EXPLORING HIGH SCHOOL PHYSICS WITH TI-Nspire™ TECHNOLOGY

Three-Day Physics Workshop

Day 1

Activity	Probe
0. Overview of TI-Nspire	
1. Force vs. Mass	Force
2. Position/Velocity/Acceleration	
3. Intro to CBR2/Graph Matching	CBR2
4. Cooling Curves	Temp
5. Entering Data, Functions, Regression	

Day 2

Activity	Probe
6. Boyle's Law	Pressure
7. Ball on a Ramp	CBR2
8. Intensity vs. Distance	Light
9. Pendulum	CBR2
10. Air Resistance	CBR2

Day 3

Activity	Probe
11. Point Charges	
12. RC Time Constant	Voltage
13. Tale of the Tape	
14. Force of Tension	Force, CBR2
15. Focusing Light	Light
16. Nuclear Binding Energy	
17. Trajectory Activity	Simulation

Three-Day Physics Workshop

Day 1

Activity	Probe	Equipment
0. Overview of TI-Nspire		
1. Force vs. Mass	7 Dual Range Force 7 EasyLink™	7 Mass Hangers, 7 Mass sets with 10 gram masses and combinations up to 100 grams
2. Distance Velocity		
3. MoveIt	7 - CBR2 with USB link cable	2 rolls of Masking tape 7 Tape measure/meter stick
4. Cooling Curves	21 - EasyTemp	7 -100 ml boiling water 7 Insulated container 3 rolls Paper towels 20 Safety goggles 7 Blank sheets of paper
5. Planetary Motion		

Day 2

Activity	Probe	Equipment
6. Boyle's Law	7 Gas Pressure 7 EasyLink™	
7. Ball on a Ramp	7 CBR2 with USB link cable	7 Inclined planes at least 1 m long 7 Small balls, such as a racket ball 7 Meter stick or tape measure
8. Light Intensity Distance	7 Light 7 EasyLink™	7 Meter stick or tape measure 7 - 1m x 1m piece of white paper 2 rolls of Masking tape 7 Flashlights 7 Vis-a-Via pens
9. Harmonic Mostion	7 CBR2 with USB link cable and clamp	7 Pendulums 1 Balance or use the Force Probes 2 rolls of fishing line
10. Air Resistance	7 CBR2 with USB link cable	35 basket-style coffee filters

Numbers based on 20 participants

Three-Day Physics Workshop

Day 3

Activity	**Probe**	**Equipment**
11. Forces Point Charges		
12. RC Time Constant	7 Vernier® Voltage Probe 7 EasyLink™	7 - 10μF capacitor 7 - 22 kΩ and 7 - 47 kΩ resistors 7 - 9 V batteries
13. Tale of the Tape		
14. Force of Tension	7 Dual Range Force Probe 7 CBR2 with USB link cable 7 EasyLink™	7 sets of Various masses 7 Mass hanger 7 Carts 7 Pulley and Clamps for edge of table 2 Balls of string/fishing line 2 Scissors
15. Focusing Light		
16. Nuclear Binding Energy		
17. Trajectory Activity		
18. Computer Lab		Registration of Software http://www.tiphysics.com/ Use Link Software – Xfer files, Screen Shot, Upgrade OS Visit/Download/Do Nspire Physics from Activity Exchange or TI Physics Print from Link and Nspire software
Other		21-Nspire 1.4 CAS software w/GO Links and mini USB adaptor Computer Lab with Internet Access and Print access 1 - Nspire Panel and Overhead Projector 1 - Computer Projector 1 - Connect-to-Class 21 Safety goggles 21 CAS handhelds with unit to unit cable and USB GraphLink Several AA and AAA batteries

Numbers based on 20 participants

Activity	TNS File
DAY 1	
0. Beginning with the TI-Nspire™ Math and Science Learning Handheld	none
1. Force Related to Mass	none
2. Distance, Velocity, and Acceleration	d-v-a.tns
3. Move It!	match.tns
4. Cooling Rates	Cooling.tns
5. Analyzing Planetary Motion	none
DAY 2	
6. Boyle's Law	Boyle.tns
7. Rolling a Ball on an Inclined Plane	ballroll.tns
8. Light, Intensity, and Distance	LightandDistance.tns
9. Simple Harmonic Motion	SimpleHarmonicMotion.tns
10. Air Resistance	AirResistance.tns
DAY 3	
11. Forces on Point Charges	PointCharges.tns
12. Resistance – Capacitance Time Constant	RCtimeconstant.tns
13. The Tale of the Tape	TaleofTape.tns
14. Force of Tension and Free Body Diagrams	Force.tns
15. Focusing on Light	Foucsinglight.tns
16. Nuclear Binding Energy	Nuclearbinding.tns
17. Trajectory Activity Scenarios	Projectile Trajectory.tns
18. Computer Lab	none

Beginning with the TI-Nspire™ Math and Science Learning Handheld

Materials

- TI-Nspire™ Math and Science Learning Handheld

Overview

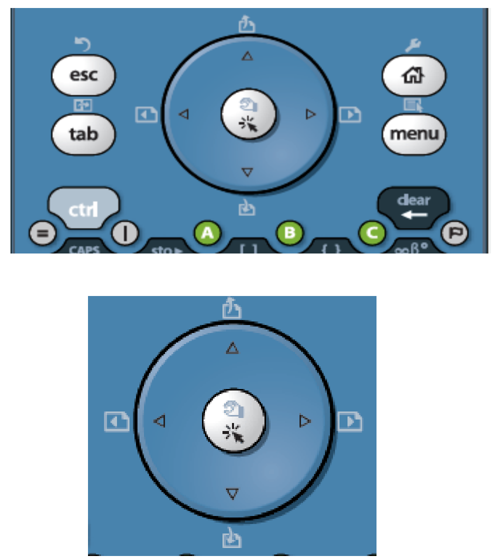
Explore the functionality of the TI-Nspire ™ math and science learning handheld.

Key Name	Function
Click Key See Figures 1 and 2.	Selects an object on the screen. ctrl + (click) grabs an object on the screen.
NavPad	Press the arrow keys to move the cursor/pointer.
esc Escape Key	Removes menus or dialog boxes from the screen.
tab Tab Key	Moves to the next entry field.
(home) Home Key	Displays the home menu.
menu Menu Key	Displays application or context menu.

The Calculator Application

1. Turn on the TI-Nspire ™ learning handheld.
   - If the screen shown in Figure 3 is not displayed, press (home) 1 for Home 1:Calculator to add a new application page (Figure 3).

Note: To select a menu option, you can highlight the option and press enter or (click). Alternatively, you can press the number key for that option.



NavPad Cursor Controls

Figures 1 and 2

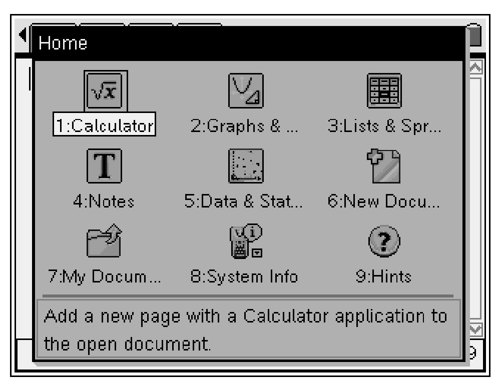


Figure 3

2. Perform the operations shown in Figures 4 to 11:

Note: Screen shots for the CAS handheld may be different.

- For an approximate value, press ctrl enter (Figure 4).
- To clear the calculator screen, press menu 1 5 for Menu 1:Actions, 5:Clear History (Figure 5).
- To solve an equation, press menu 3 1 for Menu 3:Algebra, 1:Solve (Figure 5).
- To do a numerical solve, press menu 3 5 for Menu 3:Algebra, 5: Numerical Solve (Figure 5).
- The format for nSolve is (equation, variable, low bound, up bound).

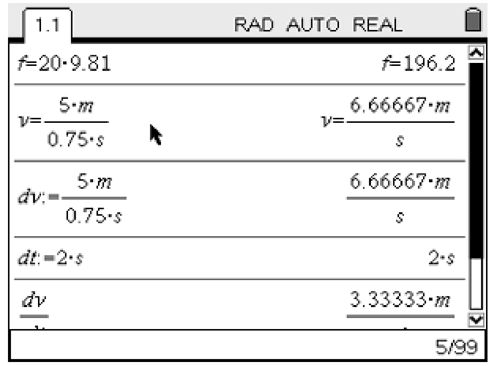


Figure 4

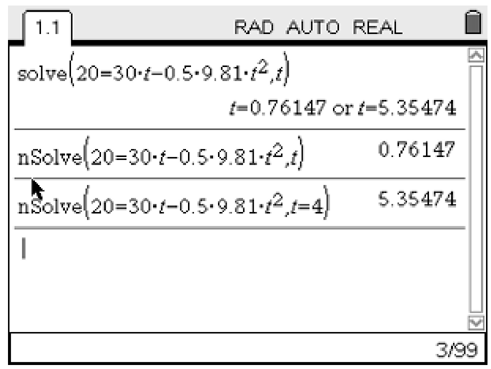


Figure 5

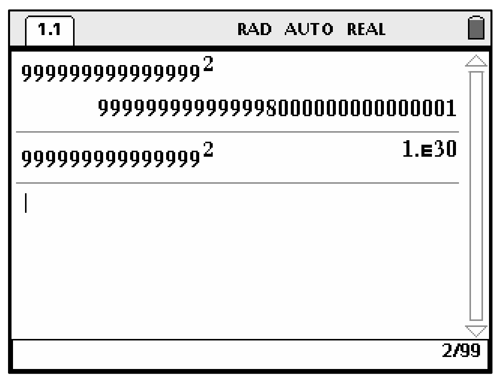


Figure 6

- To access the square root command, press ctrl, and choose the fraction template or type ctrl.
- To get a decimal output press ctrl enter (Figure 7).

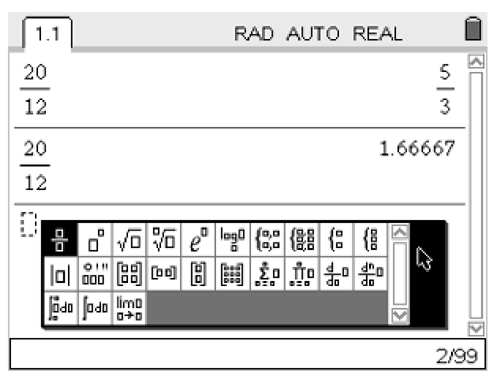


Figure 7

- To raise a number to a power, enter the base, press ctrl , and select the template shown in Figure 8.

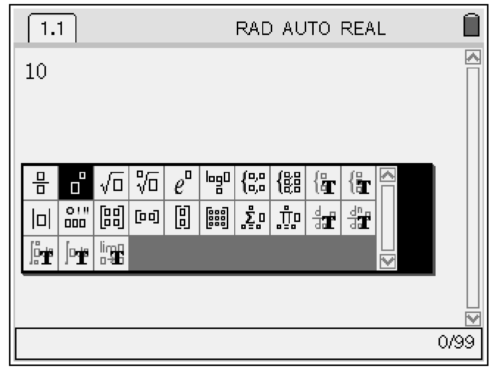


Figure 8

- Enter the power, and press enter (Figure 9).

Note: Entering the base and then pressing will produce the same result as choosing this template.

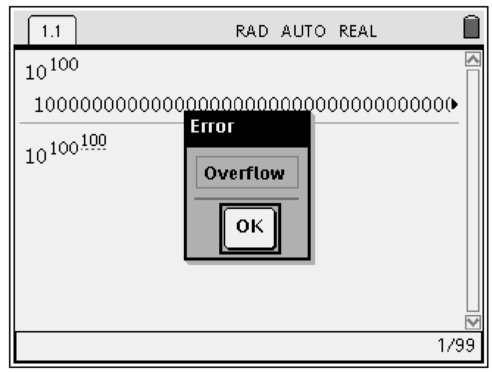


Figure 9

- To access the degree symbol, press ctrl for the symbols menu (Figure 10).
- Move the cursor to the right, and select the ° symbol.
- Press enter.

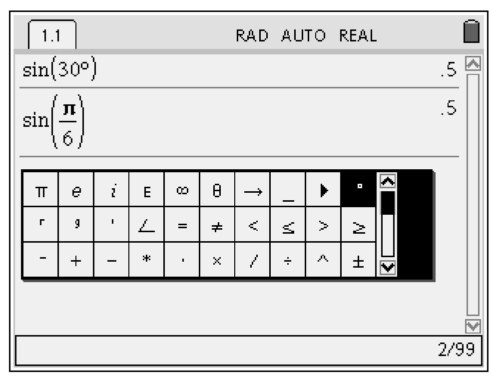


Figure 10

- Press ctrl to access the templates for matrices.

