Int. J. Engng Ed. Vol. 15, No. 6, pp. 411–416, 1999
Printed in Great Britain.

0949-149X/91 $3.00+0.00

Strategies and guidelines for using a computer algebra system in the classroom*

G. DONALD ALLEN
Workshop organizer, Department of Mathematics, Texas A&M University, College Station, TX 77843, USA. E-mail: dallen@math.tamu.edu
JIM HEROD
Department of Mathematics, Georgia Tech, Atlanta, GA 30332-0160, USA. E-mail: jherod@mail.tds.net
MARK HOLMES
Department of Mathematics, Rennslear Polytechnic Institute, Troy, NY 12180, USA. E-mail: holmes@rpi.edu
VINCE ERVIN
Department of Mathematical Sciences ,Clemson University, Clemson, SC 29634. USA. E-mail: ervin@math.clemson.edu
ROBERT J. LOPEZ
Department of Mathematics, Rose-Hulman Institute of Technology, Terra Haute, IN 47803-3999, USA. E-mail: r.lopez@rose-hulman.edu
JOE MARLIN
Department of Mathematics, North Carolina State University, Raleigh, NC 27695-8205, USA. E-mail: marlin@math.ncsu.edu
DOUGLAS MEADE
Department of Mathematics, University of South Carolina, Columbia, SC 29208, USA. E-mail: meade@math.sc.edu
DAVID SANCHEZ
Workshop organizer, Department of Mathematics, Texas A&M University, College Station, TX 77843, USA. E-mail: dsanchez@math.tamu.edu

In this note, the authors attempt to set out parameters for the usage of a Computer Algebra System in the classroom. These results are based on the common experiences of several faculty that have collectively many years of experience teaching calculus and other mathematics courses using Maple or Mathematica. While there are several styles to accommodate one's goals with the CAS, there are as well several remedies that are really just plain common sense. This paper contains such a list with explanations as to why it is so, from the faculty or student viewpoint.

INTRODUCTION

MORE AND MORE faculties of mathematics have chosen to use a Computer Algebras System (CAS), such as *Maple*, *Mathematica*, or *MATLAB*, in the classroom, substantially the calculus classroom. (The dominant tools in the calculus classroom is *Maple*, though *Mathematica* is also widely used. *MATLAB* has a strong following in engineering departments. This said, we will tend to specify *Maple* as the CAS of interest, though the reader may substitute a favorite CAS in its place.) All but a handful genuinely understand what difficulties they will encounter. The main problem is to formulate answers to the questions:

- What part of the course objectives are assisted by the use of technology?
- How do I use the technology to meet these objectives?

If these issues are not clearly addressed beforehand and clearly resolved before the fact, there is ample reason to believe they will not be later. This results in faculty using the CAS in a variety of fundamentally different ways, including not at all, which can have serious effects on the undergraduate program. Another problem is the faculty rejection of the CAS, which can be both passionate in opinion and disruptive to curricular planning. These and related issues together with a substantial collection of do's and dont's are among the main topics of this paper.

The genesis of this paper arose out of a workshop, April 3–4, 1998, on Calculus Reform and *Maple* in the Classroom, which was sponsored by the National Science Foundation and conducted by SRI International. The meeting assembled a

* Accepted 12 July 1999.

group of educators, many with extensive experience in calculus reform, more with teaching mathematics with *Maple*, and evaluation experts. Among agenda topics were:

- What are the advantages and disadvantages (both real and perceived) of using a CAS in the classroom?
- What should be the role of technology in teaching calculus?
- What standards can be developed for the use of technology in teaching?
- What guidelines should be developed for using a CAS in the classroom?

NSF coordinator, Conrad Katzenmeier, indicated that the NSF is very much interested in these and related evaluative issues of pedagogical programs in the teaching of calculus. This paper addresses some of the conclusions (and non-conclusions) connected with the first and last topics.

There is no truer statement than the following: If a CAS is to be experienced in a lab setting, then precise and well-crafted exercises and projects are essential. Students who have unpleasant experiences with the CAS are not likely to enjoy the extra effort it takes to master it and still master the pencil and paper skills of the traditional class. This desideratum is difficult to achieve.

In this paper we will consider three topics important to the CAS-experienced institution and to the institution still considering whether to become CAS involved. They are:

1. General items and strategies; human and hardware factors.
2. The levels of CAS usage.
3. The do's and dont's of CAS classroom usage.

The scope of these topics run the gamut from institutional to departmental to the classroom. Almost none are cast in concrete and therefore all require careful forethought. However, the successful program will consider the full scope and attempt to positively engage all concerned. The majority of the authors have had extensive experience using *Maple*, but have framed the following discussion so it is applicable to any CAS.

