conditions and resources: school size, curriculum, the preparation of teachers, and the interest of students, teachers, and administrators.

Success in AP Calculus is closely tied to the preparation students have had in courses leading up to their AP courses. Students should have demonstrated mastery of material from courses covering the equivalent of four full years of high school mathematics before attempting calculus. These courses include algebra, geometry, coordinate geometry, and trigonometry, with the fourth year of study including advanced topics in algebra, trigonometry, analytic geometry, and elementary functions. Even though schools may choose from a variety of ways to accomplish these studies — including beginning the study of high school mathematics in grade 8; encouraging the election of more than one mathematics course in grade 9, 10, or 11; or instituting a program of summer study or guided independent study — it should be emphasized that eliminating preparatory course work in order to take an AP course is not appropriate.

The AP Calculus Development Committee recommends that calculus should be taught as a college-level course. With a solid foundation in courses taken before AP, students will be prepared to handle the rigor of a course at this level. Students who take an AP Calculus course should do so with the intention of placing out of a comparable college calculus course. This may be done through the AP Examination, a college placement examination, or any other method employed by the college.

The Courses

Philosophy

Calculus AB and Calculus BC are primarily concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications. The courses emphasize a multirepresentational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations also are important.

Calculus BC is an extension of Calculus AB rather than an enhancement; common topics require a similar depth of understanding. Both courses are intended to be challenging and demanding.

Broad concepts and widely applicable methods are emphasized. The focus of the courses is neither manipulation nor memorization of an extensive taxonomy of functions, curves, theorems, or problem types. Thus, although facility with manipulation and computational competence are important outcomes, they are not the core of these courses.

Technology should be used regularly by students and teachers to reinforce the relationships among the multiple representations of functions, to confirm written work, to implement experimentation, and to assist in interpreting results.

Through the use of the unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics. These themes are developed using all the functions listed in the prerequisites.

Goals

- Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation and should be able to use derivatives to solve a variety of problems.
- Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change and should be able to use integrals to solve a variety of problems.
- Students should understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.

- Students should be able to communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems.
- Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.
- Students should be able to use technology to help solve problems, experiment, interpret results, and verify conclusions.
- Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
- Students should develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Prerequisites

Before studying calculus, all students should complete four years of secondary mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include those that are linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise defined. In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on) and know the values of the trigonometric functions of the numbers $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$, and their multiples.

Resources for AP Calculus Teachers

To keep up to date with changes in AP Calculus, it is strongly recommended that teachers participate in ongoing professional development opportunities. These include the many workshops and Summer Institutes focusing on curriculum, pedagogy, and technology that are sponsored or coordinated by the College Board at various locations around the country and around the world. In addition, teachers seeking advice about initiating AP courses are urged to obtain additional information from other teachers who are involved in the AP Program. The electronic discussion groups (EDGs) accessible through AP Central provide a moderated forum for exchanging ideas, insights, and practices among members of the AP professional community. Information about workshops as well as lists of Summer Institutes may be obtained from the College Board offices listed on the inside back cover, or at AP Central™.

In addition, a number of publications may be of value to both new AP teachers who are planning a course for the first time, and to experienced AP teachers. One such publication is the latest edition of the *Teacher's Guide – AP Calculus*. This book provides information that is relevant to initiating an AP program in calculus, and it goes into much greater detail about the course descriptions for Calculus AB and Calculus BC. The publication also suggests teaching strategies and resource materials and provides sample course syllabi prepared by experienced AP teachers. The Teachers' Resources section of AP Central offers reviews of textbooks, articles, Web sites, and other teaching resources.

Another useful resource for teachers — and students — is *APCD*®, the AP Calculus AB CD-ROM with multiple-choice exams, free-response tutorials, and interactive explorations of selected calculus concepts.

The AP Publications Order Form can be used to order the *Teacher's Guide – AP Calculus* as well as several other publications helpful to AP Calculus teachers, such as the 1997 Released Exams and the 1998 Released Exams. See the back of this booklet for more information on AP publications.

Up-to-date information about AP Calculus can be found at AP Central. Detailed responses to free-response questions from previous AP Calculus Examinations are also available at the site.

Topic Outline for Calculus AB

This topic outline is intended to indicate the scope of the course, but it is not necessarily the order in which the topics need to be taught. Teachers may find that topics are best taught in different orders. (See the *Teacher's Guide – AP Calculus* for sample syllabi.) Although the examination is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I. Functions, Graphs, and Limits

Analysis of graphs. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits).

- An intuitive understanding of the limiting process.
- Calculating limits using algebra.
- Estimating limits from graphs or tables of data.

Asymptotic and unbounded behavior.

- Understanding asymptotes in terms of graphical behavior.
- Describing asymptotic behavior in terms of limits involving infinity.
- Comparing relative magnitudes of functions and their rates of change. (For example, contrasting exponential growth, polynomial growth, and logarithmic growth.)

Continuity as a property of functions.

- An intuitive understanding of continuity. (Close values of the domain lead to close values of the range.)
- Understanding continuity in terms of limits.
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

II. Derivatives

Concept of the derivative.

- Derivative presented graphically, numerically, and analytically.
- Derivative interpreted as an instantaneous rate of change.
- Derivative defined as the limit of the difference quotient.
- Relationship between differentiability and continuity.

Derivative at a point.

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation.
- Instantaneous rate of change as the limit of average rate of change.
- Approximate rate of change from graphs and tables of values.

Derivative as a function.

- Corresponding characteristics of graphs of f and f'.
- Relationship between the increasing and decreasing behavior of f and the sign of f'.
- The Mean Value Theorem and its geometric consequences.
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives.

- Corresponding characteristics of the graphs of f, f', and f''.
- Relationship between the concavity of f and the sign of f''.
- Points of inflection as places where concavity changes.

Applications of derivatives.

- Analysis of curves, including the notions of monotonicity and concavity.
- Optimization, both absolute (global) and relative (local) extrema.
- Modeling rates of change, including related rates problems.
- Use of implicit differentiation to find the derivative of an inverse function.
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.

Computation of derivatives.

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- Basic rules for the derivative of sums, products, and quotients of functions.
- Chain rule and implicit differentiation.

III. Integrals

Interpretations and properties of definite integrals.

- Computation of Riemann sums using left, right, and midpoint evaluation points.
- Definite integral as a limit of Riemann sums over equal subdivisions.
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

- Basic properties of definite integrals. (Examples include additivity and linearity.)

Applications of integrals. Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the integral of a rate of change to give accumulated change or using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, and the distance traveled by a particle along a line.

Fundamental Theorem of Calculus.

- Use of the Fundamental Theorem to evaluate definite integrals.
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Techniques of antidifferentiation.

- Antiderivatives following directly from derivatives of basic functions.
- Antiderivatives by substitution of variables (including change of limits for definite integrals).

Applications of antidifferentiation.

- Finding specific antiderivatives using initial conditions, including applications to motion along a line.
- Solving separable differential equations and using them in modeling. In particular, studying the equation $y' = ky$ and exponential growth.

Numerical approximations to definite integrals. Use of Riemann and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

Topic Outline for Calculus BC

The topic outline for Calculus BC includes all Calculus AB topics. Additional topics are found in paragraphs that are marked with a plus sign (+) or an asterisk (*). The additional topics can be taught anywhere in the course that the instructor wishes. Some topics will naturally fit immediately after their Calculus AB counterparts. Other topics may fit best after the completion of the Calculus AB topic outline. (See the *Teacher's Guide – AP Calculus* for sample syllabi.) Although the examination is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I. Functions, Graphs, and Limits

Analysis of graphs. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits).

- An intuitive understanding of the limiting process.
- Calculating limits using algebra.
- Estimating limits from graphs or tables of data.

Asymptotic and unbounded behavior.

- Understanding asymptotes in terms of graphical behavior.
- Describing asymptotic behavior in terms of limits involving infinity.
- Comparing relative magnitudes of functions and their rates of change. (For example, contrasting exponential growth, polynomial growth, and logarithmic growth.)

Continuity as a property of functions.

- An intuitive understanding of continuity. (Close values of the domain lead to close values of the range.)
- Understanding continuity in terms of limits.
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

* **Parametric, polar, and vector functions.** The analysis of planar curves includes those given in parametric form, polar form, and vector form.

II. Derivatives

Concept of the derivative.

- Derivative presented graphically, numerically, and analytically.
- Derivative interpreted as an instantaneous rate of change.
- Derivative defined as the limit of the difference quotient.
- Relationship between differentiability and continuity.

Derivative at a point.

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation.
- Instantaneous rate of change as the limit of average rate of change.
- Approximate rate of change from graphs and tables of values.

Derivative as a function.

- Corresponding characteristics of graphs of f and f'.
- Relationship between the increasing and decreasing behavior of f and the sign of f'.
- The Mean Value Theorem and its geometric consequences.
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives.

- Corresponding characteristics of the graphs of f, f', and f''.
- Relationship between the concavity of f and the sign of f''.
- Points of inflection as places where concavity changes.

Applications of derivatives.

- Analysis of curves, including the notions of monotonicity and concavity.

\+ Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration vectors.

- Optimization, both absolute (global) and relative (local) extrema.
- Modeling rates of change, including related rates problems.
- Use of implicit differentiation to find the derivative of an inverse function.
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.

- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.

+ Numerical solution of differential equations using Euler's method.
+ L'Hôpital's Rule, including its use in determining limits and convergence of improper integrals and series.

Computation of derivatives.

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- Basic rules for the derivative of sums, products, and quotients of functions.
- Chain rule and implicit differentiation.

+ Derivatives of parametric, polar, and vector functions.

III. Integrals

Interpretations and properties of definite integrals.

- Computation of Riemann sums using left, right, and midpoint evaluation points.
- Definite integral as a limit of Riemann sums over equal subdivisions.
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

- Basic properties of definite integrals. (Examples include additivity and linearity.)

* **Applications of integrals.** Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the integral of a rate of change to give accumulated change or using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and the length of a curve (including a curve given in parametric form).

Fundamental Theorem of Calculus.

• Use of the Fundamental Theorem to evaluate definite integrals.
• Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Techniques of antidifferentiation.

• Antiderivatives following directly from derivatives of basic functions.
+ Antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (nonrepeating linear factors only).
+ Improper integrals (as limits of definite integrals).

Applications of antidifferentiation.

• Finding specific antiderivatives using initial conditions, including applications to motion along a line.
• Solving separable differential equations and using them in modeling. In particular, studying the equation $y' = ky$ and exponential growth.
+ Solving logistic differential equations and using them in modeling.

Numerical approximations to definite integrals. Use of Riemann and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

*IV. Polynomial Approximations and Series

* **Concept of series.** A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence or divergence.

* **Series of constants.**

+ Motivating examples, including decimal expansion.
+ Geometric series with applications.
+ The harmonic series.
+ Alternating series with error bound.
+ Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series.
+ The ratio test for convergence and divergence.
+ Comparing series to test for convergence or divergence.

* **Taylor series.**
  + Taylor polynomial approximation with graphical demonstration of convergence. (For example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve.)
  + Maclaurin series and the general Taylor series centered at $x = a$.
  + Maclaurin series for the functions e^x, $\sin x$, $\cos x$, and $\frac{1}{1-x}$.
  + Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series.
  + Functions defined by power series.
  + Radius and interval of convergence of power series.
  + Lagrange error bound for Taylor polynomials.

Use of Graphing Calculators

Professional mathematics organizations such as the National Council of Teachers of Mathematics, the Mathematical Association of America, and the Mathematical Sciences Education Board of the National Academy of Sciences have strongly endorsed the use of calculators in mathematics instruction and testing.

The use of a graphing calculator in AP Calculus is considered an integral part of the course. Students should be using this technology on a regular basis so that they become adept at using their graphing calculators. Students should also have experience with the basic paper-and-pencil techniques of calculus and be able to apply them when technological tools are unavailable or inappropriate.

The AP Calculus Development Committee understands that new calculators and computers, capable of enhancing the teaching of calculus, continue to be developed. There are two main concerns that the committee considers when deciding what level of technology should be required for the examinations: equity issues and teacher development.

Over time, the range of capabilities of graphing calculators has increased significantly. Some calculators are much more powerful than first-generation graphing calculators and may include symbolic algebra features. Other graphing calculators are, by design, intended for students studying mathematics at lower levels than calculus. The committee can develop examinations that are appropriate for any given level of technology, but it cannot develop examinations that are fair to all students if the spread in the capabilities of the technology is too wide. Therefore, the committee has found it necessary to make certain requirements of the technology that will help ensure that all students have sufficient computational tools for the AP Calculus Examinations. Examination restrictions should not be interpreted as restrictions on classroom activities. The committee will continue to monitor the developments of technology and will reassess the testing policy regularly.

Graphing Calculator Capabilities for the Examinations

The committee develops examinations based on the assumption that all students have access to four basic calculator capabilities used extensively in calculus. A graphing calculator appropriate for use on the examinations is expected to have the built-in capability to:

1) plot the graph of a function within an arbitrary viewing window,
2) find the zeros of functions (solve equations numerically),
3) numerically calculate the derivative of a function, and
4) numerically calculate the value of a definite integral.

One or more of these capabilities should provide the sufficient computational tools for successful development of a solution to any examination question that requires the use of a calculator. Care is taken to ensure that the examination questions do not favor students who use graphing calculators with more extensive built-in features.

