EXPLORATIONS

Activity 6

Mapping A Leaf

In this activity you will:

- Identify and record coordinates (ordered pairs) from a grid
- Explore symmetry about the **x** and **y** axis and the x=y line
- Create reflections, slides, transformations, and scale changes (reduction in size)
- Investigate area and perimeter and their relationship
- Use an identification key for naming trees or other plants
- Analyze symmetry in nature

Introduction

People use the shape of objects and the relationships of their parts to identify them. Often a dichotomous (two-part) key is used to discover the name of a tree or a plant. In this investigation, you will explore the shape and symmetries of leaves on a stem or branch from a tree by mapping the perimeter of one of the leaves and then plotting them on the calculator.

The Problem

How does the placement of a leaf on a stem reveal the type of tree that it came from? What types of arrangements are possible? How do these arrangements relate to the study of geometry and of the plant world?

To discover some relationships, you can start with a stem from a tree. Locate a stem from a tree that is near your school or home. You want a stem that has fallen off the tree or one that is damaged and in need of removal. Please DO NOT cut a healthy stem or branch. You need to leave those for the tree to use.

Activity

Collecting Data

1. Locate a stem from a tree with about seven leaves in good condition on it.

2. Remove one of the leaves from the stem. Pick one of the larger ones, close to the start of the stem.

3. Place the leaf on some grid paper as shown below. Be careful to start at the corner (origin) and secure the leaf for marking so that it won't move.
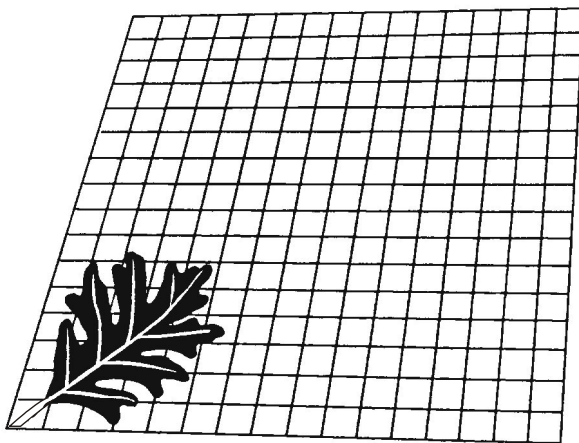

4. Place a point at the end of the stalk (called the *petiole*) of the leaf and at the point where the broad flat portion (called the *blade*) of the leaf starts. Continue to mark the points that represent the perimeter of the leaf as you move around the edge. Be very careful to get enough points to show curves in the leaf. The distance that you move along the perimeter between marks will need to be smaller on curved places so that you may collect more detail.

5. Transfer the coordinates collected along the perimeter of the leaf to Table 1 on the student data sheet. Be careful to get the coordinates in the correct order (**x,y**). In addition, when labeling the points, estimate no more than 0.05 units. Double check your numbers! Place your ending point as the place you started (0, 0).

6. Place these values into your calculator in lists named to represent the **x** and **y** values of the points.

   **a.** Press ⎣LIST⎦ and move to the top of any list in the display and then remove it from the display by pressing ⎣DEL⎦.
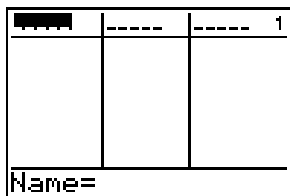
    b. Enter the names of the lists by placing your cursor in the top of the first column, pressing ⎣2nd⎦ ⎣TEXT⎦, and entering the required name for the first list, pressing ⎣ENTER⎦ for each desired letter.
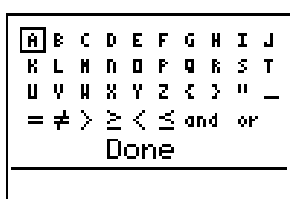
    c. Highlight **Done** when finished with the first name and press ⎣ENTER⎦ ⎣ENTER⎦.
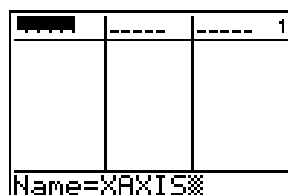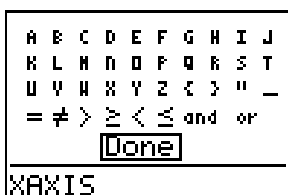
7. Repeat the process for the second list and then enter the data into the two lists, as pairs!
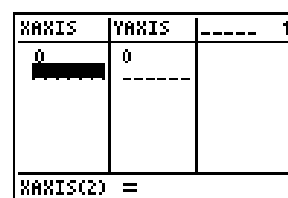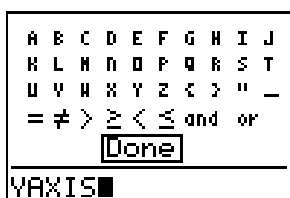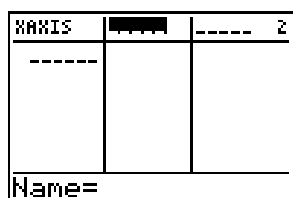
8. Double check the data in the calculator with the data in your table. If you think that the information is correct, save the data either by sending it to another individual's calculator or by using the TI-GRAPH LINK™.