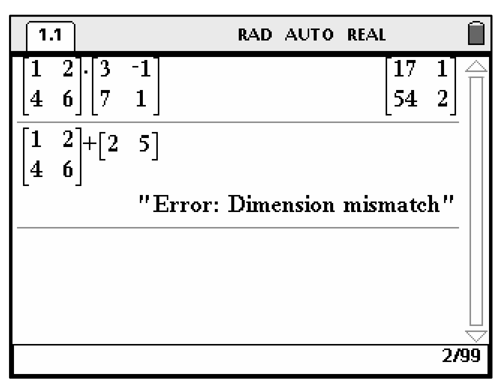


Figure 11

The Graphs & Geometry Application Basic Graphing

3. Press [home] [2] for Home 2:Graphs & Geometry to add a new application page (Figure 12).

- The Graphs & Geometry application is now page 1.2 of the document.

Note: The graphing window shown is the default window setting with a screen aspect ratio of one.

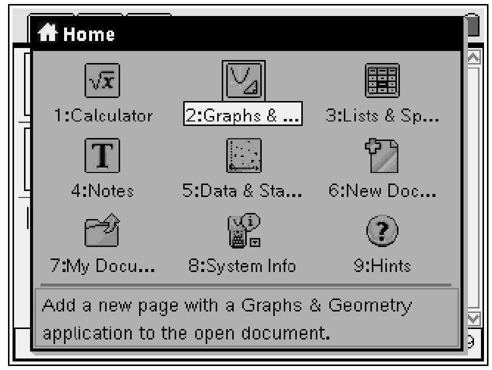


Figure 12

4. The function notation f1(x) is shown in the entry line (Figure 13).

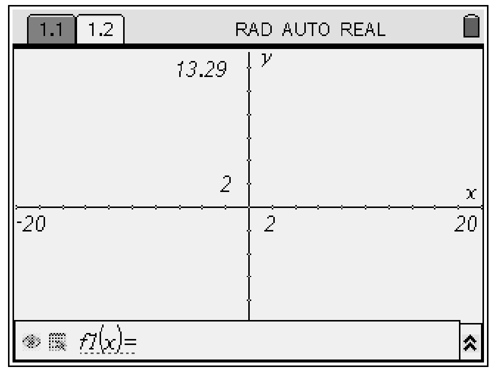


Figure 13

5. Type $2x^2$-x+1, and press [enter] (Figure 14).

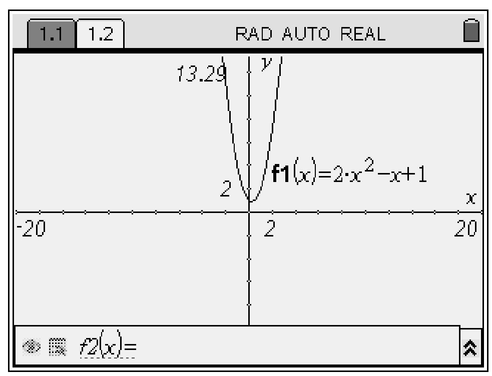


Figure 14

6. Rescaling the Axes.

- Press [tab] [tab] to exit the Equation Entry Line.
- Place the cursor on one of the axes so that the open hand appears and the tic marks flash (Figure 15).
- Press [ctrl] [click] to grab a tick mark.

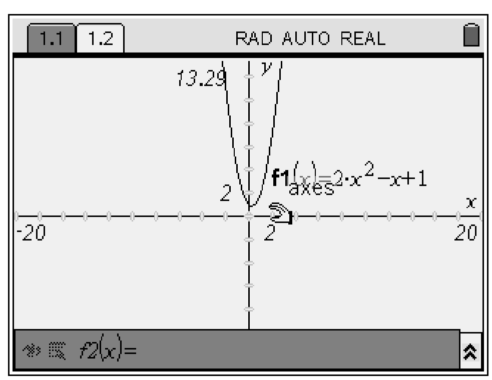


Figure 15

- Use the NavPad cursor controls to increase or decrease the maximum or minimum value and the scaling of the axes (Figure 16).
- This method will rescale both axes and will maintain the aspect ratio of the default window.

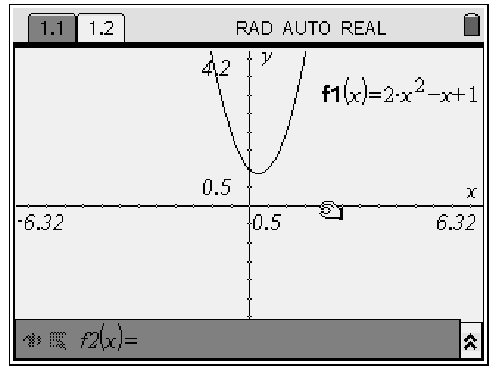


Figure 16

Alternate Graphing Method

1. Add a new Graphs & Geometry application page by pressing ⌂ 2 for Home 2:Graphs & Geometry.
2. From the Graphs & Geometry page, press menu 1 5 for Menu 1:Actions, 5:Text (Figure 17).

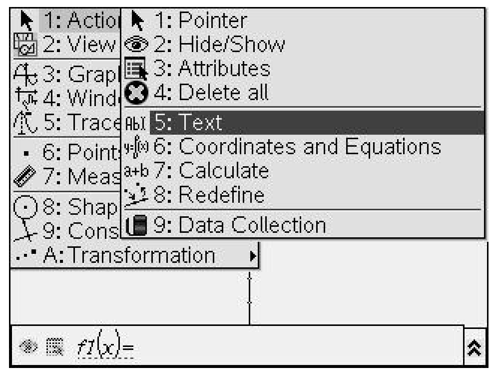


Figure 17

3. Click anywhere on the screen to open a text box (Figure 18).

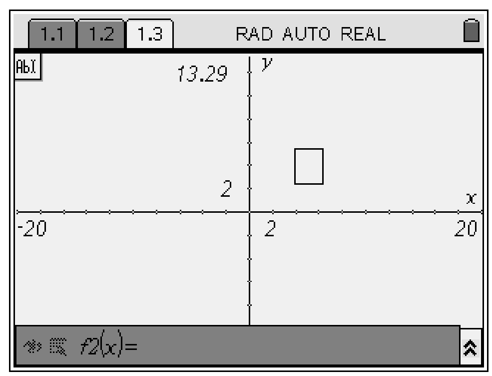


Figure 18

4. In the text box, type 2x^2-x+1, and press enter (Figure 19).
5. Press esc to exit the text mode.

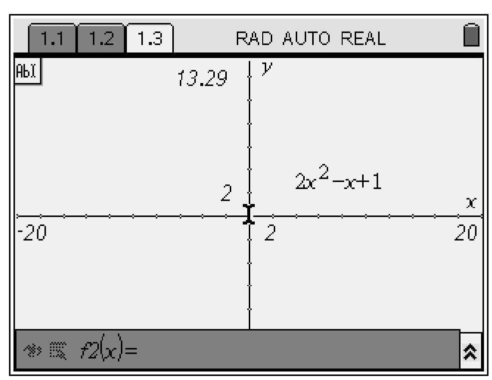


Figure 19

6. Use the NavPad to move the cursor over the equation until the hand appears (Figure 20).
   - Press ctrl , or hold down until the hand closes.

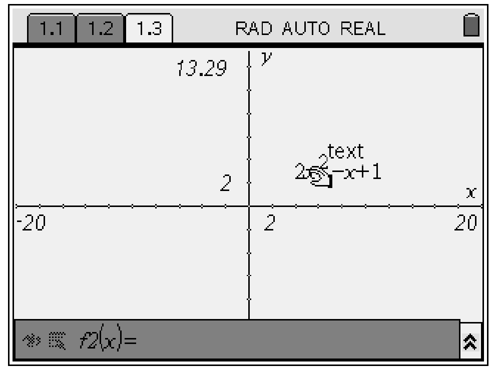


Figure 20

7. Drag the equation to the x-axis.
   - When the graph appears, press enter (Figures 21 and 22).

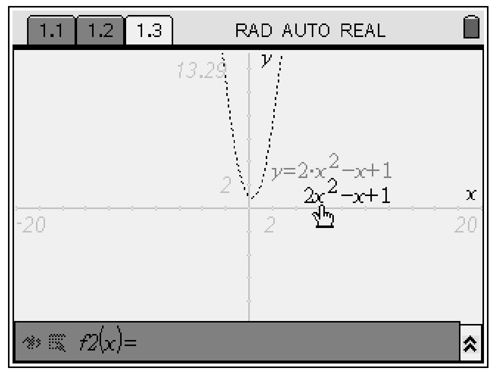


Figure 21

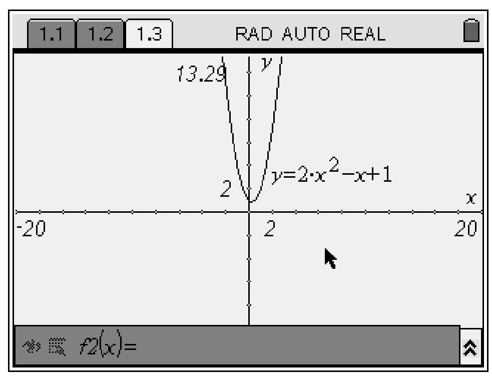


Figure 22

Document Model

While creating your calculations and graphs, you should have noticed the tabs at the top of your screen.

Moving Between Pages

1. Press ctrl ◀ (on the NavPad) to move back one page (Figure 23).

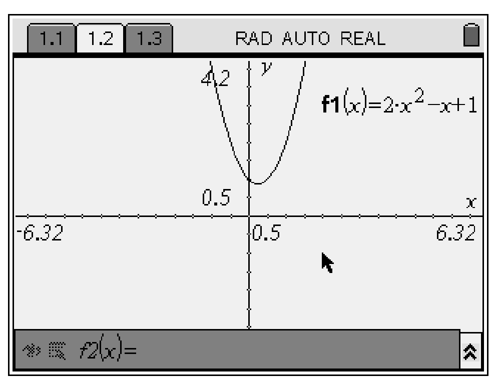


Figure 23

2. To view all of the pages of the problem, press ctrl ▲ (on the NavPad) (Figure 24).

Note: This screen allows you to move from one page to another. To open the page that you wish to view, highlight the page by moving left or right with the NavPad. When the appropriate page is highlighted, press enter.

To change the order of the pages, highlight the page by moving left or right with the NavPad, and hold the until the hand closed. Then move the page to the desired place, and press enter.

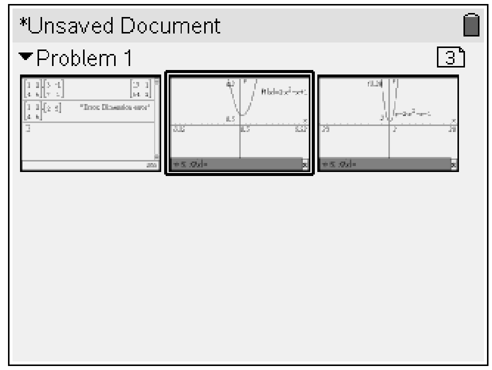


Figure 24

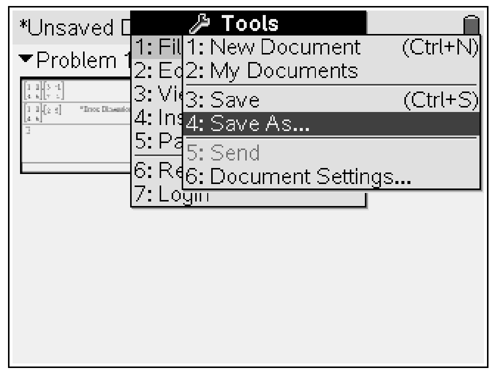


Figure 25

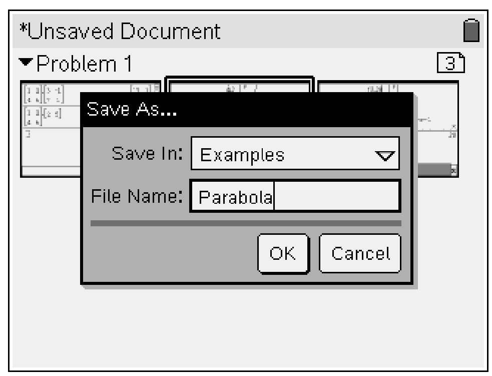


Figure 26

Saving a Document

1. Press ctrl home 1 4 for Tools 1:File, 4:Save As… (Figure 25).
2. When the Save As…dialog box appears, enter Parabola for the file name (Figure 26).
3. Press tab until OK is highlighted, and press enter to save the document.
4. Check to see if the document was saved.