Technology Restrictions on the Examinations

Nongraphing scientific calculators, computers, devices with a QWERTY keyboard, pen-input devices, and electronic writing pads are not permitted for use on the AP Calculus Examinations.

Test administrators are required to check calculators before the examination. Therefore, it is important for each student to have an approved calculator. The student should be thoroughly familiar with the operation of the calculator he or she plans to use on the examination. Calculators may not be shared, and communication between calculators is prohibited during the examination. Students may bring to the examination one or two (but no more than two) graphing calculators from the list on page 19.

Calculator memories will not be cleared. Students are allowed to bring to the examination calculators containing whatever programs they want.

Students must not use calculator memories to take test materials out of the room. Students should be warned that their grades will be invalidated if they attempt to remove test materials from the room by any method.

Showing Work on the Free-Response Sections

Students are expected to show enough of their work for readers to follow their line of reasoning. To obtain full credit for the solution to a free-response problem, students must communicate their methods and conclusions clearly. Answers should show enough work so that the reasoning process can be followed throughout the solution. This is particularly important for assessing partial credit. Students may also be asked to use complete sentences to explain their methods or the reasonableness of their answers, or to interpret their results.

For results obtained using one of the four required calculator capabilities listed on page 17, students are required to write the setup (e.g., the equation being solved, or the derivative or definite integral being evaluated) that leads to the solution, along with the result produced by the calculator. For example, if the student is asked to find the area of a region, the student is expected to show a definite integral (i.e., the setup) and the answer. The student need not compute the antiderivative; the calculator may be used to calculate the value of the definite integral without further explanation. For solutions obtained using a calculator capability other than one of the four required ones, students must also show the mathematical steps necessary to produce their results; a calculator result alone is not sufficient. For example, if the student is asked to find a relative minimum value of a function, the student is expected to use calculus and show the mathematical steps that lead to the answer. It is not sufficient to graph the function or use a built-in minimum finder.

When a student is asked to justify an answer, the justification must include mathematical (noncalculator) reasons, not merely calculator results. Functions, graphs, tables, or other objects that are used in a justification should be clearly labeled.

A graphing calculator is a powerful tool for exploration, but students must be cautioned that exploration is not a mathematical solution. Exploration with a graphing calculator can lead a student toward an analytical solution, and after a solution is found, a graphing calculator can often be used to check the reasonableness of the solution.

As on previous AP Examinations, if a calculation is given as a decimal approximation, it should be correct to three places after the decimal point unless otherwise indicated. Students should be cautioned against rounding values in intermediate steps before a final calculation is made. Students should also be aware that there are limitations inherent in graphing calculator technology; for example, answers obtained by tracing along a graph to find roots or points of intersection might not produce the required accuracy.

For more detailed information on the instructions for the free-response sections, read the "AP Calculus: Free-Response Instruction Commentary" written by the AP Calculus Development Committee. It is available in the Teachers' Corner for Calculus AB or Calculus BC at AP Central.

List of Graphing Calculators

Students are expected to bring a calculator with the capabilities listed on page 17 to the examinations. AP teachers should check their own students' calculators to ensure that the required conditions are met. If a student wishes to use a calculator that is not on the list, the AP teacher must contact ETS (609 771-7300) before April 1 of the testing year to request written permission for the student to use the calculator on the AP Examinations.

Casio
FX-6000 series
FX-6200 series
FX-6300 series
FX-6500 series
FX-7000 series
FX-7300 series
FX-7400 series
FX-7500 series
FX-7700 series
FX-7800 series
FX-8000 series
FX-8500 series
FX-8700 series
FX-8800 series
*FX-9700 series
*FX-9750 series
*CFX-9800 series
*CFX-9850 series
*CFX-9950 series
*CFX-9970 series
*FX 1.0 series
*Algebra FX 2.0 series

Hewlett-Packard
*HP-28 series
*HP-38G
*HP-39G
*HP-40G
*HP-48 series
*HP-49 series

Radio Shack
EC-4033
EC-4034
EC-4037

Sharp
EL-5200
*EL-9200 series
*EL-9300 series
*EL-9600 series
*EL-9900 series

Texas Instruments
TI-73
TI-80
TI-81
*TI-82
*TI-83/TI-83 Plus
*TI-85
*TI-86
*TI-89

Other
Micronta
Smart^2

* These graphing calculators have the built-in capabilities listed on page 17.

Note: This list will be updated, when necessary, to include new allowable calculators. An up-to-date list is available at AP Central.

Unacceptable machines include the following:
Nongraphing scientific calculators
Powerbooks and portable computers
Pocket organizers
Electronic writing pads or pen-input devices (e.g., Palm)
Devices with QWERTY keyboards (e.g., TI-92 Plus, Voyage 200)

The Examinations

The Calculus AB and BC Examinations seek to assess how well a student has mastered the concepts and techniques of the subject matter of the corresponding courses. Each examination consists of two sections, as described below.

Section I: a multiple-choice section testing proficiency in a wide variety of topics

Section II: a free-response section requiring the student to demonstrate the ability to solve problems involving a more extended chain of reasoning

The time allotted for each AP Calculus Examination is 3 hours and 15 minutes. The multiple-choice section of each examination consists of 45 questions in 105 minutes. Part A of the multiple-choice section (28 questions in 55 minutes) does not allow the use of a calculator. Part B of the multiple-choice section (17 questions in 50 minutes) contains some questions for which a graphing calculator is required.

The free-response section of each examination has two parts: one part requiring graphing calculators and a second part not allowing graphing calculators. The AP Examinations are designed to accurately assess student mastery of both the concepts and techniques of calculus. The two-part format for the free-response section provides greater flexibility in the types of problems that can be given while ensuring fairness to all students taking the exam, regardless of the graphing calculator used. See page 19 for the list of approved graphing calculators for the AP Calculus Examinations.

The free-response section of each examination consists of 6 problems in 90 minutes. Part A of the free-response section (3 problems in 45 minutes) contains some problems or parts of problems for which a graphing calculator is required. Part B of the free-response section (3 problems in 45 minutes) does not allow the use of a calculator. During the second timed portion of the free-response section (Part B), students are permitted to continue work on problems in Part A, but they are not permitted to use a calculator during this time.

In determining the grade for each examination, the scores for Section I and Section II are given equal weight. Since the examinations are designed for full coverage of the subject matter, it is not expected that all students will be able to answer all the questions.

Calculus AB Subscore Grade for the Calculus BC Examination

A Calculus AB subscore grade is reported based on performance on the portion of the examination devoted to Calculus AB topics (approximately 60% of the examination). The Calculus AB subscore grade is designed to give colleges and universities more information about the student. Although each college and university sets its own policy for awarding credit and/or placement for AP Exam grades, it is recommended that institutions apply the same policy to the Calculus AB subscore grade that they apply to the Calculus AB grade. Use of the subscore grade in this manner is consistent with the philosophy of the courses, since common topics are tested at the same conceptual level in both Calculus AB and Calculus BC.

The Calculus AB subscore grade was first reported for the 1998 Calculus BC examination. The reliability of the Calculus AB subscore grade is nearly equal to the reliabilities of the AP Calculus AB and Calculus BC Examinations.

The following tables compare the AB subscore grades with the BC grades for students taking the AP Calculus BC Examination.

Comparison of AB Subscore Grades with Calculus BC Grades — May 2002

	AB Subscore Grade				
BC Grade	1	2	3	4	5
1	25%	49%	26%		
2		9%	78%	13%	
3			26%	69%	5%
4				58%	42%
5				4%	96%

Summary of Differences between AB Subscore Grades and BC Grades (AB Subscore Grade Minus BC Grade)

	–1	0	1	2	3
1998	2.9%	59.1%	34.2%	3.8%	<0.1%
1999	2.1%	62.5%	33.0%	2.3%	<0.1%
2000	0.5%	51.6%	41.5%	6.4%	<0.1%
2001	3.6%	58.9%	34.1%	3.4%	
2002	1.7%	60.1%	33.4%	4.8%	

The Grade Setting Process

The Chief Reader for AP Calculus works in conjunction with statistical analysis and mathematics test development staff of Educational Testing Service to establish the AP grade ranges (i.e., the range of raw scores that determine a particular AP grade). Direct comparisons are made between the performance of the current year's students and that of former students on a set of previously administered multiple-choice questions. A statistical procedure called "equating" uses this information to provide grade ranges for the current exam that best represent performance equivalent to the grade ranges in previous years. Equating allows the comparison of performance of one group of students with that of groups of students taking other forms of the exam. Equating is designed to deal with the problem of adjusting scores to compensate for differences in difficulty among multiple forms of the exam. The Chief Reader also considers other pertinent data, including college validity studies and reports of table leaders from the AP Calculus Reading, to arrive at decisions on grades. The Chief Reader then determines the grade ranges that convert the total raw scores (the multiple-choice score plus the free-response score) to the AP Program 5-point scale of grades.

Additional information on "AP Grades" can be found on page 72 and at AP Central.

Calculus AB: Section I

Section I consists of 45 multiple-choice questions. Part A contains 28 questions and does not allow the use of a calculator. Part B contains 17 questions and requires a graphing calculator for some questions. Twenty-four sample multiple-choice questions for Calculus AB are included in the following sections. Answers to the sample questions are given on page 34.

Part A Sample Multiple-Choice Questions

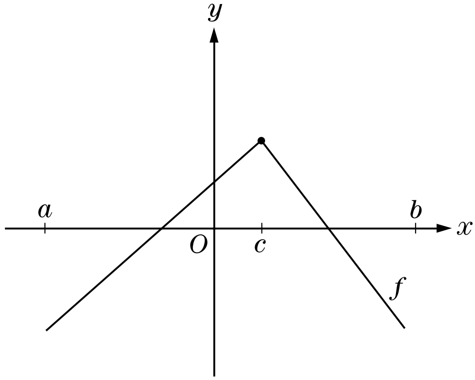
A calculator may not be used on this part of the examination.

Part A consists of 28 questions. In this section of the examination, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I Part A and a representative set of 14 questions.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:
(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1.



The function f, whose graph consists of two line segments, is shown above. Which of the following are true for f on the open interval (a, b)?

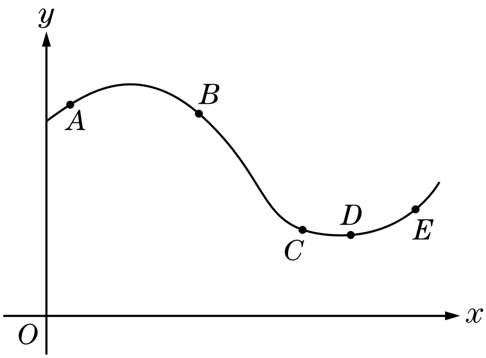
I. The domain of the derivative of f is the open interval (a, b).
II. f is continuous on the open interval (a, b).
III. The derivative of f is positive on the open interval (a, c).

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

2. What is $\lim\limits_{h \to 0} \dfrac{\cos\left(\dfrac{3\pi}{2} + h\right) - \cos\left(\dfrac{3\pi}{2}\right)}{h}$?

(A) 1
(B) $\dfrac{\sqrt{2}}{2}$
(C) 0
(D) -1
(E) The limit does not exist.

3. At which of the five points on the graph in the figure at the right are $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ both negative?



(A) A
(B) B
(C) C
(D) D
(E) E

4. The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at $(2, 1)$ is

(A) $-\dfrac{3}{2}$
(B) -1
(C) $-\dfrac{5}{14}$
(D) $-\dfrac{3}{14}$
(E) 0

5. Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

(A) The function has no relative extremum.
(B) The graph of the function has one point of inflection and the function has two relative extrema.
(C) The graph of the function has two points of inflection and the function has one relative extremum.
(D) The graph of the function has two points of inflection and the function has two relative extrema.
(E) The graph of the function has two points of inflection and the function has three relative extrema.

6. If $f(x) = \sin^2(3 - x)$, then $f'(0) =$

(A) $-2\cos 3$
(B) $-2\sin 3\cos 3$
(C) $6\cos 3$
(D) $2\sin 3\cos 3$
(E) $6\sin 3\cos 3$

7. The solution to the differential equation $\dfrac{dy}{dx} = \dfrac{x^3}{y^2}$, where $y(2) = 3$, is

(A) $y = \sqrt[3]{\frac{3}{4}x^4}$

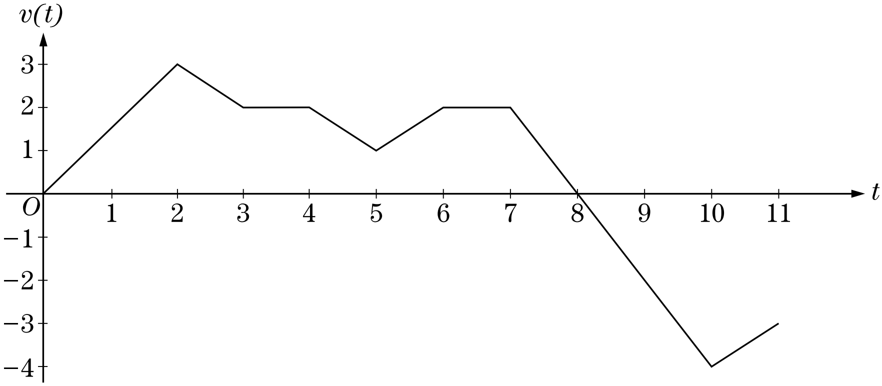
(B) $y = \sqrt[3]{\frac{3}{4}x^4} + \sqrt[3]{15}$

(C) $y = \sqrt[3]{\frac{3}{4}x^4} + 15$

(D) $y = \sqrt[3]{\frac{3}{4}x^4 + 5}$

(E) $y = \sqrt[3]{\frac{3}{4}x^4 + 15}$

Questions 8 and 9 refer to the following graph and information.