GENERAL ITEMS AND STRATEGIES: HUMAN AND HARDWARE FACTORS

The data and views expressed in the following discussion are based on a survey of the participants' institutions. Therefore, it is limited to but combines together this collective experience.

Planning your technology usage strategy

Almost any variety of instructional modes is suitable for inclusion of a CAS. Among the conference participants, the most prevalent mode was the lecture-lab format, with about 3 hours of lecture and 1–2 hours of lab, where the lab is alternately a discussion. Some labs are taught with teaching assistants; some not. This is not a serious issue until the amount of CAS inclusion is considered. Class sizes can vary widely, from lectures of 150 students to classes of size 20–30. Regarding classroom equipment, the consensus is that having a computer with a RBG (Red-Green-Blue) projector or equivalent available during the lecture is important, in fact, critical. The same can be said about projection capabilities in the lab. This allows the lecturer to integrate the CAS features and usage into the course. It demonstrates the importance of the CAS and encourages students to learn its language and capabilities. It also signifies to the student that the CAS is not some "add-on", extra-work course supplement—a serious potential problem that must be considered.

What are the quantitative aspects?

It is necessary to decide how much time is allocated for computer instruction, and what are the hardware requirements for both the students and university labs. In most cases, about 10–20% of class time is dedicated to learning the CAS and calculus through it. For linear algebra and differential equations courses the percentage is spread wider. Hardware requirements can vary widely, from university operated and equipped computer labs to a required student computer purchase—usually a Pentium-based computer (even notebooks). There is some agreement that the more readily available the computer equipment is, the more successful is the technology inclusion. This has certainly been the case with graphing calculators. As a point of interest, it is generally regarded that anywhere from 50–80% of students own their own equipment, except where it is mandatory. Some institutions require students to purchase a computer with the CAS, and other software, pre-loaded.

Faculty preparation time is another important contributor to the program's success. It is recommended that the initial teaching faculty be given some release time to prepare institutional materials. These faculty then share their experiences with others as the program evolves. It is probably not wise to force extremely negative faculty into this venture. They can damage student attitude toward the CAS in far greater proportion than the number of students they contact. Overall, it is felt that using the CAS initially requires a lot more faculty class preparation than the traditional way, but this abates somewhat as experience accrues. For the experienced instructor, development of new materials requires about four hours of preparation for each hour of classroom time. Teaching assistants must also be well versed in the CAS, to the level of being able to develop lab assignments.

Should supplementary services be available? If so, what services are the right ones?

Many services can be considered. From upperclass student mentors to teaching assistants to faculty available in labs—all types of personnel are used. Some supplementary personnel, particular at the beginning of the term, are best. In addition, some institutions provide special classroom instruction in using the computer as well as the CAS. This is recommended and is a great help to transfer students, who are beginning in the second term of the calculus sequence. Some programs begin anew with CAS instruction in each course. Whether considered supplementary or not, adequate help should be available in the lab.

What about faculty involvement?

At a small majority of institutions, the CAS-enhanced courses are taught by a select group of faculty. These faculty are strong advocates and some spend considerable time perfecting their use of technology. In most cases, faculty have wide latitude in what CAS instruction they give, and what criteria are required for their "CAS" grade. In some cases, a minimum number of labs is specified. To encourage faculty to participate, some institutions offer seminars on CAS instructions. At most places web sites with available CAS resources are maintained by the department or strong advocates. Mathematicians, though, tend to be do-it-yourself types.

When asked what the are the advantages and/or disadvantages of using a CAS in the classroom, faculty give various answers. **Advantages** given include:

- Helps develop visual/geometrical understanding.
- Can explore concepts before "hand" skills to do so are available.
- Can explore realistic problems.
- Strong request by engineering school.
- Enhances job opportunities for students.
- Allows students to concentrate on problem formulation and solution analysis.
- Easy to give math demos; introduces advanced mathematical ideas concretely.
- The CAS forces students to consciously decide what operations to use.

Disadvantages given include:

- Greater time needed for class preparation.
- Lack of familiarity with the computer and CAS; fear of making syntactical errors in class.
- Lack of administrative recognition of increasing teaching load.
- Decline of students' paper-and-pen skills.
- CAS syntax is an unreasonable burden on students.
- Learning curve is too steep; subtracts time from learning mathematics.
- The course can be victimized by equipment failure or inadequate equipment.

Each of the above is the personal opinion of some faculty member, somewhere. At least one institution has avoided most of the cons by not requiring faculty involvement, leaving the CAS instruction in the hands of teaching assistants and leaving open how much CAS use is involved in the grade computation. Many faculty teaching under this format have very little actual CAS skill. The approach has led to a very great disparity in the level of CAS use from lecture to lecture, semester to semester. Other institutions have insisted on faculty involvement and for the most part, this has met with success. But remember, some faculty will not accept this technology just because . . . .