Analyzing Data

1. To see the pattern that represents the perimeter of the leaf, set up a plot of the data.

    a. Press ⎣2nd⎦ ⎣PLOT⎦ and check to see that all three plots are **Off**. If any plot is **On,** press ⎣4⎦ and ⎣ENTER⎦.

    b. Return to the plot option and select **Plot1** to edit.

c. Set up the plot as shown below.

```
Plot1 ██ Off
Type:▨ ▨ ▨ ▨
       ⊕ ▨ ▨ ▨
Xlist:XAXIS
Ylist:YAXIS
Mark: ▫ ■ ·
```

2. Press WINDOW to set up the canvas for your graph. Set the **Xmin** and **Ymin** to zero.

```
WINDOW
 Xmin=0
 Xmax=9.4
 ΔX=.1
 Xscl=1
 Ymin=0
 Ymax=9.4
 Yscl=1
```

3. Set the **Xmax** and **Ymax** by identifying the maximum values in each list. To do this, move the cursor to the **Xmax** line and press 2nd [STAT] to display the list window.

```
██ OPS MATH CALC
██L₁
2:L₂
3:L₃
4:L₄
5:L₅
6:L₆
7↓XAXIS
```

4. From this window, move to the **MATH** menu by pressing ▶ ▶ and then selecting **2:max(** by pressing ▼ and ENTER.

```
Ls OPS ████ CALC
1:min(
██max(
3:mean(
4:median(
5:mode(
6:stdDev(
7:sum(
```

```
WINDOW
 Xmin=0
 Xmax=max(
 ΔX=.1
 Xscl=1
 Ymin=0
 Ymax=9.4
 Yscl=1
```

5. To identify the list that will be used to calculate a maximum, press 2nd [STAT].

a. Select the list that you wish to use by moving up or down to the location of the name of the list.

```
██ OPS MATH CALC
1:L₁
2:L₂
3:L₃
4:L₄
5:L₅
6:L₆
7█:XAXIS
```

b. Press ENTER when your list is highlighted. The screen shown below is based on sample data.

```
WINDOW
 Xmin=0
 Xmax=…x(∟XAXIS
 ᐃX=.1701063829…
 Xscl=1
 Ymin=0
 Ymax=9.4
 Yscl=1
```

6. Repeat this process for the **y** variable.

```
WINDOW
 Xmin=0
 Xmax=16
 ᐃX=.1702127659…
 Xscl=1
 Ymin=0
 Ymax=…(∟YAXIS)
 Yscl=1
```

✎ Answer question 1 on the student data sheet.

7. Now you need a bit more room on the graph, so increase the **x** and **y** values that appear in **Xmax** and **Ymax**. (About 3 to 4 % should do.)

```
WINDOW
 Xmin=0
 Xmax=16+0.5
 ᐃX=.1702127659…
 Xscl=1
 Ymin=0
 Ymax=15.4
 Yscl=1
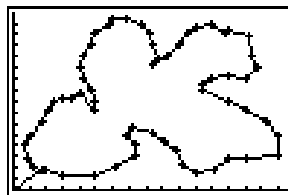```

```
WINDOW
 Xmin=0
 Xmax=16.5
 ᐃX=.1755319148…
 Xscl=1
 Ymin=0
 Ymax=15.4+0.5
 Yscl=1
```

✎ Answer question 2 on the student data sheet.

8. Press GRAPH to see the Leaf! Press TRACE and explore your leaf.

✎ Answer question 3-4 on the student data sheet.

a. Look for misdrawn lines, and check these data points in the calculator on Table 1, and on your tracing of the leaf on the grid paper. If an adjustment is made, save the new, correct data as before.

b. Set the image to be "more real" by using the Zoom Square option on the calculator. Press ZOOM and select option **5:Zsquare** from the **ZOOM** menu. This will send you back to the graph with a better image.