A bug is crawling along a straight wire. The velocity, $v(t)$, of the bug at time t, $0 \le t \le 11$, is given in the graph above.

8. According to the graph, at what time t does the bug change direction?

   (A) 2
   (B) 5
   (C) 6
   (D) 8
   (E) 10

9. According to the graph, at what time t is the speed of the bug greatest?

   (A) 2
   (B) 5
   (C) 6
   (D) 8
   (E) 10

10. $\int (x - 1)\sqrt{x}\, dx =$

(A) $\frac{3}{2}\sqrt{x} - \frac{1}{\sqrt{x}} + C$

(B) $\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$

(C) $\frac{1}{2}x^2 - x + C$

(D) $\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C$

(E) $\frac{1}{2}x^2 + 2x^{\frac{3}{2}} - x + C$

11. What is $\lim\limits_{x \to \infty} \dfrac{x^2 - 4}{2 + x - 4x^2}$?

(A) -2

(B) $-\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

(E) The limit does not exist.

12. The area of the region in the first quadrant between the graph of $y = x\sqrt{4 - x^2}$ and the x-axis is

(A) $\frac{2}{3}\sqrt{2}$
(B) $\frac{8}{3}$
(C) $2\sqrt{2}$
(D) $2\sqrt{3}$
(E) $\frac{16}{3}$

13. Which of the following are antiderivatives of $\frac{(\ln x)^2}{x}$?

I. $\frac{2\ln x - (\ln x)^2}{x^2}$

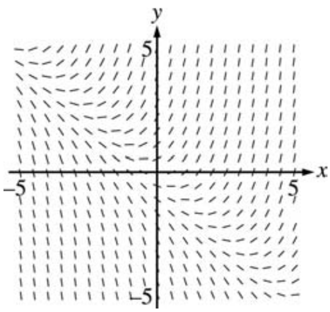
II. $\frac{(\ln x)^3}{3} + 6$

III. $\frac{(\ln x)^3}{3}$

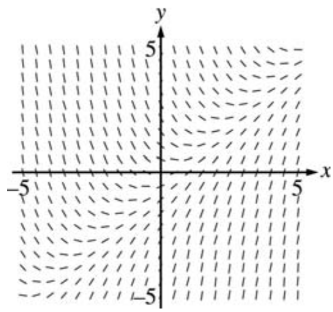
(A) I only
(B) III only
(C) I and II only
(D) I and III only
(E) II and III only

14. Which of the following is a slope field for the differential equation $\dfrac{dy}{dx} = \dfrac{x}{y}$?

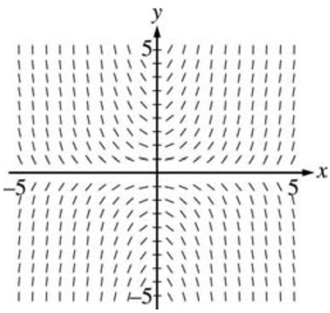
(A)



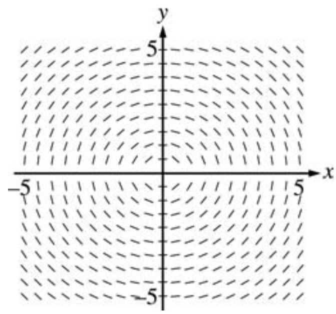
(B)



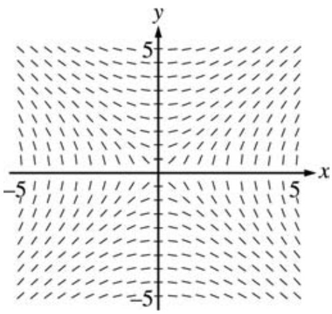
(C)



(D)



(E)



Part B Sample Multiple-Choice Questions

A graphing calculator is required for some questions on this part of the examination.

Part B consists of 17 questions. In this section of the examination, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I Part B and a representative set of 10 questions.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:
(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

15. The average value of the function $f(x) = e^{-x^2}$ on the closed interval $[-1, 1]$ is

(A) 0
(B) 0.368
(C) 0.747
(D) 1
(E) 1.494

16. Let S be the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$ for $0 \le x \le 1$. What is the volume of the solid generated when S is revolved about the line $y = 3$?

(A) $\pi\int_0^1 \left(\left(3 - 2x^2\right)^2 - \left(3 - 2x\right)^2\right) dx$

(B) $\pi\int_0^1 \left(\left(3 - 2x\right)^2 - \left(3 - 2x^2\right)^2\right) dx$

(C) $\pi\int_0^1 \left(4x^2 - 4x^4\right) dx$

(D) $\pi\int_0^2 \left(\left(3 - \frac{y}{2}\right)^2 - \left(3 - \sqrt{\frac{y}{2}}\right)^2\right) dy$

(E) $\pi\int_0^2 \left(\left(3 - \sqrt{\frac{y}{2}}\right)^2 - \left(3 - \frac{y}{2}\right)^2\right) dy$

17. Let f be defined as follows, where $a \neq 0$.

$$f(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & \text{for } x \neq a, \\ 0, & \text{for } x = a. \end{cases}$$

Which of the following are true about f?

I. $\lim\limits_{x \to a} f(x)$ exists.
II. $f(a)$ exists.
III. $f(x)$ is continuous at $x = a$.

(A) None
(B) I only
(C) II only
(D) I and II only
(E) I, II, and III

18.

x	1.1	1.2	1.3	1.4
$f(x)$	4.18	4.38	4.56	4.73

Let f be a function such that $f''(x) < 0$ for all x in the closed interval [1, 2], with selected values shown in the table above. Which of the following must be true about $f'(1.2)$?

(A) $f'(1.2) < 0$
(B) $0 < f'(1.2) < 1.6$
(C) $1.6 < f'(1.2) < 1.8$
(D) $1.8 < f'(1.2) < 2.0$
(E) $f'(1.2) > 2.0$

19. Two particles start at the origin and move along the x-axis. For $0 \leq t \leq 10$, their respective position functions are given by $x_1 = \sin t$ and $x_2 = e^{-2t} - 1$. For how many values of t do the particles have the same velocity?

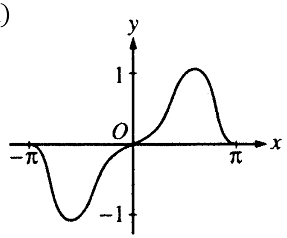
(A) None
(B) One
(C) Two
(D) Three
(E) Four

20. If the function g is defined by $g(x) = \displaystyle\int_0^x \sin(t^2)dt$ on the closed interval $-1 \leq x \leq 3$, then g has a local minimum at $x =$

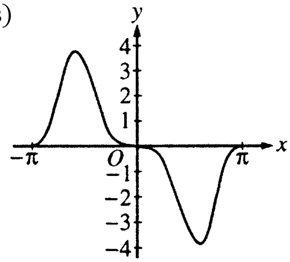
(A) 0
(B) 1.084
(C) 1.772
(D) 2.171
(E) 2.507

21. The graphs of five functions are shown below. Which function has a nonzero average value over the closed interval $[-\pi, \pi]$?

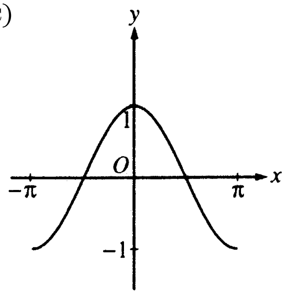
(A)



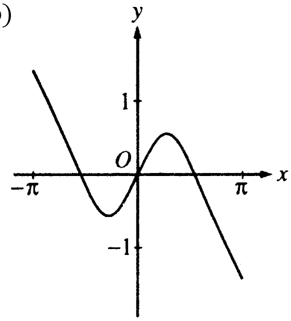
(B)



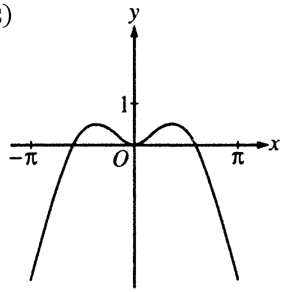
(C)



(D)



(E)



22. The region in the first quadrant enclosed by the y-axis and the graphs of $y = \cos x$ and $y = x$ is rotated about the x-axis. The volume of the solid generated is

(A) 0.484
(B) 0.877
(C) 1.520
(D) 1.831
(E) 3.040

23. Oil is leaking from a tanker at the rate of $R(t) = 2{,}000e^{-0.2t}$ gallons per hour, where t is measured in hours. How much oil has leaked out of the tanker after 10 hours?

(A) 54 gallons
(B) 271 gallons
(C) 865 gallons
(D) 8,647 gallons
(E) 14,778 gallons

24. If $f'(x) = \sin\left(\dfrac{\pi e^x}{2}\right)$ and $f(0) = 1$, then $f(2) =$

(A) -1.819
(B) -0.843
(C) -0.819
(D) 0.157
(E) 1.157

Answers to Calculus AB Multiple-Choice Questions

Part A

1. D	6. B	11. B
2. A	7. E	12. B
3. B	8. D	13. E
4. C	9. E	14. E
5. C	10. D	

Part B

15.* C	19.* D	22.* C
16. A	20.* E	23.* D
17. D	21. E	24.* E
18. D		

* Indicates a graphing calculator-active question.

Calculus BC: Section I

Section I consists of 45 multiple-choice questions. Part A contains 28 questions and does not allow the use of a calculator. Part B contains 17 questions and requires a graphing calculator for some questions. Twenty-four sample multiple-choice questions for Calculus BC are included in the following sections. Answers to the sample questions are given on page 45.

Part A Sample Multiple-Choice Questions

A calculator may not be used on this part of the examination.

Part A consists of 28 questions. In this section of the examination, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I Part A and a representative set of 14 questions.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:
(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. The line perpendicular to the tangent of the curve represented by the equation $y = x^2 + 6x + 4$ at the point $(-2, -4)$ also intersects the curve at $x =$

(A) -6

(B) $-\frac{9}{2}$

(C) $-\frac{7}{2}$

(D) -3

(E) $-\frac{1}{2}$

2. If $y = x + \sin(xy)$, then $\dfrac{dy}{dx} =$

   (A) $1 + \cos(xy)$
   (B) $1 + y\cos(xy)$
   (C) $\dfrac{1}{1 - \cos(xy)}$
   (D) $\dfrac{1}{1 - x\cos(xy)}$
   (E) $\dfrac{1 + y\cos(xy)}{1 - x\cos(xy)}$

3. Let $y = f(x)$ be the solution to the differential equation $\dfrac{dy}{dx} = \arcsin(xy)$ with the initial condition $f(0) = 2$. What is the approximation for $f(1)$ if Euler's method is used, starting at $x = 0$ with a step size of 0.5?

   (A) 2
   (B) $2 + \dfrac{\pi}{6}$
   (C) $2 + \dfrac{\pi}{4}$
   (D) $2 + \dfrac{\pi}{2}$
   (E) 3

4. What are all values of x for which the series $\displaystyle\sum_{n=1}^{\infty} \frac{n3^n}{x^n}$ converges?

   (A) All x except $x = 0$
   (B) $|x| = 3$
   (C) $-3 \le x \le 3$
   (D) $|x| > 3$
   (E) The series diverges for all x.

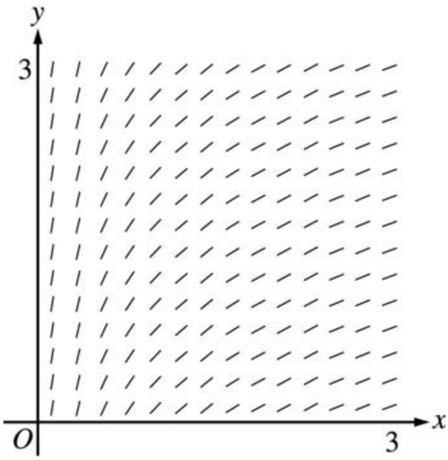
5. If $\dfrac{d}{dx} f(x) = g(x)$ and if $h(x) = x^2$, then $\dfrac{d}{dx} f(h(x)) =$

   (A) $g(x^2)$
   (B) $2xg(x)$
   (C) $g'(x)$
   (D) $2xg(x^2)$
   (E) $x^2 g(x^2)$

6. If F' is a continuous function for all real x, then $\lim_{h \to 0} \dfrac{1}{h} \displaystyle\int_a^{a+h} F'(x)\, dx$ is

(A) 0
(B) $F(0)$
(C) $F(a)$
(D) $F'(0)$
(E) $F'(a)$

7.