Who else should be involved?

Almost everyone. The department administration should strongly advocate the CAS program, watching it progress, helping to avoid pitfalls, encouraging faculty, and making certain that the hardware is continually upgraded and updated. In turn, the college dean should be on board, for the strategic importance of a senior university administrator advocating this new technology, for running interference for the department, and shielding it from the inevitable complaints. New pedagogies always incur complaints.

While support from both levels of administration are important, so also is support from the client colleges and departments. In particular, it is important to have on-board the engineering school, usually the principal client. Visitations to select faculty to explain the program, to extol the virtues of the CAS (and to show it's power) are necessary to avoid unnecessary criticism by those unaware of just what is trying to be accomplished. At one school, where this hasn't been done, the client faculty have an incorrect and limited view of what the CAS (*Maple* in this case) can do, and have begun a technology enhancement program of their own using different software (*MATLAB*). Value is gained by viewing the client departments as the customers for your product, the calculus-trained student. Good customer relations return good support over the years. As well, it can help significantly if other departments (e.g. physics) use the CAS in their courses. If this happens then the students don't necessarily think the learning curve to master a CAS is a waste of time. They also get more practice using the program and master it quicker.

LEVEL OF USAGE

The way the CAS is utilized varies widely among the institutions represented at the workshop and also within individual institutions. Whereas almost all faculty everywhere, and certainly within an institution, cover a very similar syllabus, we see no such correspondence with the use of the CAS. This is probably not good but is difficult to control. The problem at some institutions is historic. To get faculty into the venue, a great flexibility of utilization was allowed. In short, when there is no "controlling authority" willing to assure some agreement in coverage, diversity results.

For some faculty, the CAS provides something of a teaching relevation which will likely evolve to a high percentage of CAS usage, whereas other faculty see it as a drag to learning. In ways most university faculty have never seen, this issue definitely taps into the pedagogical belief systems of faculty. It is probably best to prepare the involvement with definite level-of-usage guidelines before the first machine is turned on, and then get those signing on to agree to it. Below we identify four levels of CAS usage. At the extremes, one may argue whether the use should be as simple as adding on a toolbox (say like a calculator), or as complex as creating an enveloping environment. Strong and convincing arguments can be made for each.

We list some possible levels of technology insertion into a standard science or engineering calculus class. Similar levels can be defined for other courses by altering the targeted objectives. It is assumed the levels are inclusive; that is, the objectives of Level Two contain those of Level One as well.

Level One The technology is used to enhance **visualization** of calculus concepts. Examples:

- tangent lines
- curve sketching
- optimization
- surfaces/normals.

Visualization notwithstanding, the technology also facilitates **understanding** of the:

- definition of integral using Riemann sums
- convergence of power series
- definition of arc length using polygonal approximations.

Level Two The technology is integrated to the level of being employed for symbol manipulation and the analysis of functions. Examples:

- simplifying functions
- differentiation
- integration
- solution of equations for critical points.

Level Three The technology is used to solve complex problems involving only calculus concepts that are ordinarily too complex for hand calculations. Detailed and comprehensive reports are required.

Level Four The programming language capabilities of the technology are utilized to allow the solution of even more complex, multistep problems and the creation of general algorithmic procedures.

The higher the level, the more time is invested in achieving it, and the more finely tuned should be the plan for using it. Some faculty believe that Level One is just about right; that the best use of the CAS is to impart greater understanding in the traditional sense. Others believe the opposite and with equally articulate arguments. Indeed, an excellent case has been made regarding the inclusion of the CAS as nothing less than an evolutionary step in mathematics education. In this argument the CAS becomes the tool of "first recourse" for teaching, learning, and doing mathematics. Skills learned with the CAS can replace some traditional ones (e.g. arcane trig substitutions for integration). This presents a formidable challenge to the instructor who must now think of new learning activities for the classroom.

Which should it be? A little? A lot? The jury is still out. To date, no comprehensive measure of efficacy has been devised, and therefore none has been applied. Some instructors are such good teachers that whatever they try works. They are loved and respected; their students learn well; the department chair smiles and gives rewards. In a sense, they are so good that it is difficult to remove the power of their personality from their teaching methods. Therefore, their classroom results should not necessarily provide the model or impetus for comprehensive pedagogical changes. Furthermore, probably no departments have the luxury of time and resources to experiment with pre- and post-CAS assessment, or running parallel classes, some using CAS, some not, and comparing measurable outcomes.

DO'S AND DONT'S FOR USING A CAS

OK, let's suppose you have decided to go ahead and implement a CAS, say *Maple*, in your calculus classroom. Even if you have resolved all the issues discussed above, the workshop participants agreed that there are some general guidelines which should be followed in order to optimize the CAS classroom experience. Below, we itemize the most important with brief explanations.