```
ZOOM MEMORY
1:ZBox
2:Zoom In
3:Zoom Out
4:ZQuadrant1
5:ZSquare
6:ZStandard
7↓ZoomStat
```
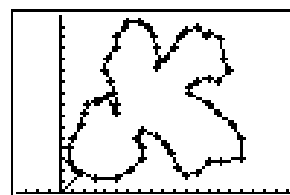

✎ Answer questions 5-6 on the student data sheet.

9. To make copies of the leaf and show how the other leaves are placed on the stem, you have several options. First, look at a reflection on the y-axis. To do this, you need to generate a list of new x-values that are the opposite of the current values. Create a list named **ʟNX** (for negative **ʟXAXIS**) by multiplying the **ʟXAXIS** list by –1.

XAXIS	YAXIS	▥ 3
0	0	------
1.75	1.75	
1.9	1.8	
3	1.35	
5	1.4	
6.1	2	
7.3	3	
NX = ⁻ʟXAXIS		

XAXIS	YAXIS	NX 3
0	0	0
1.75	1.75	⁻1.75
1.9	1.8	⁻1.9
3	1.35	⁻3
5	1.4	⁻5
6.1	2	⁻6.1
7.3	3	⁻7.3
NX(1)=0		

10. Set up a plot of **ʟNX** versus **ʟYAXIS** to go with the original plot. Set the **WINDOW** as before, with the minimum value for **X** being the opposite of the maximum value. Change the plot mark to (·) on both plots.

11. Adjust this view by using the **ZSquare** option as before.
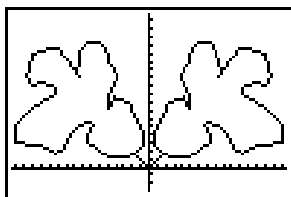
12. Save this image as a **Pic** on the TI-73 as shown below. Be careful to pick a name for an unused location since this action will record over any information stored in the selected picture location/name.

```
DRAW POINTS STO
1:ClrDraw
2:Line(
3:Horizontal
4:Vertical
5:Shade(
6:Circle(
7↓Text(
```