The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$
(B) $y = e^x$
(C) $y = e^{-x}$
(D) $y = \cos x$
(E) $y = \ln x$

8. $\displaystyle\int_0^{\infty} e^{-2t}\, dt$ is

(A) -1
(B) $-\dfrac{1}{2}$
(C) $\dfrac{1}{2}$
(D) 1
(E) divergent

9. Which of the following series converge to 2?

I. $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{n+3}$

II. $\displaystyle\sum_{n=1}^{\infty} \frac{-8}{(-3)^n}$

III. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^n}$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

10. What is the area of the closed region bounded by the curve $y = e^{2x}$ and the lines $x = 1$ and $y = 1$?

(A) $\dfrac{2 - e^2}{2}$

(B) $\dfrac{e^2 - 3}{2}$

(C) $\dfrac{3 - e^2}{2}$

(D) $\dfrac{e^2 - 2}{2}$

(E) $\dfrac{e^2 - 1}{2}$

11. Which of the following integrals gives the length of the graph of $y = \sqrt{x}$ between $x = a$ and $x = b$, where $0 < a < b$?

(A) $\displaystyle\int_a^b \sqrt{x^2 + x}\, dx$

(B) $\displaystyle\int_a^b \sqrt{x + \sqrt{x}}\, dx$

(C) $\displaystyle\int_a^b \sqrt{x + \frac{1}{2\sqrt{x}}}\, dx$

(D) $\displaystyle\int_a^b \sqrt{1 + \frac{1}{2\sqrt{x}}}\, dx$

(E) $\displaystyle\int_a^b \sqrt{1 + \frac{1}{4x}}\, dx$

12. The area of one loop of the graph of the polar equation $r = 2\sin(3\theta)$ is given by which of the following expressions?

(A) $4\int_0^{\frac{\pi}{3}} \sin^2(3\theta)\, d\theta$

(B) $2\int_0^{\frac{\pi}{3}} \sin(3\theta)\, d\theta$

(C) $2\int_0^{\frac{\pi}{3}} \sin^2(3\theta)\, d\theta$

(D) $2\int_0^{\frac{2\pi}{3}} \sin^2(3\theta)\, d\theta$

(E) $2\int_0^{\frac{2\pi}{3}} \sin(3\theta)\, d\theta$

13. The third-degree Taylor polynomial about $x = 0$ of $\ln(1 - x)$ is

(A) $-x - \frac{x^2}{2} - \frac{x^3}{3}$

(B) $1 - x + \frac{x^2}{2}$

(C) $x - \frac{x^2}{2} + \frac{x^3}{3}$

(D) $-1 + x - \frac{x^2}{2}$

(E) $-x + \frac{x^2}{2} - \frac{x^3}{3}$

14. If $\frac{dy}{dx} = y \sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$
(B) $e^{\tan x} + 5$
(C) $5e^{\tan x}$
(D) $\tan x + 5$
(E) $\tan x + 5e^x$

Part B Sample Multiple-Choice Questions

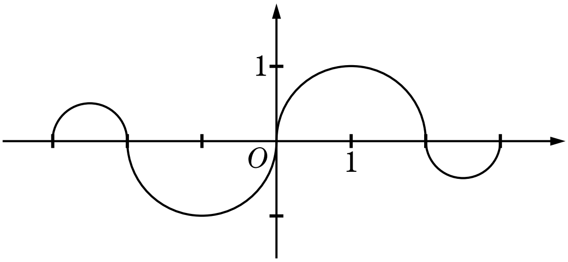
A graphing calculator is required for some questions on this part of the examination.

Part B consists of 17 questions. In this section of the examination, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I Part B and a representative set of 10 questions.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:
(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

15.



Graph of f

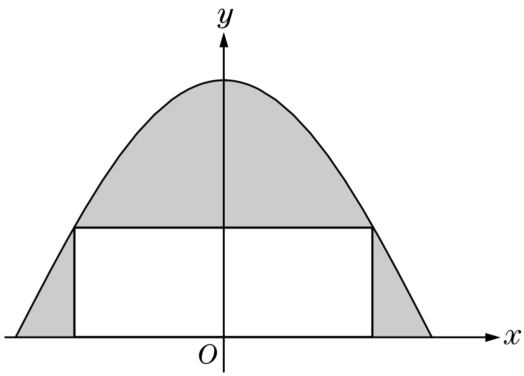
The graph of the function f above consists of four semicircles. If $g(x) = \int_0^x f(t)dt$, where is $g(x)$ nonnegative?

(A) $[-3, 3]$
(B) $[-3, -2] \cup [0, 2]$ only
(C) $[0, 3]$ only
(D) $[0, 2]$ only
(E) $[-3, -2] \cup [0, 3]$ only

16. If f is differentiable at $x = a$, which of the following could be false?

(A) f is continuous at $x = a$.
(B) $\lim_{x \to a} f(x)$ exists.
(C) $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ exists.
(D) $f'(a)$ is defined.
(E) $f''(a)$ is defined.

17.



A rectangle with one side on the x-axis has its upper vertices on the graph of $y = \cos x$, as shown in the figure above. What is the minimum area of the shaded region?

(A) 0.799
(B) 0.878
(C) 1.140
(D) 1.439
(E) 2.000

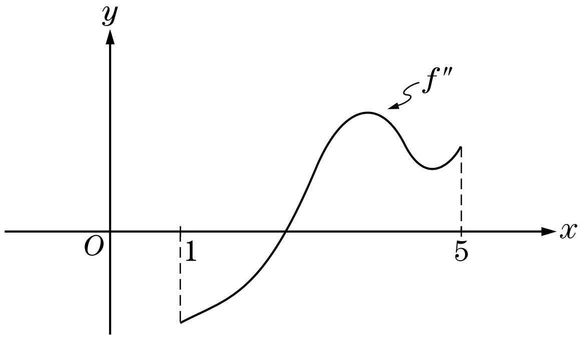
18.

Time (sec)	0	10	25	37	46	60
Rate (gal/sec)	500	400	350	280	200	180

The table above gives the values for the rate (in gal/sec) at which water flowed into a lake, with readings taken at specific times. A right Riemann sum, with the five subintervals indicated by the data in the table, is used to estimate the total amount of water that flowed into the lake during the time period $0 \le t \le 60$. What is this estimate?

(A) 1,910 gal
(B) 14,100 gal
(C) 16,930 gal
(D) 18,725 gal
(E) 20,520 gal

19.



Let f be a function whose domain is the open interval $(1, 5)$. The figure above shows the graph of f''. Which of the following describes the relative extrema of f' and the points of inflection of the graph of f'?

(A) 1 relative maximum, 1 relative minimum, and no point of inflection
(B) 1 relative maximum, 2 relative minima, and no point of inflection
(C) 1 relative maximum, 1 relative minimum, and 1 point of inflection
(D) 1 relative maximum and 2 points of inflection
(E) 1 relative minimum and 2 points of inflection

20. A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = \ln(t + 1) - 2t + 1$. The total distance traveled by the particle from $t = 0$ to $t = 2$ is

(A) 0.667
(B) 0.704
(C) 1.540
(D) 2.667
(E) 2.901

21. If the function f is defined by $f(x) = \sqrt{x^3 + 2}$ and g is an antiderivative of f such that $g(3) = 5$, then $g(1) =$

(A) -3.268
(B) -1.585
(C) 1.732
(D) 6.585
(E) 11.585

22. Let g be the function given by $g(x) = \int_1^x 100(t^2 - 3t + 2)e^{-t^2}\,dt$.

Which of the following statements about g must be true?

I. g is increasing on $(1, 2)$.
II. g is increasing on $(2, 3)$.
III. $g(3) > 0$

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

23. A point (x, y) is moving along a curve $y = f(x)$. At the instant when the slope of the curve is $-\frac{1}{3}$, the x-coordinate of the point is increasing at the rate of 5 units per second. The rate of change, in units per second, of the y-coordinate of the point is

(A) $-\frac{5}{3}$

(B) $-\frac{1}{3}$

(C) $\frac{1}{3}$

(D) $\frac{3}{5}$

(E) $\frac{5}{3}$

24. Let g be the function given by $g(t) = 100 + 20\sin\left(\frac{\pi t}{2}\right) + 10\cos\left(\frac{\pi t}{6}\right)$.

For $0 \le t \le 8$, g is decreasing most rapidly when $t =$

(A) 0.949
(B) 2.017
(C) 3.106
(D) 5.965
(E) 8.000

Answers to Calculus BC Multiple-Choice Questions

Part A

1. B	6. E	11. E
2. E	7. E	12. C
3. C	8. C	13. A
4. D	9. E	14. C
5. D	10. B	

Part B

15. A	19. E	22.* B
16. E	20.* C	23. A
17.* B	21.* B	24.* B
18. C		

* Indicates a graphing calculator-active question.

Calculus AB and Calculus BC: Section II

Section II consists of six free-response problems. The problems do NOT appear in the Section II test booklet. Part A problems are printed in the green insert only; Part B problems are printed in a separate sealed blue insert. Each part of every problem has a designated workspace in the test booklet. ALL WORK MUST BE SHOWN IN THE TEST BOOKLET. (For students taking the examination at an alternate administration, the Part A problems are printed in the test booklet only; the Part B problems appear in a separate sealed insert.)

The instructions below are from the 2003 examinations. The free-response problems are from the 2002 examinations and include commentary and information on scoring. Additional sample questions can be found at AP Central.

Instructions for Section II

PART A
(A graphing calculator is required for some problems or parts of problems.)
Part A: 45 minutes, 3 problems

During the timed portion for Part A, you may work only on the problems in Part A. The problems for Part A are printed in the green insert only. When you are told to begin, open your booklet, carefully tear out the green insert, and write your solution to each part of each problem in the space provided for that part in the pink test booklet.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

PART B
(No calculator is allowed for these problems.)
Part B: 45 minutes, 3 problems

The problems for Part B are printed in the blue insert only. When you are told to begin, open the blue insert, and write your solution to each part of each problem in the space provided for that part in the pink test booklet. During the timed portion for Part B, you may keep the green insert and continue to work on the problems in Part A without the use of any calculator.

GENERAL INSTRUCTIONS FOR SECTION II PART A AND PART B

For each part of Section II, you may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

- YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM IN THE SPACE PROVIDED FOR THAT PART IN THE PINK TEST BOOKLET. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. Clearly label any functions, graphs, tables, or other objects that you use. You will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.
- If you use decimal approximations in calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

For more detailed information on the instructions for the free-response sections, read the “AP Calculus: Free-Response Instruction Commentary” written by the AP Calculus Development Committee. It is available in the Teachers’ Corner for Calculus AB or Calculus BC at AP Central.

Calculus AB Sample Free-Response Questions

AB/BC Question 1

This problem presented a region bounded between two graphs and two vertical lines. Students were asked to use both integration and differentiation to answer some straightforward questions about this region. Part (a) required the use of a definite integral to find the area of the region. In part (b), the region is revolved about a horizontal line, resulting in a solid with cross sections in the shape of "washers," and the volume of the region was asked for, requiring another use of a definite integral. Students were expected to use the numerical integration capabilities of a graphing calculator to evaluate these definite integrals. Part (c) required the use of differentiation to find the absolute minimum height of the region.

Many students were successful in finding the area of the region but had difficulty setting up the volume integral because of the complication of rotating around a horizontal line that was not the x-axis. The students did not need to compute antiderivatives to compute the definite integrals in parts (a) and (b); the calculator may be used to calculate the value of the definite integral without further explanation once the setup of the definite integral is shown.

In part (c), students were expected to show the mathematical steps needed to analyze the location of the absolute minimum and maximum. Some students neglected to consider where $h'(x) = 0$ to find the critical point, and some did not consider both endpoints as candidates.

It is important for students to show their work as described in "Showing Work on the Free-Response Sections" on page 18. For example, when finding a critical point of a function, students should explicitly state the equation that is being solved for determining where the derivative is equal to 0, rather than expect that the equation will be inferred from a graph or a sign chart.

Question 1

Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

(a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

(b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

(c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

(a) Area $= \int_{1/2}^{1} (e^x - \ln x)\,dx = 1.222$ or 1.223	$2 \begin{cases} 1: \text{ integral} \\ 1: \text{ answer} \end{cases}$
(b) Volume $= \pi \int_{1/2}^{1} \left((4 - \ln x)^2 - (4 - e^x)^2 \right) dx$ $= 7.515\pi$ or 23.609	$4 \begin{cases} 1: \text{ limits and constant} \\ 2: \text{ integrand} \\ \quad < -1 > \text{ each error} \\ \quad \text{Note: } 0/2 \text{ if not of the form} \\ \quad k\int_a^b \left(R(x)^2 - r(x)^2 \right) dx \\ 1: \text{ answer} \end{cases}$
(c) $h'(x) = f'(x) - g'(x) = e^x - \frac{1}{x} = 0$ $x = 0.567143$ Absolute minimum value and absolute maximum value occur at the critical point or at the endpoints. $h(0.567143) = 2.330$ $h(0.5) = 2.3418$ $h(1) = 2.718$ The absolute minimum is 2.330. The absolute maximum is 2.718.	$3 \begin{cases} 1: \text{ considers } h'(x) = 0 \\ 1: \text{ identifies critical point} \\ \quad \text{and endpoints as candidates} \\ 1: \text{ answers} \end{cases}$ Note: Errors in computation come off the third point.