1. Explain to your students why a CAS system enhances and enables them to explore and understand mathematics to a higher level than before. Rationale: Convincing students that a CAS system is important for their studies and careers certainly relaxes apprehension and anxiety and eases your job of asking the students to learn even more.
2. Do use it appropriately. Rationale: Aside from introductory examples and simply learning how to use the CAS, students should develop an appreciation of when to use the CAS. They should not be required or expected to use the CAS for routine differentiations or integrations. They should view the CAS as a valuable tool, not a crutch.
3. Do prepare your CAS demonstration carefully. Rationale: Students are often "professional" about their expectations of computer interactions, and the software they are already acquainted with is likely very sophisticated. They expect perfection.
4. Do count the students' CAS work as a part of their overall course assessment. Rationale: This

enhances student motivation to study what needs to be learned. It also carries the implication that the instructor views it as valuable.
5. Do peg the demonstration/example at the students' level. Rationale: Students are not necessarily amazed at the same things you are. Overly snazzy or complex CAS demos may bewilder and intimidate all but the best students.
6. Do not make your demonstration longer than the attention span of your students. Rationale: Demonstrations can encourage passive participation. The student that 'turns off' in the middle of some lengthy point you are making profiles little or nothing at all.
7. Make available to students any worksheets and demonstrations used in the classroom.
8. Do not "wing it" in the classroom until you are an expert. Rationale: You risk losing more than you can gain if you foul up a demo. Students may interpret the CAS as being perhaps too hard for them to learn. They may also diminish their trust in you as an expert source of knowledge.
9. Do not make CAS assignments in a manner so that students regard them as "added-on" work. Rationale: the students will view them as added drudgery and any integrative benefits will be lost.

CONCLUSION

To summarize, we note that there is no set truth about the details of CAS inclusion. Rather, what one does should be carefully considered and deliberately planned from both philosophical and logistical viewpoints. Clarify what it is that the CAS course component is for and develop exercises and demonstrations to accomplish this. Too many departmental plans are little more than vague plans accompanied by rough experiments. At this point, hardly more than ten years into CAS application, the hands-down best way to include CAS instruction has not yet percolated out. Even with a CAS plan underway, advocates should be willing to make changes when ideas don't work. It is very important to be able to identify and reject bad ideas and then to begin anew in a different direction.

However, the do's and dont's are common sense remedies on which almost all practitioners agree. Following them will greatly enhance both the student and faculty experience with the CAS, no matter what level of usage is adopted.

REFERENCES

1. Carlos Espinoza and Elmer Yglesias, Calculus reform, *Proc. NSF Meeting April 3–4, 1998*, SRI International (1998).

G. Donald Allen has more than two decades experience teaching mathematics and engineering calculus at Texas A&M University. In addition to authoring a textbook on business calculus and developing a Web-based calculus course, he has developed many modules for *Maple* usage in the classroom. Dr Allen's research interests include mathematical modeling and the numerical solution of the transport equation.

Vince Ervin has been actively involved in incorporating *Maple*-based projects into his calculus classes for the past six years. Dr Ervin's research interests include numerical analysis and computational fluid dynamics.

Mark H. Holmes has been developing *Maple* labs for undergraduate courses since 1988. Some of these eventually appeared in a book he co-authored entitled *Exploring Calculus with Maple*. Dr Holmes has also written several papers on using symbolic computation in a variety of application areas, including biomechanics and chemical kinetics.

Robert J. Lopez has for more than ten years been a proponent of *Maple* in the classroom at Rose-Hulman Institute of Technology and in North America and Europe. Author of several texts on *Maple* usage, frequent conference speaker, and former editor of Maple-Tech, Dr Lopez is presently writing the textbook: *Advanced Engineering Mathematics with Maple*.

Joe A. Marlin has been teaching mathematics and engineering calculus at North Carolina State University since 1964. Working with colleagues at NCSU he has developed a Web-based interactive text of *Maple* worksheets that can be used to supplement courses in calculus and differential equations. Dr Marlin's research interests include the qualitative analysis of dynamical systems.

Douglas B. Meade, an Associate Professor in the Department of Mathematics at the University of South Carolina, has written a book and numerous papers on the use of computer algebra systems, primarily *Maple*, in engineering and mathematics courses. Dr Meade's current research interests include mathematical modeling and mathematical biology. http://www.math.sc.edu/˜meade/

David Sanchez has been a professor of mathematics at UCLA, University of New Mexico, and Texas A&M University, and has served as an administrator at both the university and federal levels. Dr Sanchez's major research interest in ordinary differential equations and he has authored or coauthored three books and numerous research papers on the subject.