5. Press (home) (7) for Home 7:My Documents (Figure 27).

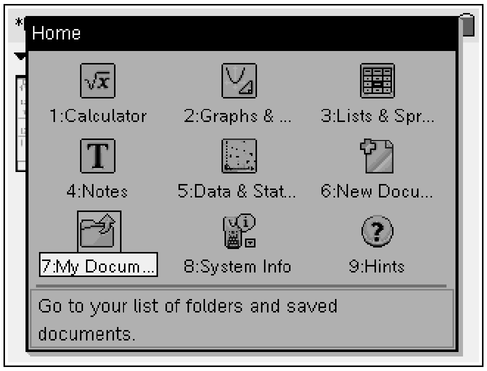


Figure 27

6. The folder will appear showing the document that you have just saved (Figure 28).

7. To reopen the document, highlight the file you wish to open, and press (enter).

Note: All work done on the TI-Nspire™ learning handheld is contained within a document. Each document contains one or more problems with a maximum of 30 problems. Each problem contains one or more pages with a maximum of 50 pages. Functions, stored variable, and data are retained throughout a problem.

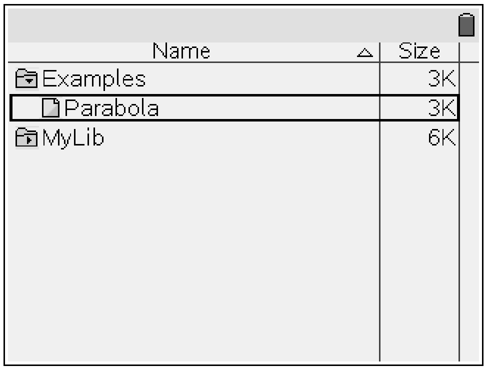


Figure 28

Force Related to Mass

Concepts

- Mass
- Force

Materials

- TI-Nspire™ Math and Science Learning Technology
- EasyLink™

Overview

This activity focuses on the relationship of mass to force.

Introduction

Open a new document, and add a Data & Statistics application page (Figure 1).

1. Press off on.
2. Press 6 for Home Menu 6:New Document.
3. If the menu pops up asking if you would like to save the document, select NO by pressing tab to move to the No button and press .
4. Press 5 for 5: Data & Statistics.
5. Connect the force probe to the Easy Link™ cable, plug the cable into the top of the TI-Nspire™ math and science learning handheld
6. When the Auto Launch menu appears, select Data & Statistics (Figure 1).

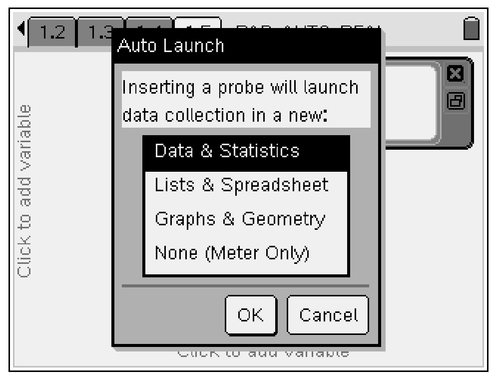


Figure 1

7. After selecting Data & Statistics, a graph should appear and the meter should be reading the force (Figure 2).
   - The sensor needs to be positioned and zeroed.
8. Hang the force probe in a vertical position with the hook downward (Figure 3).
9. Set the force probe to ±10 N.
10. Press menu 2 1 for Menu 2:Sensor, 1:Zero.
11. Setup the experiment to take Events with Entry
12. Press menu 1 3 2 for Menu 1:Experiment, 3:Set Up Collection, 2:Events with Entry.
13. Start Collecting Data.
14. Press the NavPad side arrows until the sample arrow is highlighted, and press to start.
15. Hang a 10 gram mass from the hook
16. When the meter is relatively stable, press to take the data point.
17. Enter 0.010 for the number of kilograms hanging from the hook.
18. Press tab, select OK, and press .
19. Hang another 10 grams mass, and repeat steps 14 – 16 changing the mass to 0.020.
20. Repeat for 30, 50, and 100 grams.
21. To stop sampling, press the NavPad side arrow keys until the stop button is highlighted.

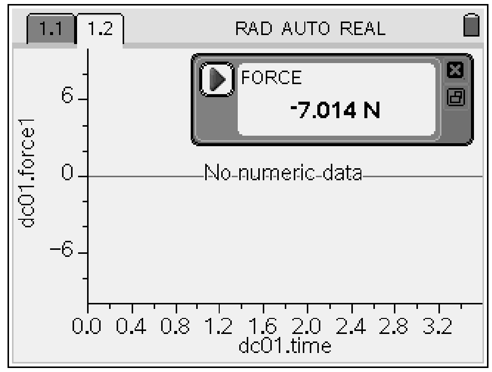


Figure 2

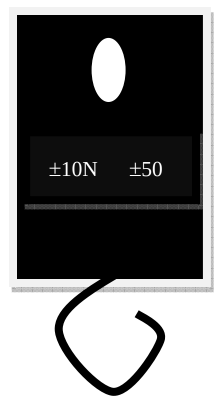


Figure 3

- Press .

22. Press tab or right and left arrows on the NavPad until the x in the data collection console is highlighted, and press to close the console.

Analysis

1. Press menu.
2. Press 4 for 4:Analyze
3. Press 6 for 6:Regression
4. Press 1 for 1:Show Linear (mx+b)
5. The linear equation will appear and be graphed on your screen.

Questions

1. What variable was manipulated (independent)?
2. What was the resultant variable (dependent)?
3. What does the *x* axis represent in this problem?
4. What does the *y* axis represent in this problem?
5. What is value of the slope of the line?
6. Rewrite the linear equation replacing *x* and *y* with the appropriate variables.
7. In the equation, what value would change if this experiment was done on the moon?
8. What does the slope represent?

9. Using the equation that was generated in question 6, answer the following questions.

a) What is the force of gravity on a 500 g sample?

b) What is the mass of an object with a force of 25.0 N?

10. Draw a picture of the force probe mass and connection between the force probe and the mass.

a) Draw an arrow from the center of the mass toward the ground, and label this F_w (Force of weight)

b) Draw an arrow from the center of the mass toward the force probe, and label it F_N (Force normal).

Distance, Velocity, and Acceleration

Concepts

- Distance
- Velocity
- Acceleration

Materials

- TI-Nspire™ Math and Science Learning Technology
- *d-v-a.tns* file

Overview

In this activity, we will explore the relationship between Distance, Velocity, and Acceleration.

Introduction

When a ball is launched in the air the ball is affected by gravity at the moment it is released. The equation for the flight of a projectile is as follows

$$d_t = d_0 + v_0*t + -.5*a*t^2$$

d_t is the distance at time t, d_0 is the initial distance or the distance at time zero, v_0 is the initial velocity or velocity at time zero, a is acceleration and t is time.

1. Open the *d-v-a.tns* file.
2. Press 7 for Home Menu 7: My Documents (Figure 1).
3. Highlight d-v-a, and press enter .
4. Read the introductory page.
   - To move to page 1.2 press ctrl ▶.
5. Move to page 1.3.

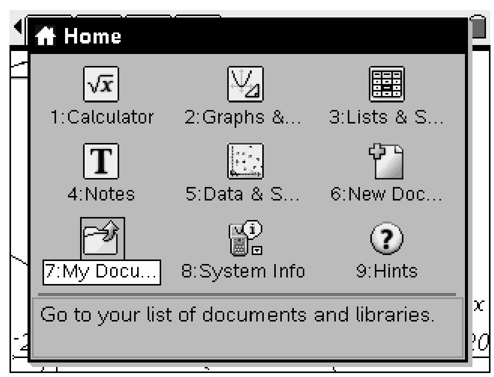


Figure 1

Questions

1. What affect does d_0 have on the flight of the object?
2. If the ground is the *x*-axis as d_o is increased, what does this do to the time at which object hits the ground?
3. Describe in a real world projectile situation what d represents.

Change v_o

4. What affect does v_o have on the graph?
5. What happens to the graph when v_o is zero?
6. Explain a real world situation where v_o is zero for the projectile.
7. What occurs when v_o is negative?
8. Describe a real problem where v_o would be negative.

Change a

9. What happens when a is greater than 9.81?
10. What is a situation where a would be greater than 9.81?
11. What happens when a is zero?

The moon has roughly 1/6 of the earth's gravitational attraction.

- If 9.81 is divided by 6, the acceleration of gravity on the moon would be determined.
- Change the a variable by dividing 9.81 by 6 (Figure 2).
- Return the v_o to 10 and d_o to 5.

12. How does this affect the height the object will fly?

13. How much longer does it take to reach the ground?

Velocity is the change in distance divided by the change in time.

- At any point on the curve, there is a different velocity.
- The slope of the line at any particular point represents the velocity.
- The tangent line attached to the curve represents the slope or velocity at any point on the curve.

Place the cursor over the point where the tangent and the curve intersect.

- Grab that point, and drag it along the curve.

To grab the point, perform the following steps.

1. Place the cursor over the point.

2. When the hand appears and the point label appears, press and hold until the hand closes.

3. At this point, use the NavPad Cursor Controls to move the point along the curve.

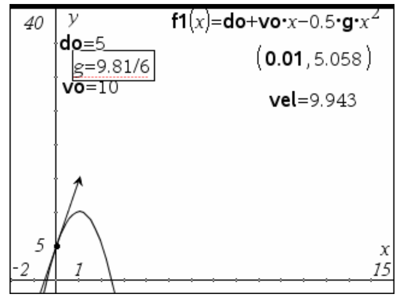


Figure 2

Questions

1. **1.** What happens to the slope of the line as you approach the peak of the curve?
2. **2.** What is the velocity at the peak?
3. **3.** Where is there the greatest positive velocity?
4. **4.** Where is there the greatest negative velocity?

To plot the velocities, move the tangent a small distance, and press ctrl ..

- This captures the time and the velocity at that time.
- Continue to move the tangent by small amounts, and capture 5 to 10 points.

5. What type of relationship is generated between time and velocity?

Create a moveable line to determine the slope of the line generated.

A. Press menu 6 4 for Menu 6: Points & Lines, 4: Line (Figure 2).

B. Click on the first velocity point, and move the cursor to the last point.

- Click to set the second point to create the line.

C. Press menu 7 3 for Menu 7: Measurement, 3: Slope (Figure 3).

D. Move the cursor over the line and click to find the slope of the line.

- Move the value to an empty spot, and click to drop the value.

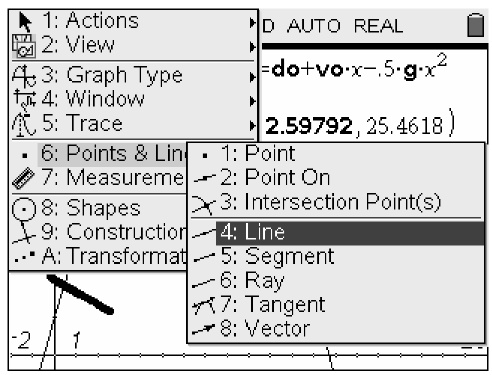


Figure 2

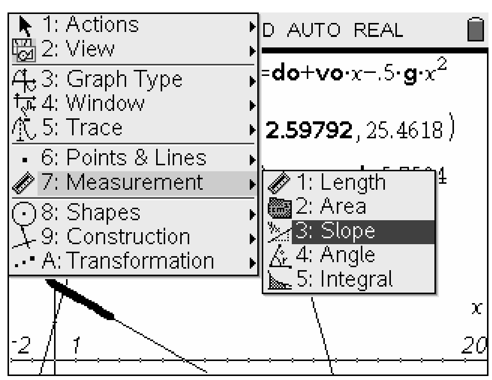


Figure 3

The velocity is the change in distance divided by time. This line represents the velocity at any time on the curve.

6. What is the slope measured in this graph?

7. Velocity is meters per second and time is in seconds, what are the units for slope?

8. What measure has the units for change in velocity divided by change in time?

9. What value does the slope match that is already on the screen?

10. What do the tangents of the distance curve represent?

11. What does the tangent of the velocity curve represent?

12. What affect does the change in gravity have on the height a projectile will travel?

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Move It!

Concepts

- Motion
- Mathematical equations

Materials

- TI-Nspire™ Math and Science Learning Technology
- CBR 2™ (Calculator-Based Ranger ™ 2)
- *Match.tns* file

Overview

In this laboratory activity, you will create graphs of your own motion. You will then try to describe the graphs conceptually, as well as, with mathematical equations. You will use a CBR 2™, which is an ultrasonic motion detector, with a TI-Nspire™ math and science learning handheld.

The study of motion is important in the study of physics. In this laboratory activity, you will create graphs of your own motion.

- You will then try to describe the graphs conceptually, as well as, with mathematical equations.
- You will use a CBR 2™, which is an ultrasonic motion detector with a TI-Nspire™ math and science learning handheld.
- The motion detector sends out an ultrasonic pulse which bounces off the nearest object and reflects back to the detector.
- Based upon the amount of time for each pulse to return, the distance from the object to the detector is computed.
- When the detector is activated, you will hear a clicking noise which indicates that the detector and CBR 2™ are collecting data.
- The range of the CBR 2™ is about 0.1 to 6 meters.