AB/BC Question 2

This problem involved a “real-life” model of people entering and leaving an amusement park, and students were expected to interpret the meanings of their calculations in that context. In part (a), students needed to recognize that the function E represented the rate of accumulation of people entering the park, and hence that the total accumulation over a time interval could be obtained by a definite integral. Part (b) took the same idea a step further, asking students to calculate ticket revenues based on different pricing over two time intervals. Part (c) presented a function H defined in terms of a definite integral. The value of $H'(17)$ was easily computed, provided students recognized that the Fundamental Theorem of Calculus could be applied. More importantly, students were expected to interpret, in words, the meanings of both $H(17)$ and $H'(17)$. These interpretations played a role in part (d), which essentially asked students to find when the maximum value of the function H is achieved.

The most common error in part (a) was to evaluate $E(17)$ or $E(17) - L(17)$, or to attempt a discrete analysis without seeing the connection to a definite integral. The most common error in calculating $H'(17)$ was not recognizing the need to use the Fundamental Theorem of Calculus or not using the theorem correctly. A large number of students used $H'(17) = E'(17) - L'(17)$. The interpretations of $H(17)$ and $H'(17)$ were difficult for students to make. Those who recognized the conceptual connection that $H(t)$ measured population in the park at time t and phrased $H(17)$ as a population had an easier time with $H'(17)$.

While students could integrate by hand or with a CAS capable calculator to find a closed form expression for the number of people who entered the park and the number of people who left the park, this approach was neither necessary nor efficient to handle the questions in this problem.

It is important for students to clearly show the setup for all definite integrals that must be evaluated numerically. As in AB/BC Question 1, when finding a critical point of a function, students should explicitly state the equation that is being solved for determining where the derivative is equal to 0, rather than expect that the equation will be inferred from a graph or a sign chart.

One of the goals of the exam is to emphasize problems that probe understanding of the fundamental concepts and not to merely test rote manipulation. This problem was an example of one that required students to make connections between calculus and the “real” world, and between topics within calculus itself.

Question 2

The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

Solution	Scoring
(a) $\int_9^{17} E(t)\,dt = 6004.270$ 6004 people entered the park by 5 pm.	$3 \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(b) $15\int_9^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt = 104048.165$ The amount collected was \$104,048. or $\int_{17}^{23} E(t)\,dt = 1271.283$ 1271 people entered the park between 5 pm and 11 pm, so the amount collected was $\$15 \cdot (6004) + \$11 \cdot (1271) = \$104,041.$	1 : setup
(c) $H'(17) = E(17) - L(17) = -380.281$ There were 3725 people in the park at $t = 17$. The number of people in the park was decreasing at the rate of approximately 380 people/hr at time $t = 17$.	$3 \begin{cases} 1: \text{value of } H'(17) \\ 2: \text{meanings} \\ \quad 1: \text{meaning of } H(17) \\ \quad 1: \text{meaning of } H'(17) \\ \quad <-1> \text{ if no reference to } t = 17 \end{cases}$
(d) $H'(t) = E(t) - L(t) = 0$ $t = 15.794$ or 15.795	$2 \begin{cases} 1: E(t) - L(t) = 0 \\ 1: \text{answer} \end{cases}$

AB Question 3

This problem presented the velocity and initial position of an object moving along the x-axis. Part (a) required a knowledge of the relationship between velocity and its derivative, acceleration. Part (b) addressed the distinction between velocity and speed by presenting students with two statements that could appear contradictory, but in fact, are both true. The words "velocity" and "speed" are often used interchangeably in everyday language, but the technical distinction between the two is highlighted in calculus. Parts (c) and (d) also focused on this distinction. The total distance traveled by the object is calculated with a definite integral of speed (absolute value of velocity). Finding the position of the object at time $t = 4$ involves displacement, calculated with a definite integral of velocity.

Most students had little trouble with part (a). Part (b), however, was more difficult because of the confusion between velocity and speed. Very few students attempted to write and analyze an analytic expression for speed. Instead, many made observations based on the graph of the velocity and/or speed.

In part (c), not realizing that the object might change direction led some students to assume that the distance traveled was the same as the displacement. In addition, some students who attempted to do the problems analytically made mistakes in both algebra and arithmetic. Students who recognized that parts (c) and (d) could be done with the calculator to evaluate definite integrals saved themselves much time and trouble.

Question 3

An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

(a) What is the acceleration of the object at time $t = 4$?

(b) Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.

Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

(c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

(d) What is the position of the object at time $t = 4$?

Solution	Scoring
(a) $a(4) = v'(4) = \frac{\pi}{3}\cos\left(\frac{4\pi}{3}\right)$ $= -\frac{\pi}{6}$ or -0.523 or -0.524	1 : answer
(b) On $3 < t < 4.5$: $a(t) = v'(t) = \frac{\pi}{3}\cos\left(\frac{\pi}{3}t\right) < 0$ Statement I is correct since $a(t) < 0$. Statement II is correct since $v(t) < 0$ and $a(t) < 0$.	3 { 1 : I correct, with reason 1 : II correct 1 : reason for II
(c) Distance $= \int_0^4 \lvert v(t)\rvert\,dt = 2.387$ OR $x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + \frac{3}{\pi} + 2$ $x(0) = 2$ $x(4) = 2 + \frac{9}{2\pi} = 3.43239$ $v(t) = 0$ when $t = 3$ $x(3) = \frac{6}{\pi} + 2 = 3.90986$ $\lvert x(3) - x(0)\rvert + \lvert x(4) - x(3)\rvert = \frac{15}{2\pi} = 2.387$	3 { 1 : { limits of 0 and 4 on an integral of $v(t)$ or $\lvert v(t)\rvert$ or uses $x(0)$ and $x(4)$ to compute distance 1 : handles change of direction at student's turning point 1 : answer 0/1 if incorrect turning point or no turning point
(d) $x(4) = x(0) + \int_0^4 v(t)\,dt = 3.432$ OR $x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + \frac{3}{\pi} + 2$ $x(4) = 2 + \frac{9}{2\pi} = 3.432$	2 { 1 : integral 1 : answer OR 2 { 1 : $x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + C$ 1 : answer 0/1 if no constant of integration

AB/BC Question 4

In this problem, students were given a graphical representation of a function f, and another function g that was defined in terms of a definite integral of f. While it was possible to find piecewise algebraic definitions for f and g, the questions asked were most efficiently and directly answered by using the Fundamental Theorem of Calculus and reasoning based on the graph of f. Part (a) asked for calculations of $g(-1)$, $g'(-1)$, and $g''(-1)$. These values could be found using, respectively, an area, ordinate, and slope related to the graph of f. Using the fact that $f = g'$, part (b) required relating the sign of f (positive or negative) to the behavior of g (increasing or decreasing). Similarly, using the fact that $f' = g''$, part (c) required relating the behavior of the slope of the graph of f to the concavity of the graph of g. Part (d) asked for a sketch of the graph of g. Utilizing previous parts of the problem helped in determining characteristics of the graph.

Students were generally successful in completing part (a). The lower limit of 0 in the definite integral for f caused errors in computing $g(-1)$ if the student did not realize that the value was negative. A small percentage of students attempted to solve the problem analytically.

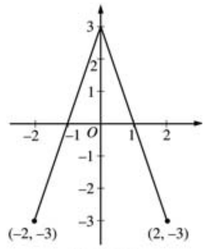
In parts (b) and (c), many students were able to identify the interval on which the function g was increasing and the interval on which the graph of g was concave down. Not all of these students were able to provide adequate reasons to support their conclusions. In particular, it was important for students to indicate the connection between the derivative of g and the function f.

Even students who did no correct work in parts (a) through (c) typically attempted to sketch the graph in part (d). A common error was to shift the correct graph up (or down) the y-axis, sometimes to be consistent with the incorrect computations in part (a).

So that students might perform better on this sort of problem, teachers are urged to continue to emphasize the Fundamental Theorem of Calculus as well as a graphical approach to problem solving. Students should work with functions defined by definite integrals in which the lower limit is not always the left endpoint of the interval of interest. Students continue to have difficulty properly justifying their conclusions and more experience with this would improve their performance on the exam. Students should practice mathematical writing skills to help communicate their reasoning and explanations in solutions.

Question 4

The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.



Graph of f

(a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

(b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

(c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

(d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.

(a) $g(-1) = \int_0^{-1} f(t)\,dt = -\int_{-1}^0 f(t)\,dt = -\frac{3}{2}$

$g'(-1) = f(-1) = 0$

$g''(-1) = f'(-1) = 3$

$$3 \begin{cases} 1: & g(-1) \\ 1: & g'(-1) \\ 1: & g''(-1) \end{cases}$$

(b) g is increasing on $-1 < x < 1$ because $g'(x) = f(x) > 0$ on this interval.

$$2 \begin{cases} 1: & \text{interval} \\ 1: & \text{reason} \end{cases}$$

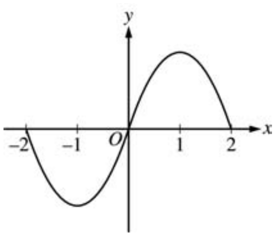
(c) The graph of g is concave down on $0 < x < 2$ because $g''(x) = f'(x) < 0$ on this interval.

or

because $g'(x) = f(x)$ is decreasing on this interval.

$$2 \begin{cases} 1: & \text{interval} \\ 1: & \text{reason} \end{cases}$$

(d)



$$2 \begin{cases} 1: & g(-2) = g(0) = g(2) = 0 \\ 1: & \text{appropriate increasing/decreasing} \\ & \text{and concavity behavior} \\ & <-1> \text{ vertical asymptote} \end{cases}$$

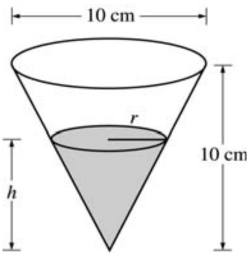
AB Question 5

This problem presented a common related rates setting with several variables (radius, depth, volume), related by geometry to the water evaporating in a conical container. Part (a) asked students to calculate the volume of water when its depth was $h = 5$ cm. The purpose of this part was to prompt students to establish the relationships among the radius, depth, and volume variables. Part (b) then asked students to relate the rate of change of the volume to the given rate of change of the depth of water. Part (c) introduced another related quantity, the exposed surface area, and asked students to verify a direct proportionality relationship between the rate of change of the volume and the exposed surface area of the water. The constant of proportionality in this case was precisely the given constant rate of change of the water's depth.

A number of students were unable to compute the volume because they failed to develop the proper relationship between radius and height. The easiest way to do part (b) was to write a formula for the volume in terms of a single variable, yet many students tried to solve this as a problem with two independent variables. This approach led to mistakes when they failed to recognize the need for the product rule or failed to correctly use the chain rule to include both $\frac{dh}{dt}$ and $\frac{dr}{dt}$.

Part (c) was a very difficult section for students because many did not know how to verify a direct proportionality relationship.

Question 5



A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

(The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

(a) When $h = 5$, $r = \frac{5}{2}$; $V(5) = \frac{1}{3}\pi\left(\frac{5}{2}\right)^2 5 = \frac{125}{12}\pi \text{ cm}^3$

$1: V$ when $h = 5$

(b) $\frac{r}{h} = \frac{5}{10}$, so $r = \frac{1}{2}h$

$$V = \frac{1}{3}\pi\left(\frac{1}{4}h^2\right)h = \frac{1}{12}\pi h^3;\ \frac{dV}{dt} = \frac{1}{4}\pi h^2\frac{dh}{dt}$$

$$\left.\frac{dV}{dt}\right|_{h=5} = \frac{1}{4}\pi(25)\left(-\frac{3}{10}\right) = -\frac{15}{8}\pi \ \text{cm}^3/\text{hr}$$

OR

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(r^2\frac{dh}{dt} + 2rh\frac{dr}{dt}\right);\ \frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}$$

$$\left.\frac{dV}{dt}\right|_{h=5,r=\frac{5}{2}} = \frac{1}{3}\pi\left(\left(\frac{25}{4}\right)\left(-\frac{3}{10}\right) + 2\left(\frac{5}{2}\right)5\left(-\frac{3}{20}\right)\right)$$

$$= -\frac{15}{8}\pi \ \text{cm}^3/\text{hr}$$

$$5\begin{cases} 1: r = \frac{1}{2}h \text{ in (a) or (b)} \\ 1: \begin{cases} V \text{ as a function of one variable} \\ \text{in (a) or (b)} \\ \qquad\text{OR} \\ \frac{dr}{dt} \end{cases} \\ 2: \frac{dV}{dt} \\ \quad <-2> \text{ chain rule or product rule error} \\ 1: \text{ evaluation at } h = 5 \end{cases}$$

(c) $\frac{dV}{dt} = \frac{1}{4}\pi h^2\frac{dh}{dt} = -\frac{3}{40}\pi h^2$

$$= -\frac{3}{40}\pi(2r)^2 = -\frac{3}{10}\pi r^2 = -\frac{3}{10}\cdot\text{area}$$

The constant of proportionality is $-\frac{3}{10}$.

$$2\begin{cases} 1: \text{ shows } \frac{dV}{dt} = k\cdot\text{area} \\ 1: \text{ identifies constant of proportionality} \end{cases}$$

---

units of cm^3 in (a) and cm^3/hr in (b)

1 : correct units in (a) and (b)

AB Question 6

This problem presented data in tabular form for a function f and its derivative f', along with information about the sign of the second derivative. Part (a) required students to use the Fundamental Theorem of Calculus and some basic properties of integrals to calculate a specific definite integral. Part (b) asked for the calculation of a linear approximation using appropriate data from the table. Students needed to interpret the sign of the second derivative in terms of concavity and relate this information to the tangent line. Part (c) required students to recognize that the Mean Value Theorem could be applied to f' to show the existence of a real number c such that $f''(c) = 6$, the average rate of change of f' over the interval [0,0.5]. Part (d) presented students with a piecewise algebraic expression for a function g that fit all of the given points on the graph of f. To determine that $f \neq g$, students needed to appeal to some inconsistency with the first or second derivatives of the respective functions.

