## Light at a Distance: Distance and Light Intensity

While traveling in a car at night, you may have observed the headlights of an oncoming vehicle. The light starts as a dim glow in the distance, but as the vehicle gets closer, the brightness of the headlights increases rapidly. This is because the light spreads out as it moves away from the source. As a result, light intensity decreases as the distance from a typical light source increases. What is the relationship between distance and intensity for a simple light bulb?

In this activity you can explore the relationship between distance and intensity for a light bulb. You will record the intensity at various distances between a Light Sensor and the bulb. The data can then be analyzed and modeled mathematically.


## OBJECTIVES

- Collect light intensity versus distance data for a point light source.
- Compare data to an inverse-square model.
- Compare data to a power law model.
- Discuss the difference between an inverse-square model and a power law model.


## MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator EasyData application data-collection interface

Light Sensor
meter stick or tape measure dc-powered point light source

## PROCEDURE

1. Arrange the equipment. There must be no obstructions between the bulb and the Light Sensor during data collection. Remove any surfaces near the bulb, such as books, people, walls or tables. There should be no reflective surfaces behind, beside, or below the bulb. The filament and Light Sensor should be at the same vertical height. This makes the light bulb look more like a point source of light as seen by the Light Sensor. While you are taking intensity readings, the Light Sensor must be pointed directly at the light bulb.
2. Set up the Light Sensor for data collection.
a. Turn on the calculator.
b. If you are using the Vernier Light Sensor, set it to 0-600 lux for a small light source, or 0-6000 lux for a larger light source.
c. Connect the Light Sensor, data-collection interface, and calculator.
3. Set up EasyData for data collection.
a. Start the EasyData application, if it is not already running.
b. Select $/$ File from the Main screen, and then select New to reset the application.
c. Select Setup from the Main screen, and then select Events with Entry.
4. Dim the lights to darken the room. A very dark room is critical to obtain good results.
5. Hold the Light Sensor about 10 cm from the light bulb filament. Move the sensor away from the bulb and watch the displayed intensity values on the calculator screen.
$\Rightarrow$ Answer Question 1 on the Data Collection and Analysis sheet.
6. To account for the particular brightness of your light source, choose a starting distance that gives a reading less than the maximum reading for your sensor ( 600 or 6000 lux for the Vernier sensor, or 1 for the TI sensor), but as large as possible. However, do not get any closer than 5 cm for small ( $<5 \mathrm{~mm}$ ) bulbs, or 10 cm otherwise. Choose the starting distance, and enter it as $\mathrm{X}_{\mathrm{L}}$ in the Data Table on the Data Collection and Analysis sheet.
7. Again place the Light Sensor at your planned starting distance from the light bulb filament. Important: The distance must be measured carefully. Be sure you measure from the filament of the lamp to the sensor tip on the Light Sensor.
8. Select $\longdiv { \text { Start } }$ from the Main screen to prepare for data collection.
9. Wait for the value displayed on the calculator to stabilize. Select $\widetilde{\text { Keep, }}$, and then enter the distance between the Light Sensor and the light source in meters on the calculator. Select OK to store the data pair.
10. Move the Light Sensor 1 cm farther away from the light source and repeat Step 9.
11. Continue moving the sensor in $1-\mathrm{cm}$ increments until the readings fall to less than $10 \%$ of the initial reading, collecting data as before. After the final data point, select $\sqrt{\text { Stop }}$ to end data collection.
12. Inspect the graph of light intensity versus distance. Trace to read the $x$ and $y$ values of the left-most point, round the values to three significant figures, and record them as $X_{L}$ and $Y_{L}$ in the Data Table.
13. Select Main to return to the Main screen. Exit EasyData by selecting $\sqrt{\text { Quit }}$ from the Main screen and then selecting $\overparen{O K}$.