```
DRAW POINTS STO
1:StorePic
2:RecallPic
```

```
StorePic 2
```
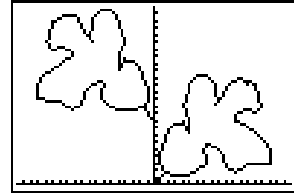
✎ Answer question 7 on the student data sheet.

13. If your branch has two leaves that come off the stem at the same location (called *nodes*) then you are ready to start the identification of your tree. In this case, the leaves are opposite. If there is an offset for the leaves as they come off of the stem (that is, there is only one leaf per node), the leaves are alternate.

14. To model the alternate type of leaves, shift (translate) the values of the **ʟYAXIS** up. The amount of the shift could be measured on the stem, and then calculated in terms of the units associated with the grid. Select some value for this translation along the y-axis and apply it to a new list called **ʟTY** (for translated **ʟYAXIS**). Set up the second plot as shown below and modify the **WINDOW** so that the graph of the data can be seen.

✎   Answer questions 8-10 on the student data sheet.

15. To make a third leaf on the "stem" (y-axis):

a. Take the original values for **x** and **y** (**ʟXAXIS** and **ʟYAXIS**) and increase the **ʟYAXIS** values by the number that represents the distance for two nodes.

b. Create a new list called **ʟTTY** (for twice translated **ʟYAXIS**) and produce a plot of all three lists as shown below.
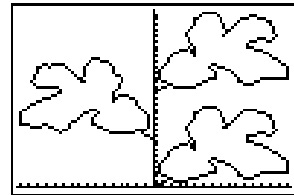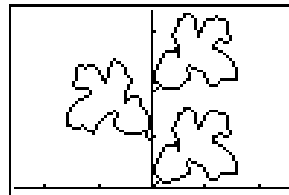
✎ Answer questions 11-12 on the student data sheet.

Going Further

1. From the data given below, create a plot of this leaf on your TI-73.

X Values	Y Values
0	0
1	0.5
2	1
3	1
5	2
6	4
6	6
4	5.5
2	5
1	3.5
0	1
0	0

2. What **WINDOW** did you use for the above plot?

    Xmin = _____   **Ymin** = _____

    Xmax = _____   **Ymax** = _____

3. Assume that the tree has an opposite node orientation for the stem. Create a plot showing four leaves on the stem, using the x-axis as the stem.

4. Give the **WINDOW** used and the translation value.

    Xmin = _____   **Ymin** = _____   Translation factor = _____

    Xmax = _____   **Ymax** = _____

5. Save this plot as a **Pic**.

6. Transfer this plot to the computer and print using Graph Link. (optional)

7. How would you produce a plot of the leaf in the 3rd quadrant?

8. What kind of reflection would this be?

9. What would you do to reflect the leaf about the line x=y?

Name(s)_____

Date _____

Activity 6

Mapping A Leaf

Table 1

X Value	Y Value
0	0
0	0

1. How did you determine the maximum values for **x** and **y** from the tracing on the grid paper?

    _____

    _____

2. Why do you increase the **Xmax** and **Ymax** by a percent rather than a given amount (like 0.5)?

    _____

    _____

3. How is the TI-73 graph of your leaf the same as, or different from, your real leaf?

    _____

    _____

4. Report the values for the **WINDOW** below:

    Xmin = _____   **Ymin** = _____

    Xmax = _____   **Ymax** = _____

5. Report the values for the **WINDOW** after the Zoom Square below:

    Xmin = _____   **Ymin** = _____

    Xmax = _____   **Ymax** = _____

6. Compare the values reported in questions 4 and 5.

    _____

    _____

7. Using the Graph Link, print or save a copy of the plot of the two leaves or use the **Pic** option to store it in your calculator.

8. How does the spacing between nodes vary for your stem? Record on Table 2 the values and identify the pattern (if any).

_____

Table 2

Node Range	Distance
1 to 2	
2 to 3	
3 to 4	
4 to 5	
5 to 6	
6 to 7	

9. How do you determine the new values for the **WINDOW** in the case of this translation up the y-axis?

_____

_____

10. What **WINDOW** did you use for this plot?

Xmin = _____   **Ymin** = _____

Xmax = _____   **Ymax** = _____

11. What **WINDOW** did you use for this plot?

Xmin = _____   **Ymin** = _____

Xmax = _____   **Ymax** = _____

12. Save a copy of the plot with three leaves as a **Pic** or transfer it to the computer using the TI-GRAPH LINK™ and save or print it there.

Teacher Notes

Math Strands: Geometry, and Algebra

Identifying and recording coordinates (ordered pairs) from a grid. Symmetry about the **x** and **y** axis and the x=y line (reflections). Slides, transformations, and scale changes (reduction in size). Area and perimeter and their relationship.

Science Strands: Life Science, Data Collection

Use of an identification key for naming trees, or other plants. Symmetry in nature.

Classroom Management and Safety

Students should work in small groups with stems from same and different trees. The stems should be acquired from the ground or from trees that are to be pruned. In addition, the climbing of trees and use of cutting tools should be monitored closely.