Most students were able to correctly evaluate the integral in part (a), and those who could not either made no use of the Fundamental Theorem of Calculus or did not antidifferentiate the constant term correctly. Most students also earned the first two points in part (b), computing a correct tangent line and finding the correct approximation at $x = 1.2$, but many were not able to provide a valid reason for the correct conclusion about the comparison between the approximation and the actual function value. In part (c), few students made explicit reference to the Mean Value Theorem. In part (d), students realized that there was a problem at $x = 0$ but very few were able to give a correct and complete reason for their conclusion. The most common error was to claim that $g'(0) = 1$.

This is an example of a problem that emphasizes the multirepresentational approach to calculus and the connections between functions represented in tabular and analytical form. Theory continues to be an important ingredient in the understanding of calculus and needs to be integrated into problem solving strategies.

Question 6

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} \left(3f'(x) + 4\right) dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

(d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0. \end{cases}$

The graph of g passes through each of the points $\left(x, f(x)\right)$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

(a) $\int_0^{1.5} \left(3f'(x) + 4\right) dx = 3\int_0^{1.5} f'(x)\,dx + \int_0^{1.5} 4\,dx$

$= 3f(x) + 4x\Big|_0^{1.5} = 3(-1 - (-7)) + 4(1.5) = 24$

$2 \begin{cases} 1: \text{antiderivative} \\ 1: \text{answer} \end{cases}$

(b) $y = 5(x - 1) - 4$

$f(1.2) \approx 5(0.2) - 4 = -3$

The approximation is less than $f(1.2)$ because the graph of f is concave up on the interval $1 < x < 1.2$.

$3 \begin{cases} 1: \text{tangent line} \\ 1: \text{computes } y \text{ on tangent line at } x = 1.2 \\ 1: \text{answer with reason} \end{cases}$

(c) By the Mean Value Theorem there is a c with $0 < c < 0.5$ such that

$f''(c) = \dfrac{f'(0.5) - f'(0)}{0.5 - 0} = \dfrac{3 - 0}{0.5} = 6 = r$

$2 \begin{cases} 1: \text{reference to MVT for } f' \text{ (or differentiability of } f'\text{)} \\ 1: \text{value of } r \text{ for interval } 0 \le x \le 0.5 \end{cases}$

(d) $\lim_{x \to 0^-} g'(x) = \lim_{x \to 0^-} (4x - 1) = -1$

$\lim_{x \to 0^+} g'(x) = \lim_{x \to 0^+} (4x + 1) = +1$

Thus g' is not continuous at $x = 0$, but f' is continuous at $x = 0$, so $f \ne g$.

OR

$g''(x) = 4$ for all $x \ne 0$, but it was shown in part (c) that $f''(c) = 6$ for some $c \ne 0$, so $f \ne g$.

$2 \begin{cases} 1: \text{answers "no" with reference to } g' \text{ or } g'' \\ 1: \text{correct reason} \end{cases}$

Calculus BC Sample Free-Response Questions

AB/BC Question 1

This problem presented a region bounded between two graphs and two vertical lines. Students were asked to use both integration and differentiation to answer some straightforward questions about this region. Part (a) required the use of a definite integral to find the area of the region. In part (b), the region is revolved about a horizontal line, resulting in a solid with cross sections in the shape of “washers,” and the volume of the region was asked for, requiring another use of a definite integral. Students were expected to use the numerical integration capabilities of a graphing calculator to evaluate these definite integrals. Part (c) required the use of differentiation to find the absolute minimum height of the region.

Many students were successful in finding the area of the region but had difficulty setting up the volume integral because of the complication of rotating around a horizontal line that was not the x-axis. The students did not need to compute antiderivatives to compute the definite integrals in parts (a) and (b); the calculator may be used to calculate the value of the definite integral without further explanation once the setup of the definite integral is shown.

In part (c), students were expected to show the mathematical steps needed to analyze the location of the absolute minimum and maximum. Some students neglected to consider where $h'(x) = 0$ to find the critical point, and some did not consider both endpoints as candidates.

It is important for students to show their work as described in “Showing Work on the Free-Response Sections” on page 18. For example, when finding a critical point of a function, students should explicitly state the equation that is being solved for determining where the derivative is equal to 0, rather than expect that the equation will be inferred from a graph or a sign chart.

Question 1

Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

(a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

(b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

(c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

(a) Area $= \int_{1/2}^{1} (e^x - \ln x)\,dx = 1.222$ or 1.223

$$2\begin{cases} 1: \text{ integral} \\ 1: \text{ answer} \end{cases}$$

(b) Volume $= \pi \int_{1/2}^{1} \left((4 - \ln x)^2 - (4 - e^x)^2\right) dx$

$= 7.515\pi$ or 23.609

$$4\begin{cases} 1: \text{ limits and constant} \\ 2: \text{ integrand} \\ \quad\; < -1 > \text{ each error} \\ \quad\; \text{Note: } 0/2 \text{ if not of the form} \\ \quad\; k\int_a^b \left(R(x)^2 - r(x)^2\right) dx \\ 1: \text{ answer} \end{cases}$$

(c) $h'(x) = f'(x) - g'(x) = e^x - \dfrac{1}{x} = 0$

$x = 0.567143$

Absolute minimum value and absolute maximum value occur at the critical point or at the endpoints.

$h(0.567143) = 2.330$
$h(0.5) = 2.3418$
$h(1) = 2.718$

The absolute minimum is 2.330.
The absolute maximum is 2.718.

$$3\begin{cases} 1: \text{ considers } h'(x) = 0 \\ 1: \text{ identifies critical point} \\ \quad\; \text{and endpoints as candidates} \\ 1: \text{ answers} \end{cases}$$

Note: Errors in computation come off the third point.

AB/BC Question 2

This problem involved a "real-life" model of people entering and leaving an amusement park, and students were expected to interpret the meanings of their calculations in that context. In part (a), students needed to recognize that the function E represented the rate of accumulation of people entering the park, and hence that the total accumulation over a time interval could be obtained by a definite integral. Part (b) took the same idea a step further, asking students to calculate ticket revenues based on different pricing over two time intervals. Part (c) presented a function H defined in terms of a definite integral. The value of $H'(17)$ was easily computed, provided students recognized that the Fundamental Theorem of Calculus could be applied. More importantly, students were expected to interpret, in words, the meanings of both $H(17)$ and $H'(17)$. These interpretations played a role in part (d), which essentially asked students to find when the maximum value of the function H is achieved.

The most common error in part (a) was to evaluate $E(17)$ or $E(17) - L(17)$, or to attempt a discrete analysis without seeing the connection to a definite integral. The most common error in calculating $H'(17)$ was not recognizing the need to use the Fundamental Theorem of Calculus or not using the theorem correctly. A large number of students used $H'(17) = E'(17) - L'(17)$. The interpretations of $H(17)$ and $H'(17)$ were difficult for students to make. Those who recognized the conceptual connection that $H(t)$ measured population in the park at time t and phrased $H(17)$ as a population had an easier time with $H'(17)$.

While students could integrate by hand or with a CAS capable calculator to find a closed form expression for the number of people who entered the park and the number of people who left the park, this approach was neither necessary nor efficient to handle the questions in this problem.

It is important for students to clearly show the setup for all definite integrals that must be evaluated numerically. As in AB/BC Question 1, when finding a critical point of a function, students should explicitly state the equation that is being solved for determining where the derivative is equal to 0, rather than expect that the equation will be inferred from a graph or a sign chart.

One of the goals of the exam is to emphasize problems that probe understanding of the fundamental concepts and not to merely test rote manipulation. This problem was an example of one that required students to make connections between calculus and the "real" world, and between topics within calculus itself.

Question 2

The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

Solution	Scoring
(a) $\int_9^{17} E(t)\,dt = 6004.270$ 6004 people entered the park by 5 pm.	$3\begin{cases} 1: \text{ limits} \\ 1: \text{ integrand} \\ 1: \text{ answer} \end{cases}$
(b) $15\int_9^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt = 104048.165$ The amount collected was \$104,048. or $\int_{17}^{23} E(t)\,dt = 1271.283$ 1271 people entered the park between 5 pm and 11 pm, so the amount collected was $\$15 \cdot (6004) + \$11 \cdot (1271) = \$104,041.$	1 : setup
(c) $H'(17) = E(17) - L(17) = -380.281$ There were 3725 people in the park at $t = 17$. The number of people in the park was decreasing at the rate of approximately 380 people/hr at time $t = 17$.	$3\begin{cases} 1: \text{ value of } H'(17) \\ 2: \text{ meanings} \\ \quad 1: \text{ meaning of } H(17) \\ \quad 1: \text{ meaning of } H'(17) \\ \quad < -1 > \text{ if no reference to } t = 17 \end{cases}$
(d) $H'(t) = E(t) - L(t) = 0$ $t = 15.794$ or 15.795	$2\begin{cases} 1: E(t) - L(t) = 0 \\ 1: \text{ answer} \end{cases}$

BC Question 3

This problem involving parametric equations described the motion of a roller coaster car. Both coordinates of the car's position were given. While the components of the velocity vector could be calculated directly, these were also stated in the problem. The emphasis in this problem was on using the velocity vector's components in a variety of ways to describe characteristics of the car's motion and its path at various times. Part (a) asked for the slope of the car's path at time $t = 2$, requiring students to formulate the evaluation of $\frac{dy}{dx}$ in terms of $\frac{dy}{dt}$ and $\frac{dx}{dt}$. Part (b) asked students to first determine the time at which the car is at a specific horizontal position, a calculation that required the numerical equation solver of a graphing calculator. Once this time was determined, students needed to use the relationship between the velocity and its derivative to determine the components of the acceleration vector. Part (c) also required the use of a graphing calculator to determine the times at which $y'(t) = 0$. At this instant, the speed of the car, given by the magnitude of the velocity vector, is $|x'(t)|$. Students may or may not have used a graphing calculator to find the times at which $y(t) = 0$ in part (d). In either case, an appropriate definite integral expression needed to be given whose evaluation would result in the average speed of the car over the interval defined by these times.

In part (a), students were generally successful in being able to find the slope at a given point from the set of parametric equations. In part (b), most students recognized that the components of the velocity vector needed to be individually differentiated, but some students then treated these components incorrectly. Some students also found it difficult to set up and/or compute an expression that represented the speed of the car at a particular time.

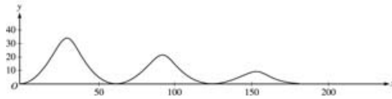
Parts (b), (c), and (d) asked students to compute the acceleration vector, speed, and average speed based on time values that also had to be computed from information given in the problem. Students needed to be careful to make sure the time values they decided to work with were within the domain specified in the stem of the problem.

A number of students reported that they had run out of time and had returned to this section of the exam without their calculators. This should not have affected their opportunity to earn all available points in part (d), where the equation was easily solved without the use of technology.

The number of students who worked at least part of this problem with their calculator set to degree mode was higher than expected. Teachers are urged to remind their students to set their calculators to radian mode before entering the examination room. Teachers are also encouraged to reinforce the difference between vector values (such as a velocity vector) and scalar values (such as speed).

Question 3

The figure above shows the path traveled by a roller coaster car over the time interval $0 \le t \le 18$ seconds. The position of the car at time t seconds can be modeled parametrically by $x(t) = 10t + 4\sin t,\ y(t) = (20 - t)(1 - \cos t)$, where x and y are measured in meters. The derivatives of these functions are given by



$x'(t) = 10 + 4\cos t,\ y'(t) = (20 - t)\sin t + \cos t - 1.$

(a) Find the slope of the path at time $t = 2$. Show the computations that lead to your answer.

(b) Find the acceleration vector of the car at the time when the car's horizontal position is $x = 140$.

(c) Find the time t at which the car is at its maximum height, and find the speed, in m/sec, of the car at this time.

(d) For $0 < t < 18$, there are two times at which the car is at ground level $(y = 0)$. Find these two times and write an expression that gives the average speed, in m/sec, of the car between these two times. Do not evaluate the expression.