## ANALYSIS

1. Redisplay your graph outside of EasyData.
a. Press 2 nd [STAT PLOT].
b. Press ENTER to select Plot1 and then press ENTER again to select On.
c. Press 200 m .
d. Press $\Theta$ until ZoomStat is highlighted; press ENTER to display a graph with the $x$ and $y$ ranges set to fill the screen with data.
2. Inspect your graph of the light intensity versus distance.
$\Rightarrow$ Answer Question 2 on the Data Collection and Analysis sheet.
3. One model for light intensity holds that the intensity is proportional to the inverse square of the distance from a point light source; that is, a graph would be of the form $y=C / x^{2}$, where $C$ is an adjustable parameter. Does your data follow this model? You can check it out by finding an approximate value for $C$, and then graphing the model with the data. First, enter the model equation.
a. Press $\gamma=$
b. Press Clear to remove any existing equation.
c. Enter $\mathrm{C} / \mathrm{x}^{2}$ in the $\mathrm{Y}_{1}$ field.
d. Press 2 nd [QUIT] to return to the home screen.
4. Set a value for the parameter $C$ and then look at the resulting graph. To obtain a good fit, you will need to try several values for $C$. Use the steps below to determine an initial guess for the parameter. One way to find an approximate value for the parameter is to use the left-most point. If you solve for $C$, then $C=y x^{2}$. Use the $x$ and $y$ values for the left-most point to calculate an initial value for $C$. Record this value in the Data Table on the Data Collection and Analysis sheet.
a. Enter a value for the parameter $C$. Press sTOD $C$ ENTER to store the value in the variable C.
b. Press $\Theta$ GRAPH to see the data with the model graph superimposed.
c. Press 2 nd [QUIT] to return to the home screen.
5. If the model is systematically high or low, you may want to adjust the value of $C$ to improve the fit. As you did before, store a new value in C, and then display the graph and model. Once you have optimized the model, record the complete equation in the Data Table.
$\Rightarrow$ Answer Question 3 on the Data Collection and Analysis sheet.
6. Another model can be used to compare to your data. The general power law of $y=a x^{b}$ may provide a better fit than the inverse square function, especially if the light source is not small or if there are reflections from walls or other surfaces. The difference between this new model and the inverse square model is that the exponent is not fixed at -2 . Instead, the exponent is now an adjustable parameter. The calculator can be used to automatically determine the parameters $a$ and $b$ in the general power law relation to the data.
a. Press sTAT and use the cursor keys to highlight CALC.
b. Press $\circledast$ repeatedly to scroll down to PwrReg. When it is highlighted, press ENTER to copy the command to the home screen.
c. Press 2 2nd $[\mathrm{L} 1] \leadsto$ 2nd $[\mathrm{L} 2] \longrightarrow$ to enter the lists containing the data.
d. Press vars and use the cursor keys to highlight Y-VARS.
e. Select Function by pressing Enter.
f. Press ENTER to copy $Y_{1}$ to the home screen.
g. On the home screen, you will now see the entry PwrReg L1, L2, Y1. This command will perform a power law regression with L 1 as the $x$ and L 2 as the $y$ values. The resulting regression curve will be stored in equation variable Y1. Press ©nter to perform the regression. Use the parameters $a$ and $b$, rounded to two significant figures, to write the power law model equation in the Data Table.
h. Press GRAPH to see the graph of your data and the power regression function.
$\Rightarrow$ Answer Questions 4 and 5 on the Data Collection and Analysis sheet.

## EXTENSIONS

1. Suppose that your patio is illuminated by an overhead light fixture with two bulbs. You decide to save on electricity by removing one of the bulbs. If the light is currently mounted 5 m off the ground, to what height should the light fixture be moved in order to retain the same amount of light on the patio with one bulb? Does your answer depend on the model you use?
2. Two identical light bulbs shine on your favorite reading chair from different locations in the room. The first bulb is 3 m from your chair and provides an intensity of $0.6 \mathrm{~mW} / \mathrm{cm}^{2}$. The second is 2 m from your chair. What intensity does this bulb provide? Does your answer depend on the model you use?

# DATA COLLECTION AND ANALYSIS 

Name
Date

## DATA TABLES

| Left-most point $X$ value $X_{\mathrm{L}}$ |  |
| :---: | :--- |
| Left-most point Y value $\mathrm{Y}_{\mathrm{L}}$ |  |
| Initial model parameter $\boldsymbol{C}$ |  |
| Optimized inverse-square model $y=C / \boldsymbol{x}^{2}$ |  |
| Power law model $y=a \boldsymbol{x}^{\boldsymbol{b}}$ |  |

## QUESTIONS

1. What is your prediction for the relationship between intensity and the distance to a light source?
2. Is the graph of the light intensity versus distance consistent with your earlier prediction?
3. How well does the inverse-square model $y=C / x^{2}$ fit the experimental data?
4. How well does the power law model fit the data? Could it fit any better than the inversesquare model? Could it fit more poorly?
5. How would using a brighter light bulb affect the parameters $a, b$, and $C$ in the two models?
