Application of Polynomials
TIMATH.COM: ALGEBRA

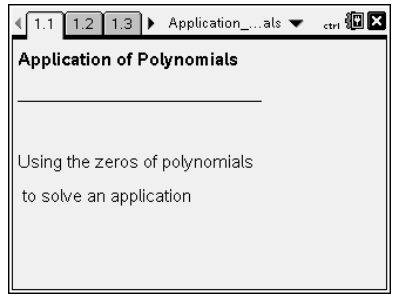
TEACHER NOTES

Math Objectives
- Students will formulate cubic polynomial equations to represent the volume of right rectangular prisms.
- Using graphs, students will find and test zeros of polynomial equations using substitution.
- Using graphs, students will recognize that not all cubic equations have three zeros.
- Students will interpret zeros of cubic polynomials in the context of the application.

Vocabulary
- zeros
- volume

About the Lesson
- This lesson is a follow-up lesson to the activity *Polynomials – Factors, Roots, and Zeros*.
- This activity uses the volume formula to find cubic polynomials in order to determine the dimensions of four different-size boxes used for packaging trash bags. Graphical representations are used to find the zeros of the polynomials.

Related Lessons
- Prior to this lesson: Polynomials – Factors, Roots, and Zeros



TI-Nspire™ Technology Skills:
- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:
- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity
- Application_of_Polynomials_Student.PDF
- Application_of_Polynomials_Student.DOC

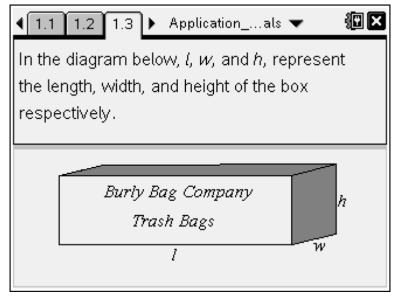
TI-Nspire document
- Applications_of_Polynomials.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: Press (esc) to hide the entry line if students accidentally click the chevron.

Move to page 1.3.



1. Using the information given, represent the length, width, and height of the box. Use x for the variable.

   <u>Answer:</u> Length = $x + 8$, Width = x, Height = $x + 1$

2. Using the results from Question 1, what formula would be used to find the volume of the box? Showing your work, write the expression in expanded form.

   <u>Answer:</u> $V = l \cdot w \cdot h = (x + 8) \cdot x \cdot (x + 1)$

   $= (x^2 + 8x)(x + 1) = x^3 + x^2 + 8x^2 + 8x$

   $= x^3 + 9x^2 + 8x$

Teacher Tip: Students may need to be reminded of the volume formula for a right rectangular prism and/or the FOIL method for multiplying binomials.

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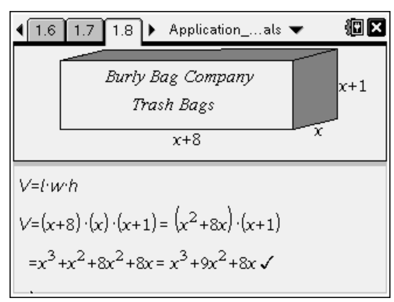


3. The results from Questions 1 and 2 are shown in the diagram on page 1.8. Why was x chosen to represent the width of the box and not the length or the height?

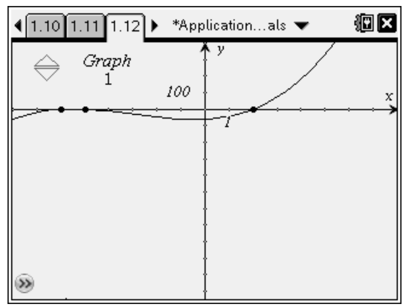
   <u>Answer:</u> The descriptions of both the height and length include comparisons to the width. The width is the dimension with the least amount of given information; therefore, x is used to represent the width.

4. Using the information above and the volume formula from page 1.8, represent the volume of the box for each type of bag as an equation. What type of polynomial functions are these equations?

Answer: The completed table is below. These equations are cubic polynomials.

Type of Bag	Volume Equation
small	$x^3 + 9x^2 + 8x = 60$
tall kitchen	$x^3 + 9x^2 + 8x = 240$
large lawn	$x^3 + 9x^2 + 8x = 390$
contractor	$x^3 + 9x^2 + 8x = 840$

Move to page 1.12.



5. On page 1.12, the points on the graphs indicate where the polynomial intersects the x-axis (zeros). Write the zero(s) for each graph in the table below. What do these values represent?

Sample Answers: These values represent x-intercepts, zeros, and widths. Answers may vary.

Graph #	Zero(s)
1	–6, –5, 2
2	4
3	5
4	7

6. Showing your work, determine which zeros belong to each of the polynomial equations from Question 4 using substitution.

Graph #	Polynomial Equation	Box Type	Substitution	Zeros
1	$x^3 + 9x^2 + 8x = 60$	**small**	$(-6)^3 + 9(-6)^2 + 8(-6) = 60$ $-216 + 324 - 48 = 60$ $60 = 60$ ✓ $(-5)^3 + 9(-5)^2 + 8(-5) = 60$ $-125 + 225 - 40 = 60$ $60 = 60$ ✓ $(2)^3 + 9(2)^2 + 8(2) = 60$ $8 + 36 + 16 = 60$ $60 = 60$ ✓	**–6, –5, 2**

2	$x^3 + 9x^2 + 8x = 240$	**tall kitchen**	$(4)^3 + 9(4)^2 + 8(4) = 240$ $64 + 144 + 32 = 240$ $240 = 240$ ✓	**4**
3	$x^3 + 9x^2 + 8x = 390$	**large lawn**	$(5)^3 + 9(5)^2 + 8(5) = 390$ $125 + 225 + 40 = 390$ $390 = 390$ ✓	**5**
4	$x^3 + 9x^2 + 8x = 840$	**contractor**	$(7)^3 + 9(7)^2 + 8(7) = 840$ $343 + 441 + 56 = 840$ $840 = 840$ ✓	**7**

7. Which of the three zeros on graph 1 represents the width of the box? How can you tell?

   Answer: The 2 is the only solution that makes sense in the context of the application problem since a width would not be negative.

8. Now that the width of each of the four box sizes has been determined, find the other dimensions for each box. Show your work in the table.

   Answer: Completed table is below.

Box Type	**Length ($x + 8$)**	**Width x**	**Height ($x + 1$)**
small	**2 + 8 = 10**	**2**	**2 + 1 = 3**
tall kitchen	**4 + 8 = 12**	**4**	**4 + 1 = 5**
large lawn	**5 + 8 = 13**	**5**	**5 + 1 = 6**
contractor	**7 + 8 = 15**	**7**	**7 + 1 = 8**

9. Check the dimensions you found for each of the four different-size boxes by using the volume formula ($V = l \cdot w \cdot h$). The volumes should match the values given on page 1.9. Show your work.

Box Type	$V = l \cdot w \cdot h$
small	$V = 10 \cdot 2 \cdot 3 = 60$
tall kitchen	$V = 12 \cdot 4 \cdot 5 = 240$
large lawn	$V = 13 \cdot 5 \cdot 6 = 390$
contractor	$V = 15 \cdot 7 \cdot 8 = 840$

Wrap Up:

Upon completion of the discussion, the teacher should ensure that students are able to:

- Understand how to use graphs to find zeros of polynomials.
- Understand how to test zeros of polynomial equations using substitution.
- Recognize that not all cubic equations have three zeros.
- Understand how to interpret zeros of cubic polynomials in the context of an application.
- Recognize that not all zeros of polynomials are solutions in the context of an application.
- Understand how polynomials can be used to solve real-world applications.