(a) Slope $= \left.\dfrac{dy}{dx}\right|_{t=2} = \dfrac{y'(2)}{x'(2)} = \dfrac{18\sin 2 + \cos 2 - 1}{10 + 4\cos 2}$

$= 1.793$ or 1.794

$1:$ answer using $\dfrac{dy}{dx} = \dfrac{dy}{dt}\Big/\dfrac{dx}{dt}$

(b) $x(t) = 10t + 4\sin t = 140;\ t_0 = 13.647083$

$x''(t_0) = -3.529,\ y''(t_0) = 1.225$ or 1.226

Acceleration vector is $\langle -3.529, 1.225 \rangle$

or $\langle -3.529, 1.226 \rangle$

$2 \begin{cases} 1: \text{identifies acceleration vector as derivative of velocity vector} \\ 1: \text{computes acceleration vector when } x = 140 \end{cases}$

(c) $y'(t) = (20 - t)\sin t + \cos t - 1 = 0$

$t_1 = 3.023$ or 3.024 at maximum height

Speed $= \sqrt{\left(x'(t_1)\right)^2 + \left(y'(t_1)\right)^2} = |x'(t_1)|$

$= 6.027$ or 6.028

$3 \begin{cases} 1: \text{sets } y'(t) = 0 \\ 1: \text{selects first } t > 0 \\ 1: \text{speed} \end{cases}$

(d) $y(t) = 0$ when $t = 2\pi$ and $t = 4\pi$

Average speed $= \dfrac{1}{2\pi}\displaystyle\int_{2\pi}^{4\pi} \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\, dt$

$= \dfrac{1}{2\pi}\displaystyle\int_{2\pi}^{4\pi} \sqrt{(10 + 4\cos t)^2 + ((20 - t)\sin t + \cos t - 1)^2}\, dt$

$3 \begin{cases} 1: t = 2\pi, t = 4\pi \\ 1: \text{limits and constant} \\ 1: \text{integrand} \end{cases}$

AB/BC Question 4

In this problem, students were given a graphical representation of a function f, and another function g that was defined in terms of a definite integral of f. While it was possible to find piecewise algebraic definitions for f and g, the questions asked were most efficiently and directly answered by using the Fundamental Theorem of Calculus and reasoning based on the graph of f. Part (a) asked for calculations of $g(-1)$, $g'(-1)$, and $g''(-1)$. These values could be found using, respectively, an area, ordinate, and slope related to the graph of f. Using the fact that $f = g'$, part (b) required relating the sign of f (positive or negative) to the behavior of g (increasing or decreasing). Similarly, using the fact that $f' = g''$, part (c) required relating the behavior of the slope of the graph of f to the concavity of the graph of g. Part (d) asked for a sketch of the graph of g. Utilizing previous parts of the problem helped in determining characteristics of the graph.

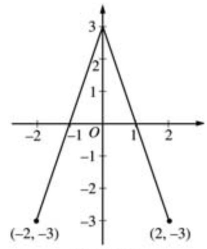
Students were generally successful in completing part (a). The lower limit of 0 in the definite integral for f caused errors in computing $g(-1)$ if the student did not realize that the value was negative. A small percentage of students attempted to solve the problem analytically.

In parts (b) and (c), many students were able to identify the interval on which the function g was increasing and the interval on which the graph of g was concave down. Not all of these students were able to provide adequate reasons to support their conclusions. In particular, it was important for students to indicate the connection between the derivative of g and the function f.

Even students who did no correct work in parts (a) through (c) typically attempted to sketch the graph in part (d). A common error was to shift the correct graph up (or down) the y-axis, sometimes to be consistent with the incorrect computations in part (a).

So that students might perform better on this sort of problem, teachers are urged to continue to emphasize the Fundamental Theorem of Calculus as well as a graphical approach to problem solving. Students should work with functions defined by definite integrals in which the lower limit is not always the left endpoint of the interval of interest. Students continue to have difficulty properly justifying their conclusions and more experience with this would improve their performance on the exam. Students should practice mathematical writing skills to help communicate their reasoning and explanations in solutions.

Question 4



Graph of f

The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

(b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

(c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

(d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.

(a) $g(-1) = \int_0^{-1} f(t)\,dt = -\int_{-1}^{0} f(t)\,dt = -\frac{3}{2}$

$g'(-1) = f(-1) = 0$

$g''(-1) = f'(-1) = 3$

$$3 \begin{cases} 1: & g(-1) \\ 1: & g'(-1) \\ 1: & g''(-1) \end{cases}$$

(b) g is increasing on $-1 < x < 1$ because $g'(x) = f(x) > 0$ on this interval.

$$2 \begin{cases} 1: & \text{interval} \\ 1: & \text{reason} \end{cases}$$

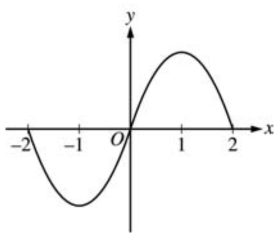
(c) The graph of g is concave down on $0 < x < 2$ because $g''(x) = f'(x) < 0$ on this interval.

or

because $g'(x) = f(x)$ is decreasing on this interval.

$$2 \begin{cases} 1: & \text{interval} \\ 1: & \text{reason} \end{cases}$$

(d)



$$2 \begin{cases} 1: & g(-2) = g(0) = g(2) = 0 \\ 1: & \text{appropriate increasing/decreasing} \\ & \text{and concavity behavior} \\ & < -1 > \text{ vertical asymptote} \end{cases}$$

BC Question 5

A differential equation was presented in this problem. Part (a) asked for two solution curves sketched against a slope field provided for the differential equation. Part (b) took one of the initial conditions from part (a) and asked for a demonstration of the use of Euler's method to approximate another point on the solution curve. In part (c), students were asked to find the y-intercept of a linear solution to the differential equation. One of the two solution curves sketched in part (a) should be a straight line, so there was a strong visual clue for students to use in checking the reasonableness of their answers. The slope field also provided strong visual clues for part (d), but students needed to provide more than a visual explanation to justify that the solution curve passing through the origin has a local maximum there. The second derivative test was the most straightforward way to justify this result.

It can be difficult to get the precise solution curve when sketching on a slope field. Therefore the points in part (a) were awarded for the right "location," the right "extent," and the right "shape," with close calls going in the student's favor. Some students had difficulty because they seemed to be drawing their curves from left to right instead of working from the given initial point outward. In part (b), there were some students who did not know how to calculate the value of the derivative from the differential equation for use in Euler's method.

In part (c), it was important for the students to argue from the differential equation rather than just graphically. While many students determined that $b = 1$, few could present a complete, correct argument to earn both points.

It was extremely difficult to earn the justification points in part (d) via a first derivative test based on changes in the sign of the derivative of the function g. Students needed to make use of the differential equation to justify a conclusion, not just appeal to behavior discerned from the slope field.

More attention should be given to the concept of a differential equation, emphasizing the fundamental fact that a solution to a differential equation is a function that satisfies the equation. In addition, students should be presented with many opportunities to use the second derivative test instead of the first derivative test, including cases where the second derivative test is much easier to use or the first derivative test is not applicable.

It is important for students to clearly indicate the steps involved in using Euler's method. When asked to justify a local extremum, students should show why all appropriate conditions about the first and/or second derivatives are satisfied, and explain in words the conclusion obtained from a first or second derivative test.

Question 5

Consider the differential equation $\frac{dy}{dx} = 2y - 4x$.

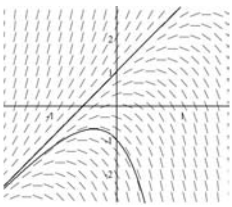
(a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point $(0, 1)$ and sketch the solution curve that passes through the point $(0, -1)$.

(b) Let f be the function that satisfies the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

(c) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.

(d) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point $(0, 0)$? If so, is the point a local maximum or a local minimum? Justify your answer.

(a)



$2 \begin{cases} 1: \text{ solution curve through } (0,1) \\ 1: \text{ solution curve through } (0,-1) \end{cases}$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

(b) $f(0.1) \approx f(0) + f'(0)(0.1)$

$= 1 + (2 - 0)(0.1) = 1.2$

$f(0.2) \approx f(0.1) + f'(0.1)(0.1)$

$\approx 1.2 + (2.4 - 0.4)(0.1) = 1.4$

$2 \begin{cases} 1: \text{ Euler's method equations or equivalent table applied to (at least) two iterations} \\ 1: \text{ Euler approximation to } f(0.2) \text{ (not eligible without first point)} \end{cases}$

(c) Substitute $y = 2x + b$ in the DE:

$2 = 2(2x + b) - 4x = 2b$, so $b = 1$

OR

Guess $b = 1$, $y = 2x + 1$

Verify: $2y - 4x = (4x + 2) - 4x = 2 = \frac{dy}{dx}$.

$2 \begin{cases} 1: \text{ uses } \frac{d}{dx}(2x + b) = 2 \text{ in DE} \\ 1: \ b = 1 \end{cases}$

(d) g has local maximum at $(0, 0)$.

$g'(0) = \left.\frac{dy}{dx}\right|_{(0,0)} = 2(0) - 4(0) = 0$, and

$g''(x) = \frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 4$, so

$g''(0) = 2\,g'(0) - 4 = -4 < 0$.

$3 \begin{cases} 1: \ g'(0) = 0 \\ 1: \text{ shows } g''(0) = -4 \\ 1: \text{ conclusion} \end{cases}$

BC Question 6

This problem presented students with an explicit Maclaurin series for a function f. In part (a), students were asked to determine its interval of convergence. Part (b) then asked students to derive a Maclaurin series for f' by manipulating the given series. Finally, in part (c), the evaluation of the Maclaurin series for f' at the specific value $x = -1/3$ resulted in a geometric series whose sum could be found by a simple calculation.

Most students got a good start on part (a) with a ratio test setup, algebraic work, and a computation of the limit. Some then made algebra mistakes, however, in solving the inequality for x. Students were explicitly asked to justify their answer for the interval of convergence. The majority knew that they needed to test the endpoints but had trouble earning the justification points. A common mistake occurred when students tried to use a direct comparison test for the right endpoint to compare their series with the harmonic series. The comparison goes the wrong way to conclude divergence. Many in fact failed to recognize that their series at the right endpoint was exactly the harmonic series.

In part (b), students usually stated the first four terms correctly but many failed to get the correct general term because of a chain rule error. In part (c), students tended to sum the first four terms only.

Using an infinite series to find the value of a function is a concept that some students still need practice with. It is also important to stress the use of appropriate and correct mathematical notation to indicate the steps in a ratio test, particularly when taking the limit.

Question 6

The Maclaurin series for the function f is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n+1} = 2x + \frac{4x^2}{2} + \frac{8x^3}{3} + \frac{16x^4}{4} + \cdots + \frac{(2x)^{n+1}}{n+1} + \cdots$$

on its interval of convergence.

(a) Find the interval of convergence of the Maclaurin series for f. Justify your answer.

(b) Find the first four terms and the general term for the Maclaurin series for $f'(x)$.

(c) Use the Maclaurin series you found in part (b) to find the value of $f'\left(-\frac{1}{3}\right)$.

(a) $$\lim_{n\to\infty}\left|\frac{\dfrac{(2x)^{n+2}}{n+2}}{\dfrac{(2x)^{n+1}}{n+1}}\right| = \lim_{n\to\infty}\left|\frac{(n+1)}{(n+2)}2x\right| = \left|2x\right|$$

$\left|2x\right| < 1$ for $-\frac{1}{2} < x < \frac{1}{2}$

At $x = \frac{1}{2}$, the series is $\sum_{n=0}^{\infty}\frac{1}{n+1}$ which diverges since this is the harmonic series.

At $x = -\frac{1}{2}$, the series is $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{1}{n+1}$ which converges by the Alternating Series Test.

Hence, the interval of convergence is $-\frac{1}{2} \le x < \frac{1}{2}$.

5 points:
- 1 : sets up ratio
- 1 : computes limit of ratio
- 1 : identifies interior of interval of convergence
- 2 : analysis/conclusion at endpoints
  - 1 : right endpoint
  - 1 : left endpoint
- $< -1 >$ if endpoints not $x = \pm\frac{1}{2}$
- $< -1 >$ if multiple intervals

(b) $f'(x) = 2 + 4x + 8x^2 + 16x^3 + \ldots + 2(2x)^n + \ldots$

2 points:
- 1 : first 4 terms
- 1 : general term

(c) The series in (b) is a geometric series.

$$\begin{aligned} f'\left(-\frac{1}{3}\right) &= 2 + 4\left(-\frac{1}{3}\right) + 8\left(-\frac{1}{3}\right)^2 + \ldots + 2\left(2\cdot\left(-\frac{1}{3}\right)\right)^n + \ldots \\ &= 2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \ldots + 2\left(-\frac{2}{3}\right)^n + \ldots \\ &= \frac{2}{1+\frac{2}{3}} = \frac{6}{5} \end{aligned}$$

OR

$f'(x) = \frac{2}{1-2x}$ for $-\frac{1}{2} < x < \frac{1}{2}$. Therefore,

$$f'\left(-\frac{1}{3}\right) = \frac{2}{1+\frac{2}{3}} = \frac{6}{5}$$

2 points:
- 1 : substitutes $x = -\frac{1}{3}$ into infinite series from (b) or expresses series from (b) in closed form
- 1 : answer for student's series

AP® Program Essentials

The AP Reading

In June, the free-response sections of the exams, as well as the Studio Art portfolios, are scored by college faculty and secondary school AP teachers at the AP Reading. Thousands of readers participate, under the direction of a Chief Reader in each field. The experience offers both significant professional development and the opportunity to network with like-minded educators.

If you are an AP teacher or a college faculty member and would like to serve as a reader, you can visit AP Central for more information on how to apply. Alternatively, send an e-mail message to apreader@ets.org, or call Performance Scoring Services at 609 406-5383.

