

# Projectile Trajectory Scenarios

Student Worksheet

Name \_\_\_\_\_

Class \_\_\_\_\_

Note: Sections of this document are numbered to correspond to the pages in the TI-Nspire™ .tns document “ProjectileTrajectory.tns.”

## 1.1 Trajectories and Velocity Components

What determines the trajectory of a projectile? What happens to its motion during its flight? We can investigate the basics of projectile motion using vectors and parametric equations to define and describe the motion.

A projectile may be traveling both horizontally and vertically at the same time. These two simultaneous motions can be combined with vectors and described separately with a set of parametric equations.

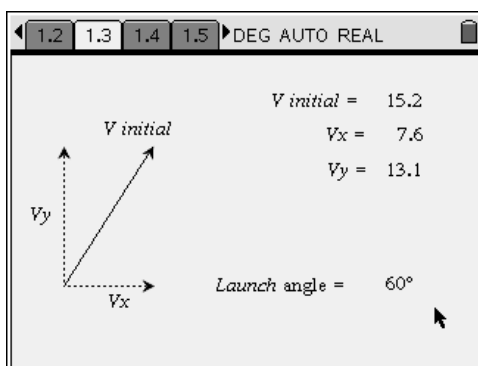
Let's begin by looking a simple projectile launched upwards from "ground level" across a horizontal surface, starting with the initial velocity vectors.

## 1.2

The path of a projectile depends on the *initial velocity vector*. This initial velocity can have different magnitudes and direction. This causes the  $x$  (or horizontal) and the  $y$  (or vertical) components to change.

1. On the next page, drag the tip of the initial velocity vector, and note how the launch angle and the components  $V_x$  and  $V_y$  change.

## 1.3

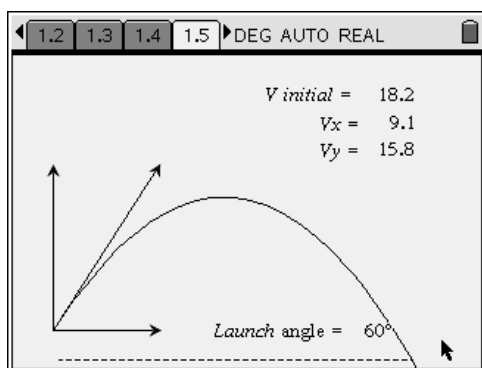


## 1.4

A projectile trajectory determined by the initial velocity is shown on page 1.4.

2. Drag the tip of the initial velocity, and note how the trajectory changes.
3. Press **menu** **5** **1** for Menu 5: Trace, 1: Graph Trace.
4. Move the trace point along the graph to see the  $x$  (horizontal distance), the  $y$  (vertical distance), and the  $t$  (time of flight) for each point on the trajectory.
  - Clicking will lock a trace point, giving the  $x$ - and  $y$ -coordinates of that point.

## 1.5



## 1.6

5. By experimenting with the *initial velocity vector*, what two things tend to make the projectile travel higher?

## 1.7

6. What two things can you do to make the projectile travel farther?

## 1.8

7. Set a suitable trajectory back on page 1.5.
  - What is the maximum height of that trajectory?
8. Lock in a trace point.

## 1.9

9. When did the projectile reach the maximum height, and what was its horizontal displacement at that time?

**1.10**

10. When and where did the projectile land on the ground?

**2.1 Changing Velocity Components**

As a projectile flies through the air, its motion changes. Vertical and horizontal motions are separate and independent. For simple projectile motion consideration (ignoring air resistance, earth movement, curvature of the earth, etc...), horizontal motion is uniform and based on the  $V_x$  (horizontal component of the initial velocity). The vertical motion is controlled by gravity and based on  $V_y$  (vertical component of the initial velocity).

**2.2**

11. What is the equation to determine distance for *uniform horizontal motion*?

**2.3**

12. What is the equation for vertical height in freefall motion?

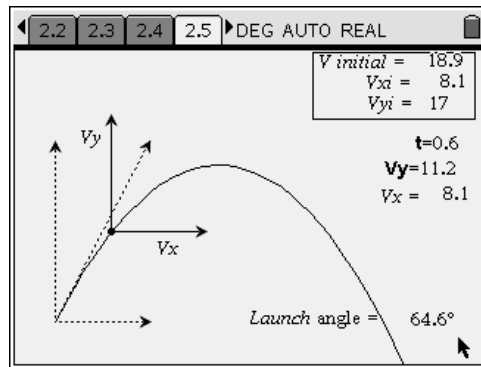
**2.4**

Parametric equations relate two separate relations to a third variable—usually *time*. This works perfectly for projectiles because we can determine horizontal and vertical displacements from different velocities—both at the same times.

Note: The parametric equations for vertical and horizontal motions are stepped through chosen time intervals to create the trajectories in this document.

On the next page of the TI-Nspire™ document is a trajectory that you can vary by changing the initial velocity. The projectile is attached to the trajectory path, and the velocity of the projectile is shown as it moves along the trajectory.

## 2.5



## 2.6

13. What happens to the horizontal component of velocity ( $V_x$ ) as the projectile moves along the trajectory?

## 2.7

14. What happens to the vertical velocity ( $V_y$ ) as the projectile moves along the trajectory?

## 2.8

15. When does the vertical velocity become zero? Why? What is this point of the trajectory called?

## 2.9

16. What happens to the vertical velocity ( $V_y$ ) after the projectile reaches maximum height? Explain.

## 2.10



On the next page of the TI-Nspire™ document is a spreadsheet with columns for the time and vertical velocity component as the projectile moves along the trajectory.