Part I: Making Positions versus Time Graphs

1. Use masking tape to set a scale on the floor.

- Place a piece of tape ½ meter from the CBR 2™ and then each ½ meter for 2 meters.

2. Connect the CBR 2™ to the TI-Nspire™ handheld with the USB link cable.

3. Select Data & Statistics as the application for data collection (Figure 1).

4. Press enter to begin collecting data.

5. Make position versus time graphs by walking back and forth in front of the motion detector.

- Make a sketch of two of the sample graphs obtained (Figures 2 and 3).
- Name these trials Motion A and Motion B.
- Write a description of how each graph was created.
- Include the starting position and direction that the walker moved.
- Motion A description: ___________________
  ____
  ____
  ____
- Motion B description: ___________________
  ____
  ____
  ____

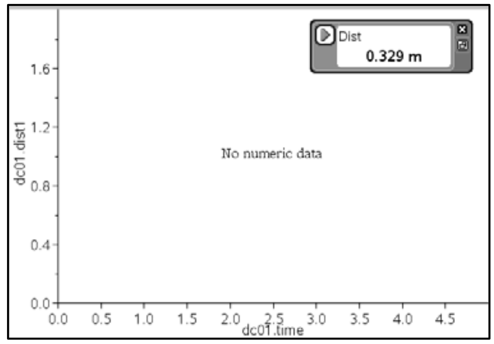


Figure 1

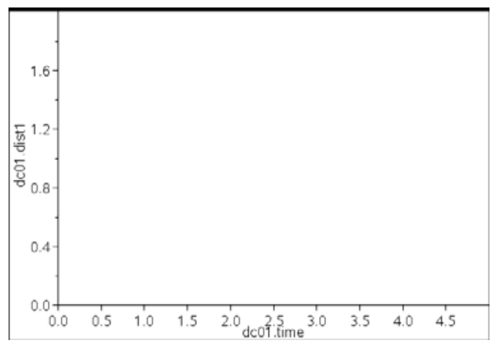


Figure 2

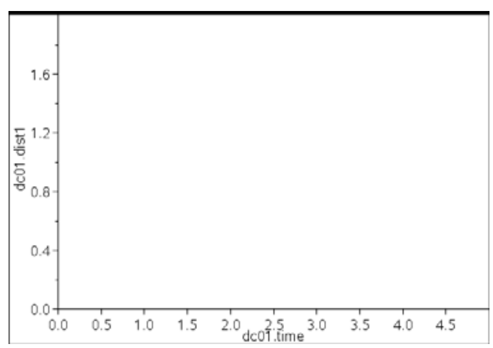


Figure 3

6. Predict and describe the position vs time graph of each of the following:

A. A person standing still 1.0 meter away from the motion detector.

B. A person walking with a constant speed toward the motion detector.

C. A person walking with a constant speed away from the motion detector.

D. A person first walks away from the detector and then walks back toward the motion detector.

7. Check your predictions.

- Explain any differences between your predictions and the graphs produced by the CBR 2™ and TI-Nspire™ system.

Part II: Match Me

1. Open the file *Match* on your TI-Nspire™ handheld.

- In this activity, you will try to match the graph of your motion to the given graph which will be shown on the TI-Nspire™ screen.
- The goal is to match the graph so that your graph is exactly on top of the one shown on the screen.
- After the graph is displayed, press enter to start sampling [FORMAT].
- If you are not satisfied with the graph, press enter to match again.

NOTE: To match the graph, you may either use the walk back and forth from a wall, aiming the CBR 2™ at the wall; or you may use the Viewscreen™ panel, and project the image on the TI-Nspire™ screen so that you can see it from the overhead projection screen.

2. From the MATCH activity that you have just completed, explain what you needed to consider in order to match the graph.

- Include as many considerations as possible.
- Sketch the graph in Figure 4.

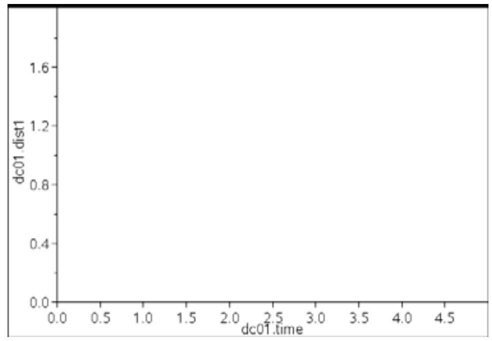


Figure 4

3. Once you have matched the first graph, proceed to the other graphs and match them.

4. Which graph was the most challenging to match?

- Explain why.

Part III: What's My Line

What happens when you move away from the CRB 2™ at a constant speed?

- Make a sketch of your prediction in Figure 5.
- Compare it with other members of your group.
- Explain why this graph should represent the motion of a walker at a constant speed moving away from the CBR 2™.

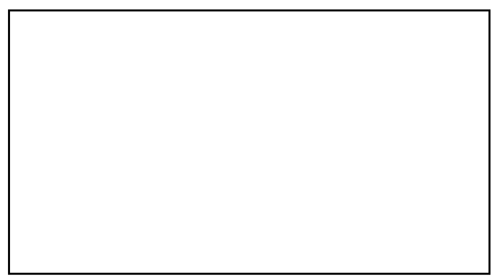


Figure 5

1. In the earlier sections of this laboratory activity, you described the position versus time graphs conceptually in writing.

- You will now describe motion mathematically with an equation.
- Open a new TI-Nspire™ document.

2. Connect the CBR 2™ to the TI-Nspire™ handheld with the USB link cable.
3. Select Data & Statistics as the application for data collection.
4. Make a position versus time graph by moving away from the CBR 2™ at a constant speed.
   - The walker in your group should move slowly away from the motion detector.
   - The walker should begin walking before the person managing the data collection begins collecting.
5. Begin walking, and then press enter to begin data collection.
6. If the graph is linear, begin analyzing the data.
   - If not, press enter and collect the data again.
   - Discard the first set of data.

Data Analysis

1. Disconnect the CBR 2™.
   - Use the tab key to select and close the data collection console.
2. Sketch a graph of the motion in Figure 6.
3. Your graph should show a line with positive slope.
   - Note that the units on the y-axis are meters and the units on the x-axis are seconds.
   - Select two points on the graph which are not close together, and find the slope.

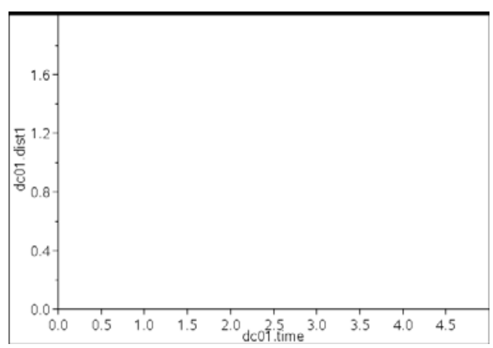


Figure 6

- To select the points, move the cursor over the point; and hold the key.
- Record your values below:

Selected points: t_1 = __________

d_1= __________

t_2 = __________

d_2 = __________

4. Add a Calculator application page to your TI-Nspire™ document.

- Calculate the slope of the line between the two points.
- The equation for the slope is the change in position divided by the corresponding change in time $slope = \frac{\Delta d}{\Delta t}$.
- Show your method below along with your answer.
- Include units in your answer:

Slope = ________________

5. What does the slope represent physically?

- Hint: look at the units.

6. How would the slope change if the walker moved faster?

7. To fully describe the original motion mathematically, you need the slope and the starting position.

- Find the starting position, and record it below:

Starting position = __________

8. The equation of motion for an object or person moving with constant velocity is $d = d_o + v\ t$, where d is the position or distance at any time from the detector, d_o is the starting position, v is the velocity, and t is the time.
    - Use the values that you found above to write an equation that fits your data.
    - Since the values of *d* and t vary, they should be written as variables.
    - Record the equation.
9. Check to see if your equation matches one that you can find in another manner using the features of the TI-Nspire™ handheld.
    - Go back to the graph of the motion.
    - From the Analyze menu, select Add a Movable Line (Figure 7).
    - Grab the line at the ends to change the slope.
    - You will see a circular icon with arrows to indicate that you can rotate the line to change the slope.
10. Grab the line in the center, and move the arrows in the shape of a cross to move the line up and down.
    - Select escape at the end of each move.
11. Record the equation of the line.
12. How is the mathematical equation on the TI-Nspire™ different from the physics form of the equation for the motion of the walker?

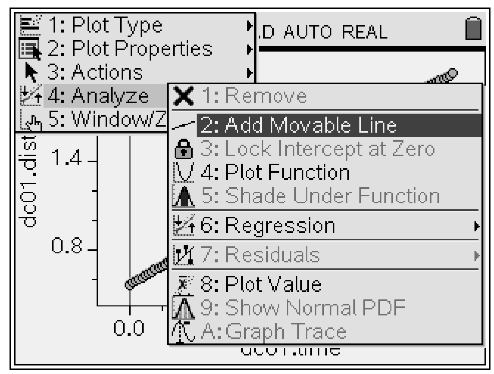


Figure 7

Move the Other Way

1. Make a prediction about what happens when you move toward the CBR 2™ at a constant speed.
   - Sketch your prediction in Figure 8.
2. From the Tools menu, select 4:Insert, and then 1:Problem (Figure 9).
3. Connect the CBR 2™ to the TI-Nspire™ with the USB link cable.
   - Select Data & Statistics as the application for data collection.
4. Make a position versus time graph by moving toward from the CBR 2™ at a constant speed.
   - The walker in your group should move slowly toward from the motion detector.
   - The walker should begin walking before the person managing the data collection begins collecting.
5. Begin walking, and then press enter to begin collecting data.
6. If graph is linear, begin analyzing the data.
   - If not, press enter; and collect the data again.
   - Discard the first set of data.

Data Analysis

1. Disconnect the CBR 2™.
   - Use the tab key or the NavPad arrows to select and close the data collection console.
2. Sketch a graph of the motion (Figure 10).

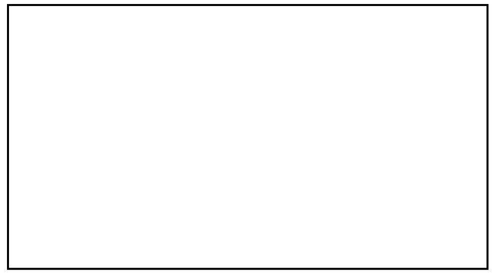


Figure 8

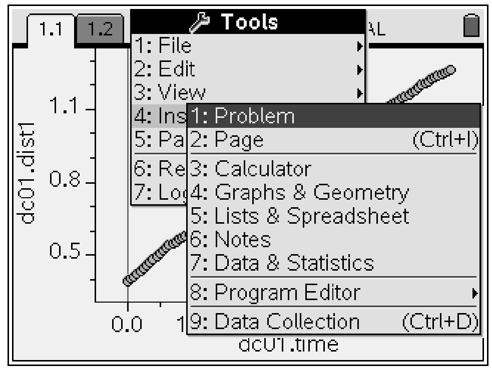


Figure 9

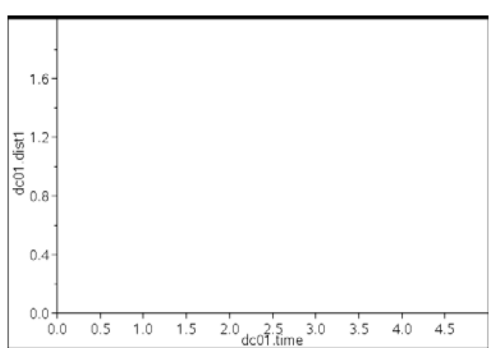


Figure 10

3. Your graph should show a line with negative slope.

- Note that the units on the y-axis are meters and the units on the x-axis are seconds.
- Select two points on the graph which are not close together, and find the slope.
- To select the points, move the cursor over the point and hold the  key.
- Record your values below:

Selected points: t_1 = __________

d_1= __________

t_2 = __________

d_2 = __________

4. Add a Calculator application page to your TI-Nspire™ document.

- Calculate the slope of the line between the two points.
- The equation for the slope is the change in position divided by the corresponding change in time $slope = \frac{\Delta d}{\Delta t}$.
- Show your method below along with your answer.
- Include units in your answer:

Slope = _______________

5. Find the starting position, and record it below.

- Starting position = ____________

6. Use the values that you found above to write an equation that fits your data in the form $d = d_0 + v\,t$.

- Record the equation.

7. Check the equation you wrote this time using a linear regression feature of the TI-Nspire™.

- Select the Data & Statistics page.
- From the menu, select 4:Analyze, and then 6:Regresion (Figure 11).