AP Grades

The readers' scores on the essay and problem-solving questions are combined with the results of the computer-scored multiple-choice questions, and the total raw scores are converted to AP's 5-point scale:

AP GRADE	QUALIFICATION
5	Extremely well qualified
4	Well qualified
3	Qualified
2	Possibly qualified
1	No recommendation

Grade Distributions

Many teachers want to compare their students' grades with the national percentiles. Grade distribution charts are available at AP Central, as is information on how the cut-off points for each AP grade are calculated. Grade distribution charts are also available on the AP student site at www.collegeboard.com/apstudents.

Earning College Credit and/or Placement

Credit, advanced placement, or both are awarded by the college or university, not the College Board or the AP Program. The best source of specific and up-to-date information about an individual institution's policy is its catalog or Web site.

Why Colleges Grant Credit and/or Placement for AP Grades

Colleges know that the AP grades of their incoming students represent a level of achievement equivalent to that of students who take the same course in the colleges' own classrooms. That equivalency is assured through several Advanced Placement Program processes:

- College faculty serve on the committees that develop the course descriptions and examinations in each AP subject.
- College faculty are responsible for standard setting and are involved in the evaluation of student responses at the AP Reading.
- AP courses and exams are updated regularly, based on both the results of curriculum surveys at up to 200 colleges and universities and the interactions of committee members with professional organizations in their discipline.
- College comparability studies are undertaken in which the performance of college students on AP Exams is compared with that of AP students to confirm that the AP grade scale of 1–5 is properly aligned with current college standards.

In addition, the College Board has commissioned studies that use a "bottom-line" approach to validating AP Exam grades by comparing the achievement of AP versus non-AP students in higher-level college courses. For example, in the 1998 Morgan and Ramist "21-College" study, AP students who were exempted from introductory courses and who completed a higher-level course in college were compared favorably, on the basis of their college grades, with students who completed the prerequisite first course in college, then took the second, higher-level course in the subject area. Such studies answer the question of greatest concern to colleges — are AP students who are exempted from introductory courses as well prepared to continue in a subject area as students who took their first course in college? To see the results of several college validity studies, go to AP Central. (The Morgan and Ramist study can be downloaded from the site in its entirety.)

Guidelines on Granting Credit and/or Placement for AP Grades

If you are an admissions administrator and need guidance on setting an AP policy for your college or university, you will find the *College and University Guide to the Advanced Placement Program* useful; see the back of this booklet for ordering information. Alternatively, contact your local College Board office, as noted on the inside back cover of this Course Description.

Finding Colleges That Accept AP Grades

In addition to contacting colleges directly for their AP policies, students and teachers can use College Search, an online resource maintained by the College Board through its Annual Survey of Colleges. College Search can be accessed via the College Board's Web site (www.collegeboard.com). It is worth remembering that policies are subject to change. Contact the college directly to get the most up-to-date information.

AP Awards

The AP Program offers a number of awards to recognize high school students who have demonstrated college-level achievement through AP courses and exams. Although there is no monetary award, in addition to an award certificate, student achievement is acknowledged on any grade report sent to colleges following the announcement of the awards. For detailed information on AP Awards, including qualification criteria, visit AP Central or contact the College Board's National Office. Students can find this information at www.collegeboard.com/apstudents.

AP Calendar

The *AP Program Guide* and the *Bulletin for AP Students and Parents* provide education professionals and students, respectively, with information on the various events associated with the AP year. Information on ordering and downloading these publications can be found at the back of this booklet.

Test Security

The entire AP Exam must be kept secure at all times. Forty-eight hours after the exam has been administered, the green and blue inserts containing the free-response questions (Section II) can be made available for teacher and student review.* **However, the multiple-choice section (Section I) MUST remain secure both before and after the exam administration.** No one other than students taking the exam can ever have access to or see the questions contained in Section 1 — this includes AP Coordinators and all teachers. The multiple-choice section must never be shared, copied in any manner, or reconstructed by teachers and students after the exam.

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*The alternate form of the free-response section (used for late testing administration) is NOT released.

Selected multiple-choice questions are reused from year to year to provide an essential method of establishing high exam reliability, controlled levels of difficulty, and comparability with earlier exams. These goals can be attained only when the multiple-choice questions remain secure. This is why teachers cannot view the questions and students cannot share information about these questions with anyone following the exam administration.

To ensure that all students have an equal opportunity to demonstrate their abilities on the exam, AP Exams must be administered in a uniform manner. **It is extremely important to follow the administration schedule and all procedures outlined in detail in the most recent *AP Coordinator's Manual*.** Please note that Studio Art portfolios and their contents are not considered secure testing materials; see the *AP Coordinator's Manual* for further information. The manual also includes directions on how to deal with misconduct and other security problems. Any breach of security should be reported to Test Security immediately (call 800 353-8570, fax 609 406-9709, or e-mail tsreturns@ets.org).

Teacher Support

You can find the following Web resources at AP Central:

- Teachers' Resources (reviews of classroom resources).
- Institutes & Workshops (a searchable database of professional development opportunities).
- The most up-to-date and comprehensive information on AP courses, exams, and other Program resources.
- The opportunity to exchange teaching methods and materials with the international AP community using electronic discussion groups (EDGs).
- An electronic library of AP publications, including released exam questions, the *AP Coordinator's Manual*, Course Descriptions, and sample syllabi.
- Opportunities for professional involvement in the AP Program.
- Information about state and federal support for the AP Program.
- AP Program data, research, and statistics.
- FAQs about the AP Program.
- Current news and features about the AP Program, its courses and teachers.

AP teachers can also use a number of AP publications, CD-ROMs, and videos that supplement these Web resources. Please see the following pages for an overview and ordering information.

Pre-AP®

Pre-AP® is a suite of K–12 professional development resources and services to equip middle and high school teachers with the strategies and tools they need to engage their students in high-level learning, thereby ensuring that every middle and high school student has the depth and understanding of the skills, habits of mind, and concepts they need to succeed in college.

Pre-AP rests upon a profound hope and heartfelt esteem for teachers and students. Conceptually, Pre-AP is based on two important premises. The first is the expectation that all students can perform at rigorous academic levels. This expectation should be reflected in curriculum and instruction throughout the school such that all students are consistently being challenged to expand their knowledge and skills to the next level.

The second is the belief that we can prepare every student for higher intellectual engagement by starting the development of skills and acquisition of knowledge as early as possible. Addressed effectively, the middle and high school years can provide a powerful opportunity to help all students acquire the knowledge, concepts, and skills needed to engage in a higher level of learning.

Since Pre-AP teacher professional development supports explicitly the goal of college as an option for every student, it is important to have a recognized standard for college-level academic work. The Advanced Placement Program (AP) provides these standards for Pre-AP. Pre-AP teacher professional development resources reflect topics, concepts, and skills found in AP courses.

The College Board does not design, develop, or assess courses labeled "Pre-AP." Courses labeled "Pre-AP" that inappropriately restrict access to AP and other college-level work are inconsistent with the fundamental purpose of the Pre-AP initiatives of the College Board. We encourage schools, districts, and policymakers to utilize Pre-AP professional development in a manner that ensures equitable access to rigorous academic experiences for all students.

Pre-AP Professional Development

Pre-AP professional development is administered by Pre-AP Initiatives, a unit in K–12 Professional Development, and is available through workshops and conferences coordinated by the regional offices of the College Board. Pre-AP professional development is divided into two categories:

1. **Articulation of content and pedagogy across the middle and high school years —** The emphasis of professional development in this category is aligning curriculum and improving teacher communication. The intended outcome from articulation is a coordinated program of teaching skills and concepts over several years.
2. **Classroom strategies for middle and high school teachers —** Various approaches, techniques, and ideas are emphasized in professional development in the category.

For a complete list of Pre-AP Professional Development offerings, please contact your regional office or visit AP Central at apcentral.collegeboard.com.

AP Publications and Other Resources

A number of AP resources are available to help students, parents, AP Coordinators, and high school and college faculty learn more about the AP Program and its courses and exams. To identify resources that may be of particular use to you, refer to the following key.

AP Coordinators and Administrators .... **A**
College Faculty .... **C**
Students and Parents .... **SP**
Teachers .... **T**

Ordering Information

You have several options for ordering publications:

- **Online.** Visit the College Board store at store.collegeboard.com.
- **By mail.** Send a completed order form with your payment or credit card information to: Advanced Placement Program, Dept. E-06, P. O. Box 6670, Princeton, NJ 08541-6670. If you need a copy of the order form, you can download one from AP Central.

- **By fax.** Credit card orders can be faxed to AP Order Services at 609 771-7385.
- **By phone.** Call AP Order Services at 609 771-7243, Monday through Friday, 8:00 a.m. to 9:00 p.m. ET. Have your American Express, Discover, JCB, MasterCard, or VISA information ready. This phone number is for credit card orders only.

Payment must accompany all orders not on an institutional purchase order or credit card, and checks should be made payable to the College Board. The College Board pays UPS ground rate postage (or its equivalent) on all prepaid orders; delivery generally takes two to three weeks. Please do not use P.O. Box numbers. Postage will be charged on all orders requiring billing and/or requesting a faster method of delivery.

Publications may be returned for a full refund if they are returned within 30 days of invoice. Software and videos may be exchanged within 30 days if they are opened, or returned for a full refund if they are unopened. No collect or C.O.D. shipments are accepted. Unless otherwise specified, orders will be filled with the currently available edition; prices and discounts are subject to change without notice.

In compliance with Canadian law, all AP publications delivered to Canada incur the 7 percent GST. The GST registration number is 13141 4468 RT. Some Canadian schools are exempt from paying the GST. Appropriate proof of exemption must be provided when AP publications are ordered so that tax is not applied to the billing statement.

Print

Items marked with a computer mouse icon can be downloaded for free from AP Central.

Bulletin for AP Students and Parents **SP**

This bulletin provides a general description of the AP Program, including how to register for AP courses, and information on the policies and procedures related to taking the exams. It describes each AP Exam, lists the advantages of taking the exams, describes the grade reporting process, and includes the upcoming exam schedule. The *Bulletin* is available in both English and Spanish.

AP Program Guide **A**

This guide takes the AP Coordinator step-by-step through the school year — from organizing an AP program, through ordering and administering the AP Exams, payment, and grade reporting. It also includes information on teacher professional development, AP resources, and exam schedules. The *AP Program Guide* is sent automatically to all schools that register to participate in AP.

College and University Guide to the AP Program **C, A**

This guide is intended to help college and university faculty and administrators understand the benefits of having a coherent, equitable AP policy. Topics included are validity of AP grades; developing and maintaining scoring standards; ensuring equivalent achievement; state legislation supporting AP; and quantitative profiles of AP students by each AP subject.

Course Descriptions **SP, T, A, C**

Course Descriptions provide an outline of the AP course content, explain the kinds of skills students are expected to demonstrate in the corresponding introductory college-level course, and describe the AP Exam. They also provide sample multiple-choice questions with an answer key, as well as sample free-response questions. Note: The Course Description for AP Computer Science is available in electronic format only.

Pre-AP **A, T**

This brochure describes the Pre-AP concept and the professional development opportunities available to middle school and high school teachers.

Released Exams **T**

About every four to five years, on a rotating schedule, the AP Program releases a complete copy of each exam. In addition to providing the multiple-choice questions and answers, the publication describes the process of scoring the free-response questions and includes examples of students' actual responses, the scoring guidelines, and commentary that explains why the responses received the scores they did.

Teacher's Guides **T**

For those about to teach an AP course for the first time, or for experienced AP teachers who would like to get some fresh ideas for the classroom, the Teacher's Guide is an excellent resource. Each Teacher's Guide contains syllabi developed by high school teachers currently teaching the

AP course and college faculty who teach the equivalent course at colleges and universities. Along with detailed course outlines and innovative teaching tips, you'll also find extensive lists of suggested teaching resources.

AP Vertical Team Guides **T, A**

An AP Vertical Team (APVT) is made up of teachers from different grade levels who work together to develop and implement a sequential curriculum in a given discipline. The team's goal is to help students acquire the skills necessary for success in AP. To help teachers and administrators who are interested in establishing an APVT at their school, the College Board has published these guides: *A Guide for Advanced Placement English Vertical Teams*; *Advanced Placement Program Mathematics Vertical Teams Toolkit*; *AP Vertical Teams in Science*, *Social Studies*, *Foreign Language*, *Studio Art*, *and Music Theory: An Introduction*; *AP Vertical Teams Guide for Social Studies; AP Vertical Teams Guide for Fine Arts*, *Vol.1: Studio Art*; *AP Vertical Teams Guide for Fine Arts*, *Vol. 2: Music Theory*; and *AP Vertical Teams Guide for Fine Arts*, *Vol.1 and 2* (set).

Multimedia

APCD® (home version),
(multi-network site license) **SP, T**

These CD-ROMs are available for Calculus AB, English Language, English Literature, European History, Spanish Language, and U.S. History. They each include actual AP Exams, interactive tutorials, and other features, including exam descriptions, answers to frequently asked questions, study-skill suggestions, and test-taking strategies. There is also a listing of resources for further study and a planner to help students schedule and organize their study time.

The teacher version of each CD, which can be licensed for up to 50 workstations, enables you to monitor student progress and provide individual feedback. Included is a Teacher's Manual that gives full explanations along with suggestions for utilizing the APCD in the classroom.

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