The Set Up

♦ Each group should have enough people to do all the leaves.

♦ In some cases it might be wise to have more than one group do the same type of tree, or have the whole class do one type of tree.

♦ With younger students the number of points marked on the grid for a leaf can be smaller. To maintain as much information as possible, have these students select "simpler" leaves. This concept could also apply to differentiation for any class. Just point one group to more complex leaves, and others to a simpler shape.

♦ The size of the grid can be used to force students to use rational numbers. If the grid is large they will need to give the points on the perimeter in decimal form, and if the grid is small they be able to place many points at grid "corners" getting whole number values for their coordinates.

♦ The seasons will have a bit to do with access, the teacher may want to "Ziploc®" some data for use in the non-leaf season.

♦ Placement of the leaf on the grid is critical to the complexity of the problem. If the leaf starts at the origin and lays in the first quadrant about the x=y line then the symmetry is rather straightforward. If the leaf straddles the x or y-axis, then addition symmetries may be addressed. In the case of random placement on the grid, the concept of translation of coordinates will be the focus.

♦ The variations in types of leaf margins, which range from a very simple "entire" to a very complex "doubly serrate", can be used to help

differentiate for the age or ability of students. A sugar maple leaf would be challenging, while a black locust leaf would be far less complex.

♦ For more advanced applications, students could use the stored pictures and the three active plots to produce a stem of many leaves. In addition the use of **augment(** to combine lists could expand the options. The traceability of a shape could be addressed here.

Student Data Collection and Analysis Sheet – Key

Table 1: See the program **LEAF.73p** for a typical set of values.

This and all other programs are available for download on TI's web site at: **http://www.ti.com/calc**.

1. Look at the **X** and **Y** axes and identify the maximum numbers.

2. Small leaves would change more than big ones.

3. Not as detailed in showing the shape.

4. Answers will vary.

5. Answers will vary.

6. The values in question 4 are closer to the real maximum values of the sketch.

7. Name of **Pic** or the printout.

8. Table 2 of values; answers will vary.

9. Leave the x-values alone and just add the spacing between nodes to the **Ymax** value.

10. Answers will vary.

11. Answers will vary.

12. Optional, or the printouts

*Note***:** Sample data is in a program named **LEAF.73p** and **LEAFMAP.73p**.

Data from the post oak (*Quercus stellata*) leaf shown in the sample data.

Going Further - Key

1.
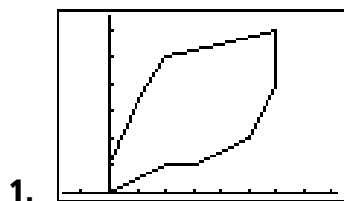
```
WINDOW
 Xmin=-1.677419…
 Xmax=8.1774193…
 ⊿X=.1048387096…
 Xscl=1
 Ymin=0
 Ymax=6.5
 Yscl=1
```

2.

3. There should be a **Pic** used as background for the plot saved on the final window. To get the three new leaves you have (x, -y), (x+c, y), and (x+c, -y). In the example below, the translation factor is c = 5.
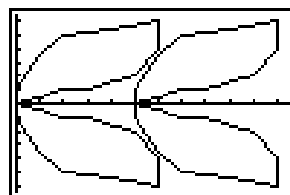
```
WINDOW
 Xmin=0
 Xmax=11.5
 ⊿X=.1223404255…
 Xscl=1
 Ymin=-6.5
 Ymax=6.5
 Yscl=1
```
```
STAT PLOTS
1:Plot1…On
   ∠ TESTX ATY
2:Plot2…On
   ∠ TXC  TESTY
3:Plot3…On
   ∠ TXC  ATY
4↓PlotsOff
```
```
TESTY  TXC  ATY   2
 0      5    0
 .5     6   -.5
 1      7   -1
 1      8   -1
 2     10   -2
 4     11   -4
 6     11   -6
TESTY(1)=0
```

4. Shown above.

5. Name of **Pic**, copy on the computer or in your calculator.

6. Optional, or file name and location.

7. To place in the 3rd quadrant graph (-x, -y).

8. This is a reflection about the origin.

9. To reflect on the line x=y, the **x** and **y** values should be reversed so plot (**y**, **x**).

Additional Questions

♦ Examine the stem from the tree and explain how the sizes of the leaves change as they move away from the base of the branch?

♦ How would you use the data collected to find the area of the leaf? How do the areas of the leaf change as they move away from the base of the tree?

♦ How would you use the data collected to find the perimeter of the leaf? How do the perimeters of the leaf change as they move away from the base of the tree?

♦ Are the ratios of the areas of the leaves as they change size the same as the ratios of the perimeters of these leaves?

♦ How do leaves and branches help trees? How do they help us?

References

Trees of Arkansas, Dwight M. Moore, Arkansas Forestry Commission, January 1981, Little Rock, Arkansas.