8. From the list of regression options, choose 1:Show Linear (mx +b).

- See Figure 12.

9. Record the equation.

10. How does it compare with the equation that you found from using two points?

11. What does it mean to have a negative velocity?

- Positive velocity?
- Zero velocity?

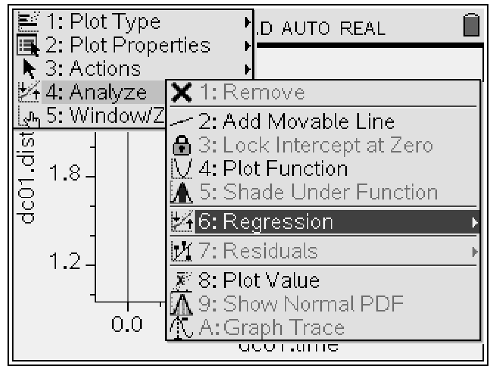


Figure 11

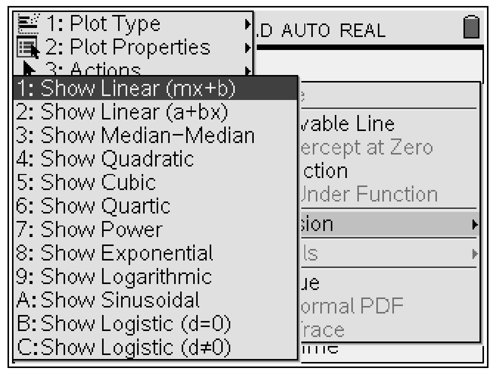


Figure 12

Cooling Rates

Student Worksheet

Name ____________________

Class ____________________

In this activity, you will explore the following:

- how the temperature of an object decreases with time
- how to make a mathematical model of a physical phenomenon

Introduction

Predict what the graph will look like for an object as it cools.

- In this activity, you will heat a temperature probe in hot water and then take data as it cools.
- Sketch a graph of your prediction for the data in Figure 1.

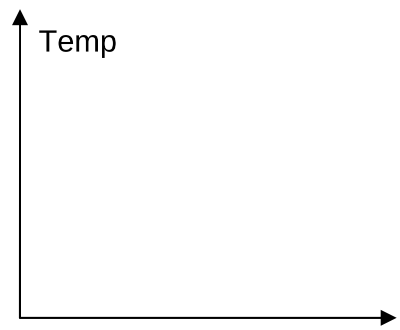


Figure 1

- Explain why you think the graph will look like your sketch.

Open the file **Cooling in the Physics folder** on your handheld or computer, and follow along with your teacher for the first two pages (Figure 2).

- Move to page 1.2 and wait for further instructions from your teacher.

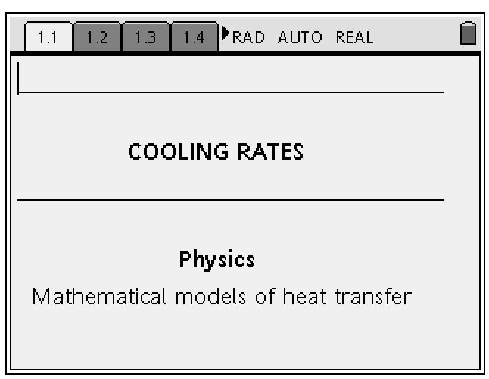


Figure 2

In this activity, you will collect temperature data for a cooling metal sensor (Figure 3).

- Then, you will attempt to fit an equation to the data you collect.
- Finally, you will explore the relationship between cooling rate and temperature difference.

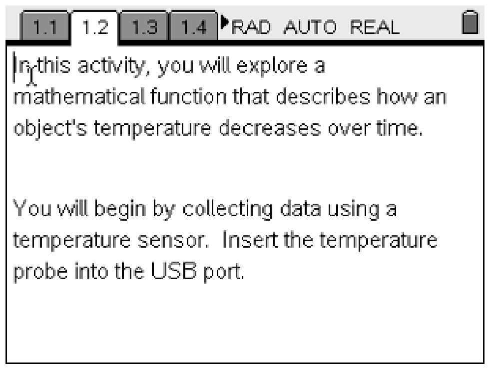


Figure 3

Part 1: Collecting Temperature Data

1. Put on a pair of safety goggles, and connect a Vernier® EasyTemp™ or Go!™Temp temperature sensor to your handheld or computer.

- Choose to collect data in the Data & Statistics application page.
- The light on the sensor should come on, and the display in the data collection box on a blank graph screen should show the current temperature reading (Figure 4).

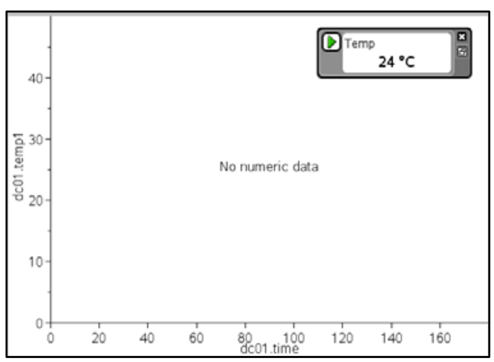


Figure 4

- Wait until the temperature reading stabilizes.
- This is the ambient (room) temperature.

Q1. What is the ambient (room) temperature?

2. Your teacher will give you a container of hot water. **CAUTION**: Be very careful handling the hot water.

- Make sure your work area is clear, and be careful not to spill the water.
- Place the end of the temperature sensor into the boiling water.
- Watch the temperature readout until it has stabilized.

3. Remove the sensor from the hot water, and wipe it off with a paper towel.

- Immediately after you wipe the water off the sensor, click the ▶ button to start the data collection.
- Data collection has started when data begins to be displayed on the graph.
- Data collection will continue for three minutes.
- During this time, the sensor will record one data point each second.
- After three minutes, close the Data Collection Console to view just the graph.

Note: The temperature reading in the box will continue to change even after data collection has stopped. The ▶ symbol will reappear when data collection has finished.

Q2. Describe the shape of the graph.

- How does it compare with your prediction?

Part 2: Fitting a Curve to the Data

4. Sketch your graph or print a copy for this lab report (Figure 5).

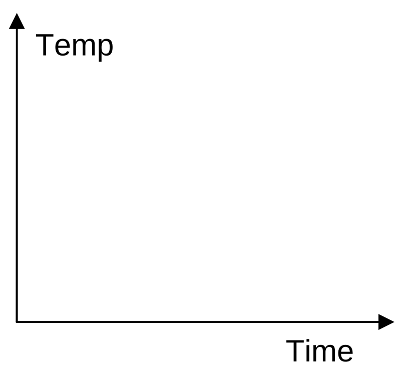


Figure 5

- The shape of the graph should be consistent with an exponential function of the form $y = a + b \cdot x^c$.
- Next, you will attempt to find the values of *a*, *b*, and *c* that produce a curve that best fits your recorded data.

Q3. Should *c* be greater than, less than, or equal to 1?

- Explain your answer, and give your prediction for the value of *c*.

Q4. What value should *a* have?

- Hint: What will happen to the temperature of the sensor as time approaches infinity?

Q5. What value should *b* have?

- Explain your reasoning.

5. Move to the Graphs & Geometry application page, and set up a stat plot (Figure 6).

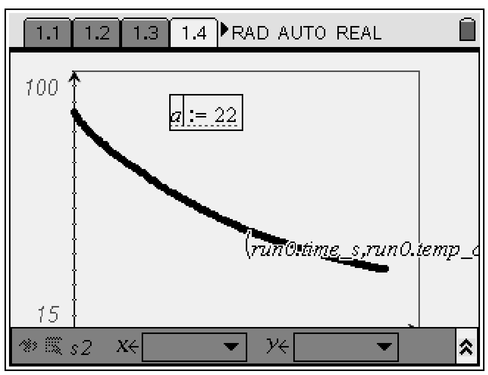


Figure 6

- Choose dc01.time for the x-axis and dc01.temp for the y-axis.
- Next, you will define three variables, **a**, **b**, and **c**, to represent each of the values in the equation.
- Add a text box, press (menu) (1) (6) for Menu 1:Actions, 6:Text.
- Type in the value you determined for *a* above, and press (enter).
- Then, press (esc), and move the cursor so that it is hovering above the value you just typed.
- Click once to select the text box, and then press (sto► var).
- Select **Store Var**, and then type **a** in the text box when it becomes highlighted.
- Press (enter) to define the variable.
- Repeat this process for *b* and *c*.

6. Next, set the function **f1(x)** equal to **a** + **b·c**^{x} (Figure 7).

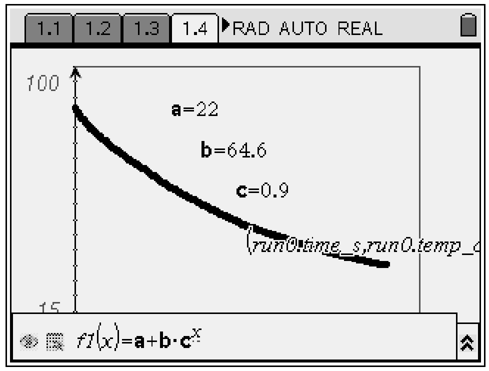


Figure 7

- Change the graph type to **Function**.
- In the function line, type in the expression above and press enter.
- A graph of the function should now be displayed on your screen. It will probably not fit your data very well.
- Press esc to exit out of the function line.
- Adjust the value of **c** by clicking on the **c=** text box and typing in new values.
- Vary **c** until the curve fits your data reasonably well.
- Make changes to **a** and **b** if necessary.

Q6. What value of **c** gave you the best fit to your data?

- What does c tell you about how the temperature is changing?

Part 3: Exploring the Relationship between Cooling Rate and Temperature Difference

In the previous steps, you determined the function that describes the cooling of the sensor. The slope of this curve at any point is the cooling rate at that time, as shown by the following equation:

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta(\text{temperature})}{\Delta(\text{time})} = \text{cooling rate}$$

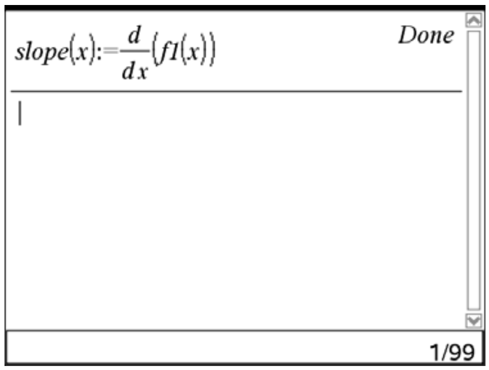
To find the slope algebraically, you would find the differences in temperature of the sensor and the time at two points and divide them.

When you calculate slope in this way, you are limited by how close together your data points are. A tool of calculus, the derivative, allows you to take the difference between two points that are infinitesimally close together. Therefore, the derivative allows you to find the slope of the curve at any single point.

In this part of the activity, you will plot the slope of the curve (the cooling rate) versus the difference in temperature between the sensor and the room to learn how temperature difference affects cooling rate.

7. First, you must define the *slope* function.

- Move to page 1.6, which contains a Calculator application page.
- Type **slope(x):=** (Figure 8).



`

Figure 8

Note: Make sure you type **:=**, not just **=**.

- Then, press ctrl to open the templates menu, and select the template.
- Enter **x** in the box in the denominator of the template and **f1(x)** in the large box to the right of the template.
- Then, press enter.
- This command defines the function **slope(x)** as the derivative of **f1(x)**.

8. Insert a blank Lists & Spreadsheet application page.

- In the formula bar (gray square) of column A, press =, press sto► var, and choose **dc01.time** (Figure 9).

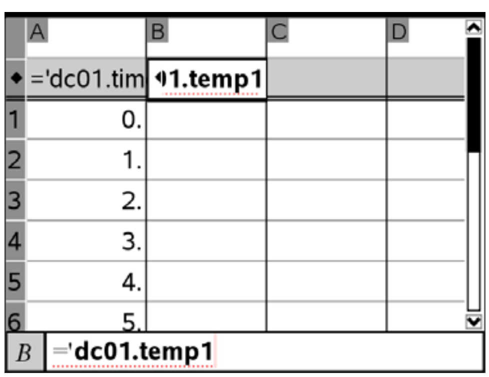


Figure 9

- Repeat this process for column B, but set it equal to the temperature readings.

9. Next, make column C equal to the slope of the curve at each point.

- In the formula bar for column C, type **=slope(a[]),** and press enter (Figure 10).