17. Go back to the last trajectory on page 2.5.

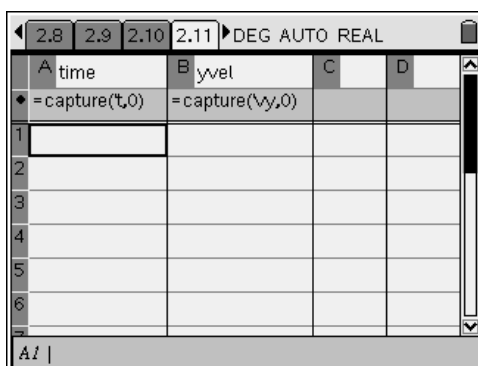
18. Set a suitable trajectory, and move the projectile near to the beginning of the trajectory.

19. Press  .

- These keystrokes will capture data from this point for the spreadsheet.

20. Move the projectile along the trajectory, and press   again to gather another set of data at a new point.
21. Do this for at least ten points along the range of the trajectory.
22. Look at the spreadsheet on page 2.11, and then move on to page 2.12.

## 2.11



The screenshot shows a TI-84 Plus calculator spreadsheet window. The window title bar includes page numbers 2.8, 2.9, 2.10, and 2.11, along with mode indicators DEG, AUTO, and REAL. The spreadsheet has four columns: A (time), B (yvel), C, and D. Row 1 contains the formulas  $=\text{capture}(t,0)$  in column A and  $=\text{capture}(v_y,0)$  in column B. Rows 2 through 6 are empty. The status bar at the bottom shows 'A1 |'.

	A time	B yvel	C	D
1	$=\text{capture}(t,0)$	$=\text{capture}(v_y,0)$		
2				
3				
4				
5				
6				

## 2.12

In the spreadsheet on the previous page, you captured data of time and  $y$ -velocity along the trajectory. As you can see, the  $y$ -velocity changes. By plotting this data, we can find the how the  $y$ -velocity changes with time.

## 2.13

On the bottom half of this page is a data plot.

- Press **ctrl** **tab** to toggle between the two work areas of this page.

23. Move the cursor, click below the x-axis, and select the variable "time".

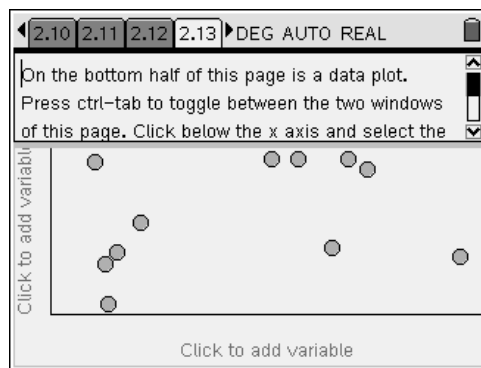
24. Click beside the y-axis, and select "yvel".

- This plots the captured data from the spreadsheet.

25. Click in the data plot, and press **menu** **3** **5** **2** for Menu 3: Actions,

5: Regression, 2: Show Linear (ax+b).

- Note the equation that appears.
- Click on the line if the equation disappears.



## 2.14

26. What does the slope of a velocity vs. time graph represent? What is the slope of this equation? What units should the slope have? Explain this value.

## 2.15

27. What is the vertical axis intercept (y-intercept)? What does this mean?

## 2.16

28. How does the y-intercept relate to the original trajectory on page 2.5?

## 2.17

29. What does the point where the line crosses the x-axis represent? Where is this seen on the trajectory?

**2.18**

30. What happens to the vertical velocity at the point where the graph crosses the x-axis?

Explain what is going on here.

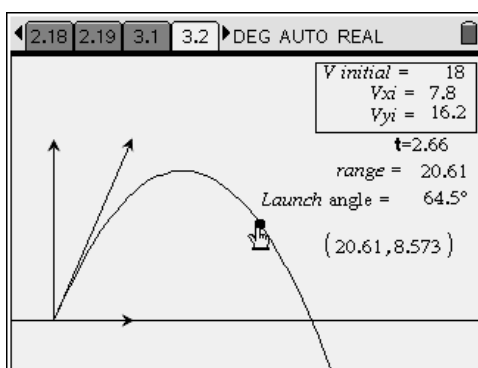
**2.19**

31. Why is the vertical acceleration negative throughout the entire trajectory?

**3.1 Maximum Range**

How can you maximize the range of a projectile? Shooting it faster is a first, obvious solution; what else can you do?

On the next page is a trajectory for a projectile launched at a fixed, maximum speed. Explore how to make the projectile go farther before landing on the ground. You can drag the initial velocity vector, and you can drag the trace point on the trajectory.

**3.2****3.3**

32. In this example, what launch angle gives the maximum range?

**3.4**

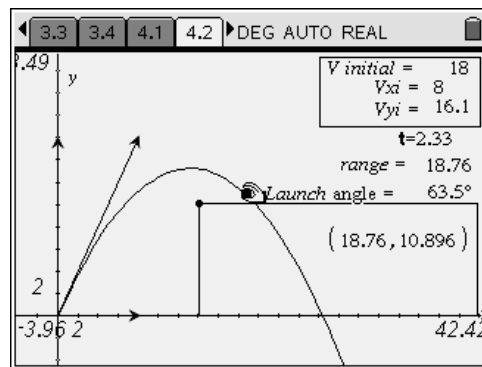
33. What is the maximum range?

## 4.1 Optimum Angles?

You now launch the projectile towards an elevated landing area.

34. Does this affect the optimum angle for maximum range?
35. Explore this by dragging the corner of the elevated surface, the tip of the initial velocity vector, and the trace point.

## 4.2



## 4.3

36. As the elevated landing area becomes higher, what happens to the angle needed for maximum range?

## 4.4

37. As an extension to this activity, try to create another trajectory where the launch point is elevated above the landing area.
38. What does this do to the range and to the optimum angle for maximum range?