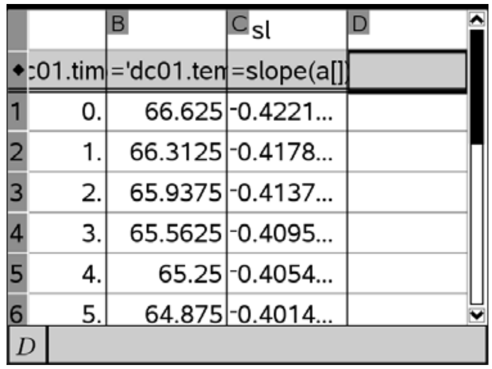


Figure 10

- To insert the brackets after **a**, press ctrl [ ] and then press ▶ (right)on the NavPad Cursor Controls.
- Assign this series to the variable **sl** by typing **sl** in the title bar of column C (the white box next to the C label), and pressing enter.

Q7. Why is the variable for the slope function in column C time, not temperature?

- That is, why do you have to type **a[]** and not **b[]** into the function?

10. Now, set column D equal to the difference in temperature between the probe and the room.

- In the formula bar for column D, type **=b[]-***temp*, but substitute the ambient temperature you recorded in part 1 for *temp* (Figure 11).

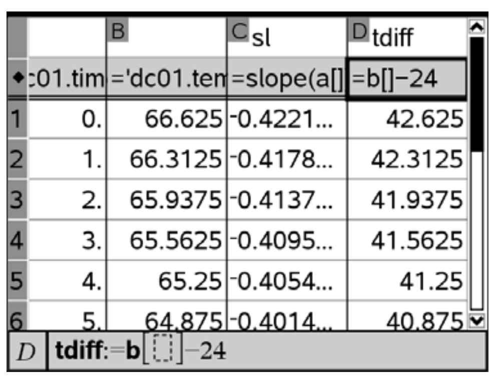


Figure 11

- For example, if your ambient temperature was 19ºC, you would type **=b[]-19**.
- Press enter to populate the list.
- Assign this series to the variable **tdiff** by typing **tdiff** in the title bar of column D and pressing enter.

11. Move to the next page (Figure 12).

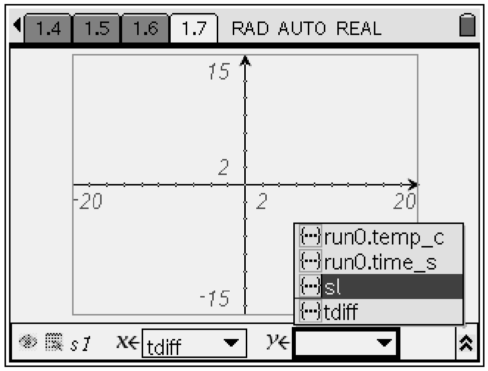


Figure 12

- Create a scatter plot of slope (**sl**) versus difference in temperature (**tdiff**).
- You may need to resize the window, press menu 4 9 for Menu 4: Window, 9: Zoom-Data.
- Describe the shape of the graph.

Q8. What does the shape of the graph tell you about the relationship between cooling rate and temperature difference?

Q9. If the graph were extended, what would its *x*- and *y*-intercepts be?

- What does this tell you about the relationship between cooling rate and temperature difference?

Q10. Which would you expect to cool more quickly, a 90ºC sensor in a 10ºC room, or a 50ºC sensor in a 20ºC room?

- Explain your answer.

Cooling Rates

Concepts

- Heat transfer by convection
- Mathematical models of physical phenomena

Materials

- TI-Nspire™ CAS Math and Science Learning Technology
- 50 ml to 100 ml water in an insulated container
- Copy of the student worksheet
- Pen or pencil
- Paper towels
- Vernier® Easy Temp™ or Go! ™ Temp temperature sensor
- Safety goggles
- Blank sheet of paper
- *Cooling.tns* file

Overview

In this activity, students will collect data on the temperature of a cooling metal sensor. They will then fit a model to their collected data, and attempt to find numerical values for the variables in the modeled equation. Finally, they will use the derivative to investigate the rate of temperature change as a function of the difference in temperature between the probe and the environment.

Teacher Preparation

Students should be familiar with the idea of heat transfer and with the fact that it does not occur at the same rate in all situations. Exponential cooling, which is explored in this activity, applies to heat transfer by convection only. Furthermore, it applies only to situations in which the ambient temperature remains effectively constant.

1. As a point of interest, you may wish to explain that forensic scientists sometimes use exponential cooling models to estimate time of death.

- You could also have students brainstorm situations in which these models do or do not apply.

2. The screenshots on pages 4–8 demonstrate expected student results.
- Refer to the screenshots on pages 9–10 for a preview of the student .tns file.

Classroom Management

1. This activity is designed to be **student-centered**, with the teacher acting as a facilitator while students work cooperatively.
- The student worksheet guides students through the main steps of the activity and includes questions to guide their exploration.
- Students should record their answers to the questions on a separate sheet of paper.

2. The ideas contained in the following pages are intended to provide a framework as to how the activity will progress.
- Suggestions are also provided to help ensure that the objectives for this activity are met.

3. In some cases, these instructions are specific to those students using TI-Nspire™ CAS math and science learning handheld devices, but the activity can easily be done using TI-Nspire™ CAS computer software.

The following questions will guide student exploration during this activity:

- How does the temperature of a hot object change when it is placed in a cool environment?
- Students are asked to make a prediction of the graph prior to data collection.
- This engages them to think about what will happen before performing the experiment.
- You should walk around and look at the predictions because so that they can be discussed in the final debriefing.
- You may want to ask students to share their predictions with other members of their group before beginning the data collection.
- How can we make a mathematical model of such a temperature change?

How is the rate of temperature change related to the difference in temperature between the object and the environment?

Part 1: Collecting Temperature Data

1. Students will use a Vernier® EasyTemp™ or Go!™Temp temperature sensor to collect temperature data.

- They should wait until they are told to connect the temperature probe to their handhelds or computers.
- This should activate the temperature sensor, and a temperature display should appear in the data collection box.
- As instructed, choose Data & Statistics to collect data.

Q1. What is the ambient temperature?

Answer: The ambient temperature will vary.

- Make sure students' values are reasonable.

2. After making sure all students are wearing safety goggles, give each student approximately 100 mL of boiling water in an insulated container.

- Students should place the metal ends of their temperature sensors into the water, and wait for the temperature reading to stabilize.
- Then, they should remove the sensor from the water and wipe it off.

3. Immediately after removing the sensor from the water, students will begin data collection.

- Data collection will run for three minutes.
- When it is complete and students close the data collection box, their screens should look like the one in Figure 1.
- At this point, students can disconnect the temperature sensors.
- Collect the containers of water from the students and dispose of them properly. After you have taken all the containers of water away, students may remove their safety goggles.

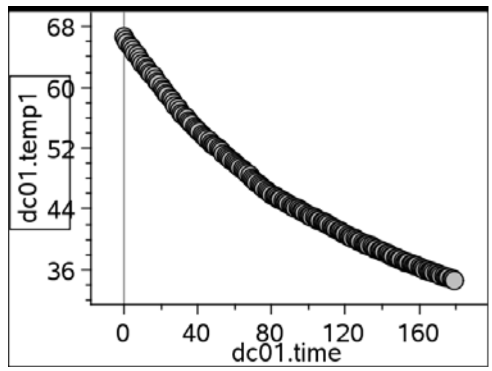


Figure 1

4. Next, students will advance to page 1.5, and make a scatter plot of the temperature and time data.

Q2. Describe the shape of the graph.

Answer: The data curve downward, indicating that temperature decreased over time at a nonuniform rate.

- The curve should be approximately exponential, as shown in Figure 2.

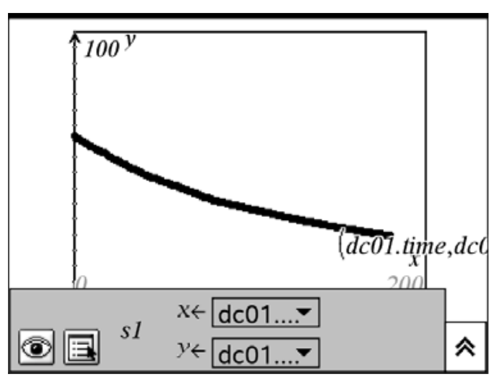


Figure 2

Part 2: Fitting a Curve to the Data

1. Next, students will attempt to fit an exponential curve of the form $y = a + b \cdot c^x$ to their data.

Q3. Should *c* be greater than, less than, or equal to 1?

- Explain your answer, and give your prediction for the value of *c*.

Answer: The variable c should be less than 1 because temperature decreases as time increases.

- Students may struggle with this reasoning.
- You may wish to give them several simple examples

  (e.g., $\left(\frac{1}{2}\right)^x$ *vs.* 2^x *)*

  to illustrate why a decreasing curve implies a base that is less than 1.
- Their predictions about the value of c will vary.

Q4. What value should *a* have?

- Hint: What will happen to the temperature of the sensor as time approaches infinity?

Answer: The variable *a* should be equal to the ambient temperature.

- You can help students understand why this is so by reminding them that the temperature of the sensor will eventually (i.e., as time approaches infinity) equal the ambient temperature and showing them how the equation requires that y approach a as x approaches infinity.

Q5. What value should *b* have?

Answer: The variable *b* should equal the initial temperature of the sensor minus the ambient temperature.

2. Next, students define the variables **a**, **b**, and **c** using the text box tool, the sto→ var key, and the **Store Var** command (Figure 3).

3. Next, students set the function **f1(x)** equal to **a** + **b** · **c**^{x}.

- They change the graph to a **Function** graph, and enter the expression in the function line.
- They should then vary the value of **c** to obtain the best possible fit to their data set.

Q6. What value of *c* gave you the best fit to your data?

Answer: The value of **c** will vary from student to student, but it should be very close to 1.

Part 3: Exploring the Relationship between Cooling Rate and Temperature Difference

1. Students will now define the function **slope(x)** as the derivative of **f1(x)** (Figure 4).

- Students may have a hard time understanding the concept of the derivative.
- You may wish to discuss the idea of infinitesimally small intervals with them, or show a few simple examples.

Note: Make sure that students use the **:=** notation (not just =) when defining the slope function.

- The = notation will cause the handheld to calculate and display the actual equation for **slope(x)**.

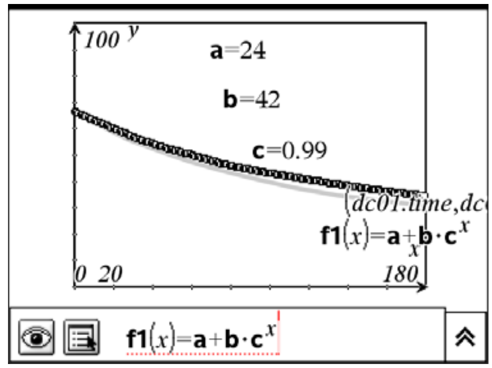


Figure 3

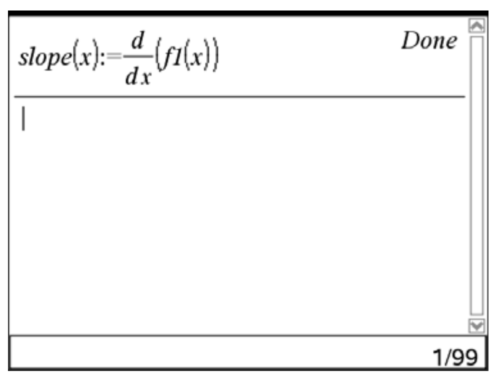


Figure 4

2. Next, students set up a Lists & Spreadsheet application page to store slope and temperature-difference data before plotting them.

- They set column A equal to the time data collected in part 1 and column B equal to the temperature data collected in part 1 (Figure 5).

3. Next, students use the **slope** function to fill column C with slope data (Figure 6).

- They name the slope series **sl**.
- Remind students that they can resize the columns to make the formulae easier to read.

Note: The handheld will run slowly after the first function is entered.

- Students should be patient, and wait until the clock icon disappears before entering new commands.

Q7. Why is the variable for the **slope** function in column C time, not temperature?

- That is, why do you have to type **a[]** and not **b[]** into the function?

Answer: The data in column A are passed into the **slope** function because **slope** is a function of time (x), not temperature.

4. Next, students use a function to fill column D with temperature-difference data.

- They name the series **tdiff** (Figure 7).

Note: Make sure students use their ambient measured temperatures in the **tdiff** formula (i.e., that they do not just copy the formula shown in the worksheet).

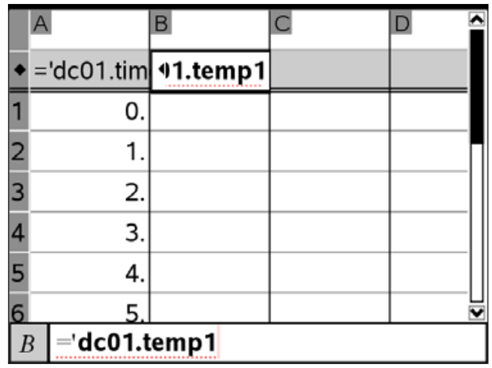


Figure 5

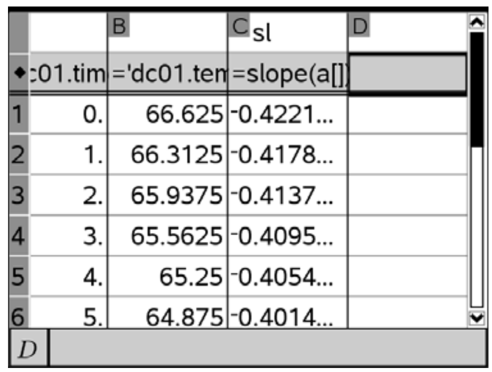


Figure 6

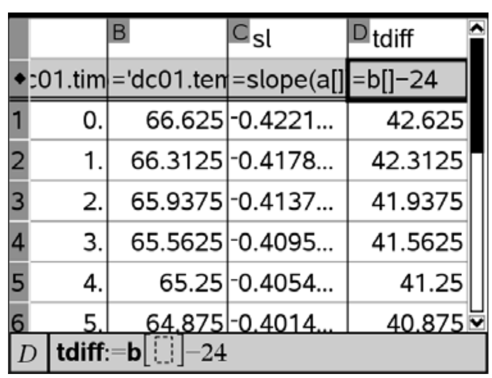


Figure 7

5. Next, students make a scatter plot of slope versus temperature difference (Figure 8).

Q8. Describe the shape of the graph.

Answer: The data lie on a nearly straight line.

Q9. What does the shape of the graph tell you about the relationship between cooling rate and temperature difference?

Answer: The shape of the graph implies that cooling rate decreases linearly as temperature difference decreases.

Q10. If the graph were extended, what would its x- and y-intercepts be?

- What does this tell you about the relationship between cooling rate and temperature difference?

Answer: The graph appears to pass through the origin.

- This implies that, when the temperature difference is 0, the cooling rate is also 0—exactly what would be expected.
- If students have a hard time with this concept, remind them that an object will never cool to below the ambient temperature.

Q11. Which would you expect to cool more quickly, a 90ºC sensor in a 10ºC room, or a 50ºC sensor in a 20ºC room?

- Explain your answer.

Answer: A 90ºC sensor in a 10ºC room will cool more quickly than a 50ºC sensor in a 20ºC room because the temperature difference is greater in the first example.

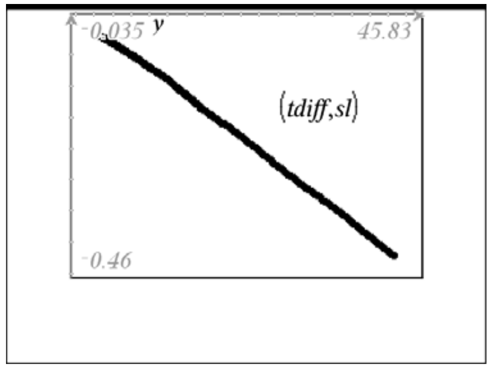


Figure 8

Student TI-Nspire™ File: *cooling.tns*

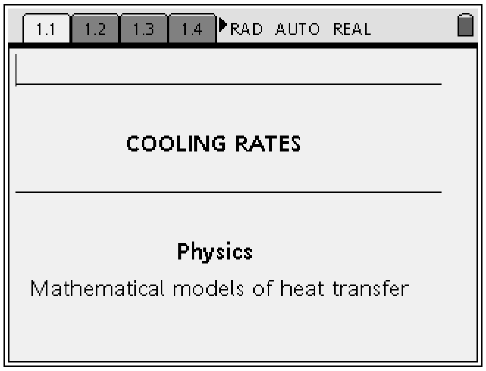


Figure 9

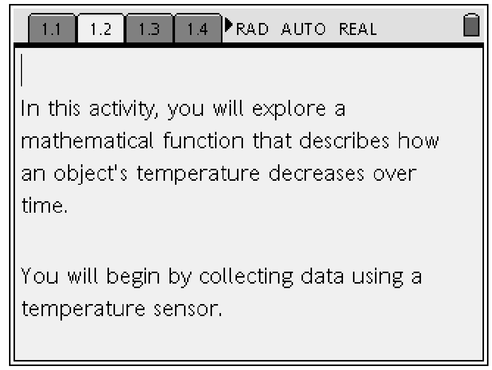


Figure 10

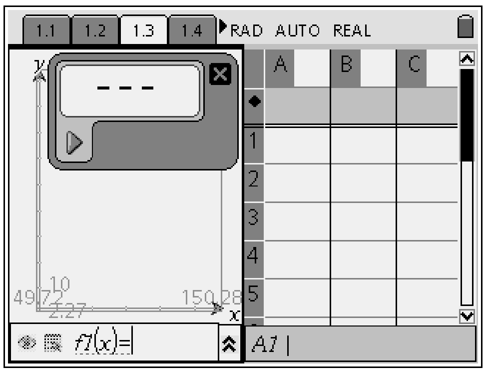


Figure 11

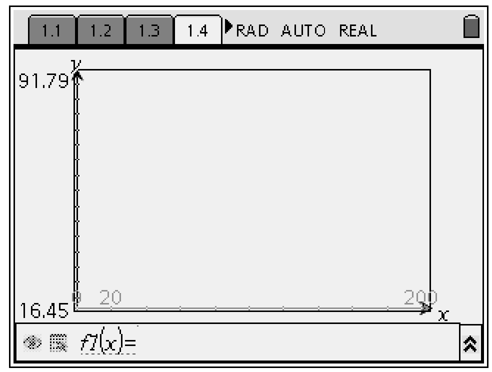


Figure 12

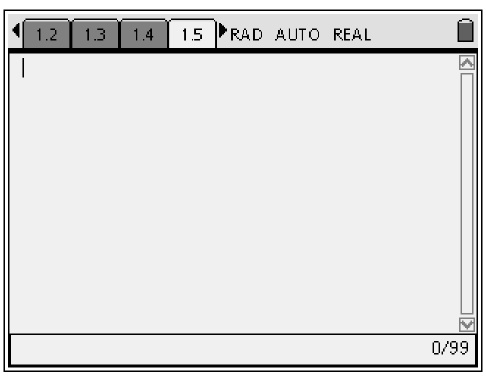


Figure 13

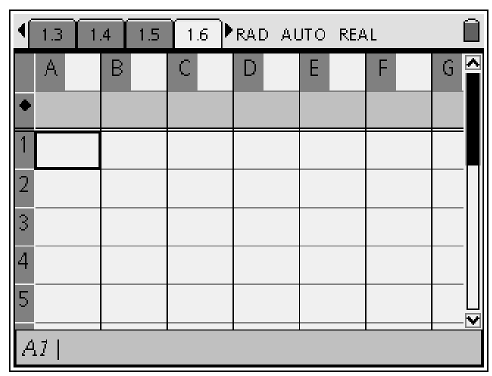


Figure 14

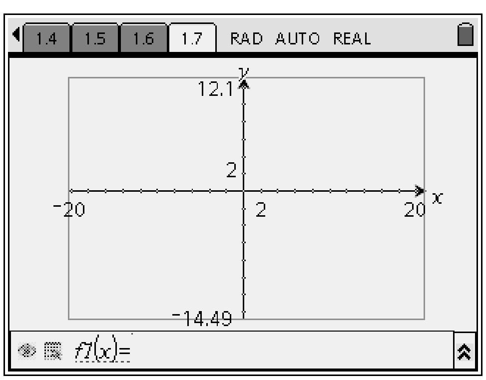


Figure 15

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Analyzing Planetary Motion

Concepts

- Mean
- Distance
- Motion

Materials

- TI-Nspire™ Math and Science Learning Technology

Overview

In this activity, you will examine the relationship between the mean distance from the planet to the sun and its period of revolution.

Introduction

People have studied the motion of planets, stars and other celestial objects for many years. Astronomer, Johannes Kepler, spent 16 years analyzing the motion of the planets, trying to find a mathematical relationship. In this activity, you will examine the relationship between the mean distance from the planet to the sun and its period of revolution. The data table below gives the mean distance from the sun to the planets and their corresponding period of revolution. The distances are given in astronomical units, where one AU equals the mean distance from the sun to earth; and the period is given in years.

Planet	**Mean Distance from Sun (AU)**	**Period of Revolution (Years)**
Mercury	0.3871	0.24
Venus	0.7233	0.62
Earth	1	1
Mars	1.524	1.88
Jupiter	5.203	11.86
Saturn	9.539	29.46
Uranus	19.19	84.01
Neptune	30.06	164.79
Pluto	39.48	248.54

Creating a List

1. Open a new document by selecting (home) (6) for Home 6: New Document (Figures 1 and 2).

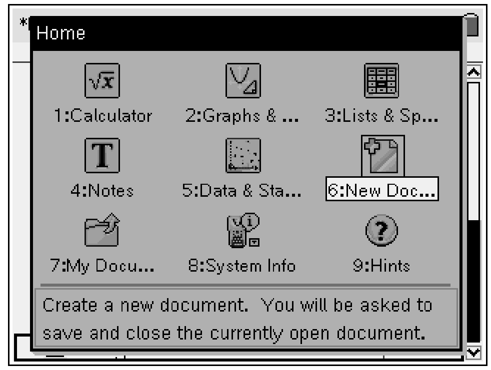


Figure 1

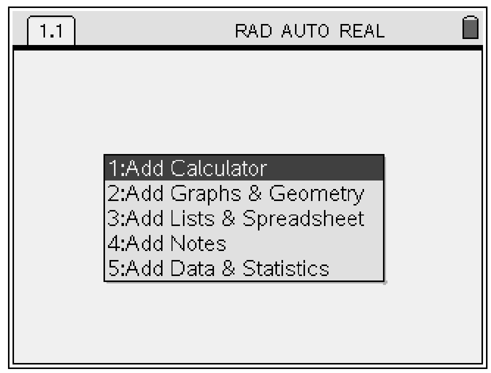


Figure 2

2. Press (3) to add 3: Add Lists & Spreadsheets (Figure 3).

- Label the columns.

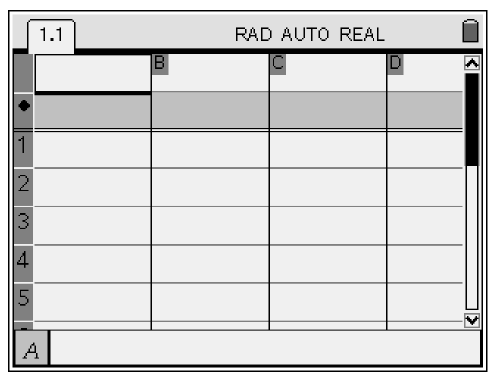


Figure 3

3. Using the NavPad cursor controls, move up twice.

- Be sure the cursor is left to the white space next to the "A" column heading.
- Type "dist," and press (enter) (Figure 4).

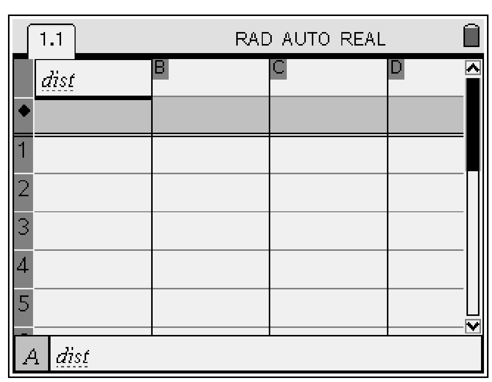


Figure 4

4. Move to the column "B."

- Label it "period" (Figure 5).

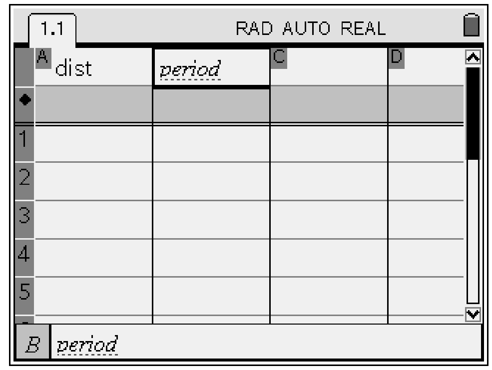


Figure 5

5. Enter the distances in column A and the periods in column B (Figure 6).

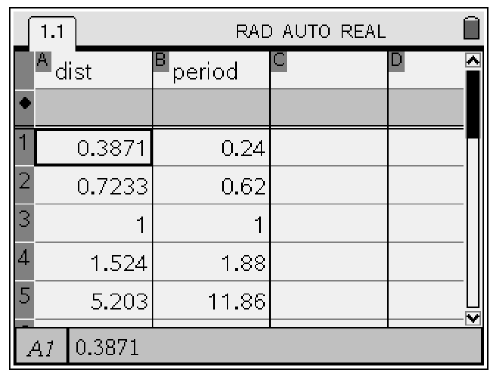


Figure 6

Graphing the Data

1. To graph the data, add a new Graphs & Geometry application page to this problem.

- Press the 2 for Home 2: Graphs & Geometry (Figures 7 and 8).

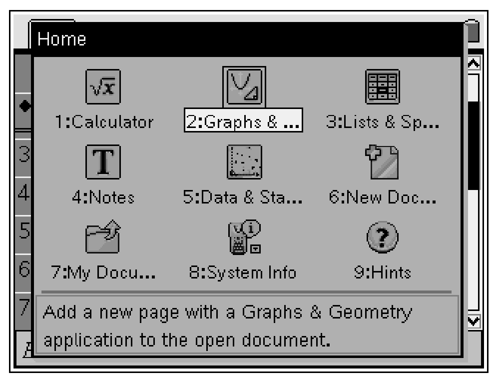


Figure 7

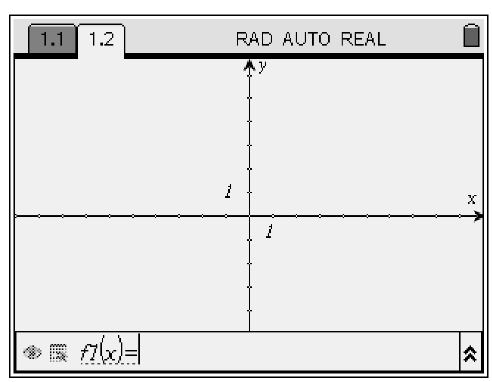


Figure 8

2. Press menu 3 4 for Menu 3: Graph Type, 4: Scatter Plot (Figure 9).

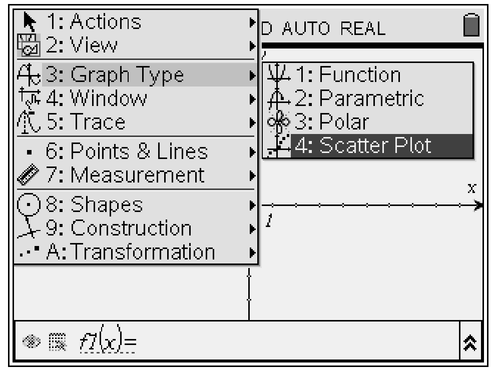


Figure 9

3. Use the NavPad to select the *x* box, and choose "dist".
   - Tab to move the cursor to the *y* box, and choose "period" (Figure 10).

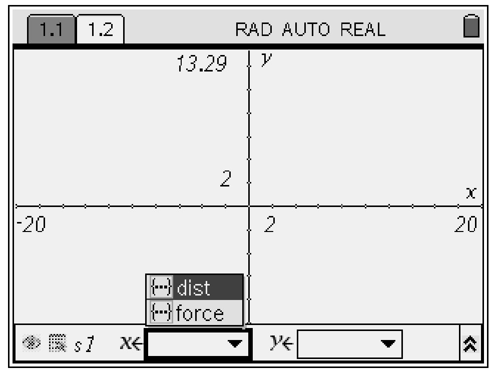


Figure 10

4. To set a window, press menu 4 9 for Menu 4: Window, 9: Zoom – Data (Figure 11).

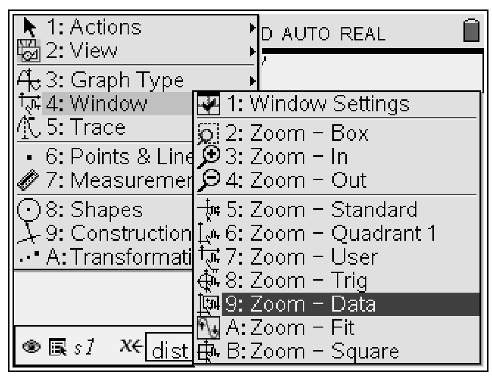


Figure 11

   - The Zoom-Data sets a window that shows all data points for the plots selected (Figure 12).

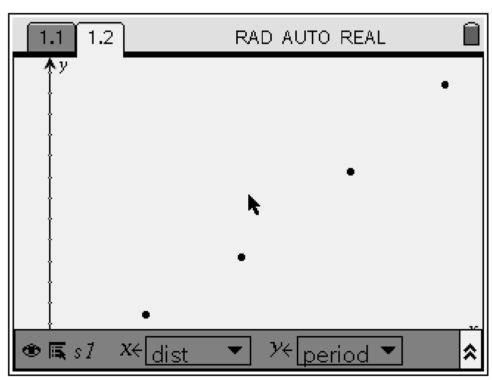


Figure 12

5. Hide the statistics plot line by pressing ctrl G (Figure 13).

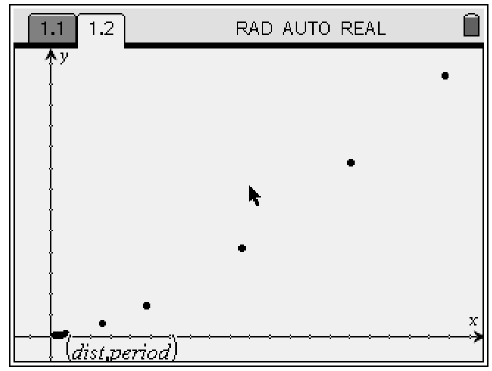


Figure 13

6. You may want to go back and readjust the YMin value in the window so that you can see all points without the function line covering up the bottom of the plot.
   - Press menu 4 1 for Menu 4: Window, 1: Window Settings.
   - Set XMin to 0 and YMin to -50 (Figure 14).
   - Select OK (Figure 15).

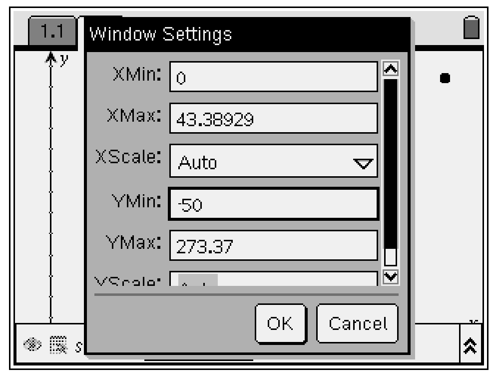


Figure 14

By just viewing the data, it appears that it may be a parabola with a vertex going through the origin. If it is, we know that the formula should be $T = Ad^2$ where T is the period and d is the mean distance.

- Try a parabola with the data.
- One method to find a value for "A" is to graph the equation $f(x) = x^2$, and adjust the equation until it fits the data.

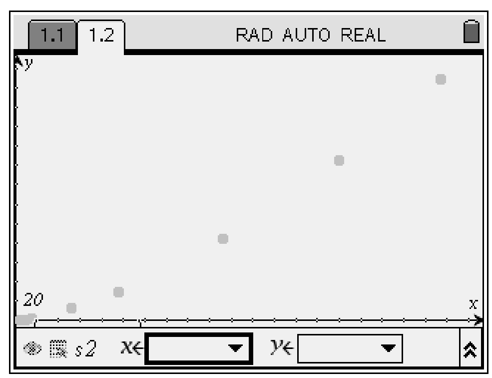


Figure 15

7. Press menu 3 1 for Menu 3: Graph Type, 1: Function.
8. Enter *f1(x)* = x^2, and press enter (Figure 16).

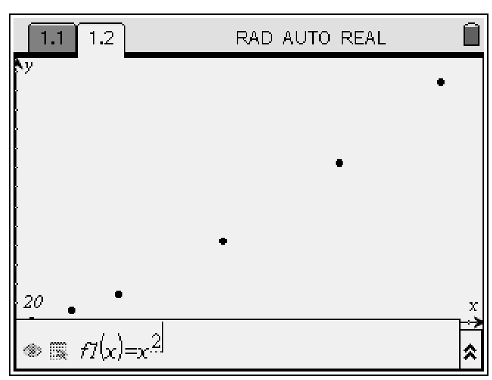


Figure 16

9. Press esc to move the cursor from the equation entry line to the graph.
10. Use the NavPad to move the cursor to the right corner of the graph.
    - The cursor looks like a slanted line with arrows when it is on the graph of the equation (Figure 17).
11. Hold down the click key until the hand closes.
12. Use the NavPad to transform the graph.
    - As you pull the graph, it seems that it does not exactly match the graph (Figure 18).
    - While it seems to be a power function, it looks like the equation is not $T = Ad^2$.

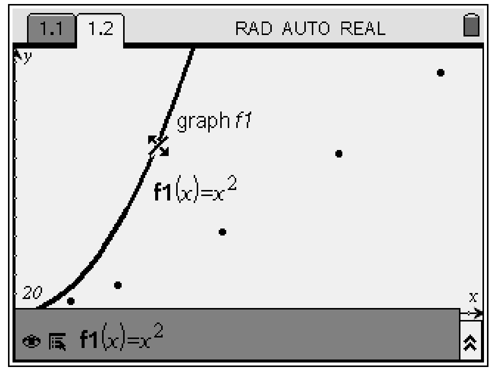


Figure 17

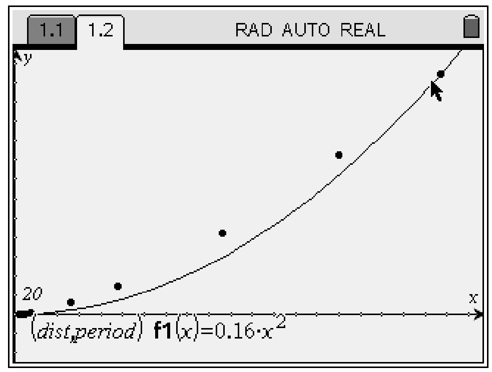


Figure 18

Regression Analysis

1. Press home 1 for Home 1: Calculator (Figure 19).

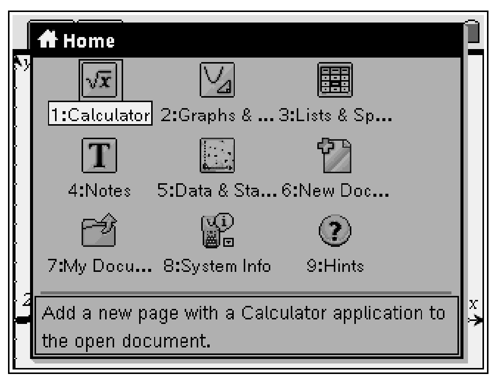


Figure 19

2. Press menu 6 1 9 for Home 6: Statistics, 1: Stat Calculations, 9: Power Regression (Figure 20).
3. Select the lists, and press enter.
4. Record the equation here_________________.
5. What does this tell you about the relationship between the period and mean distance?

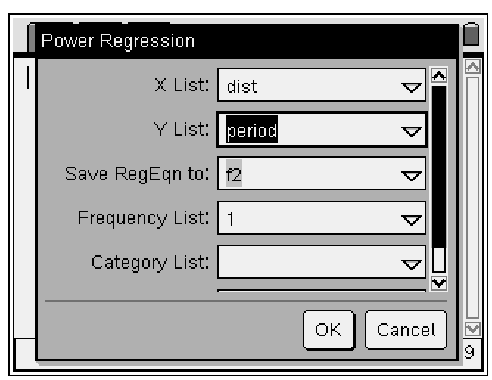


Figure 20

6. Move back to the graph (Figure 21).
   - Select the equation line.
   - Press enter on f2(x) to see the regression equation with the data.
   - You can also see your original quadratic equation with the data.

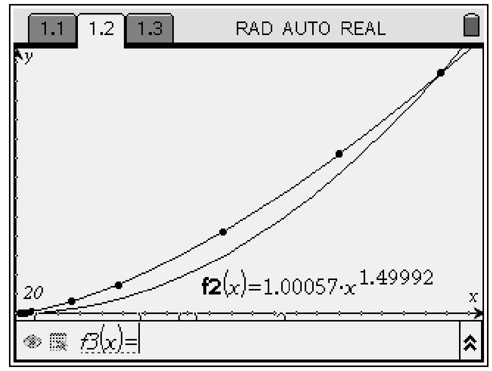
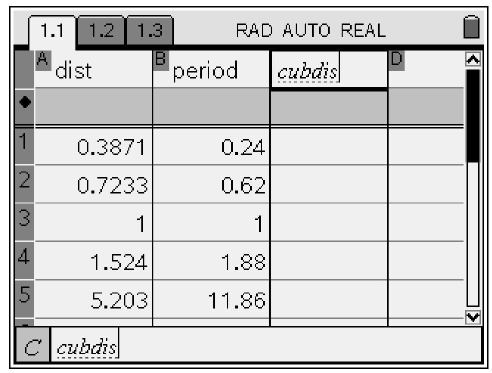


Figure 21

Transforming the Data

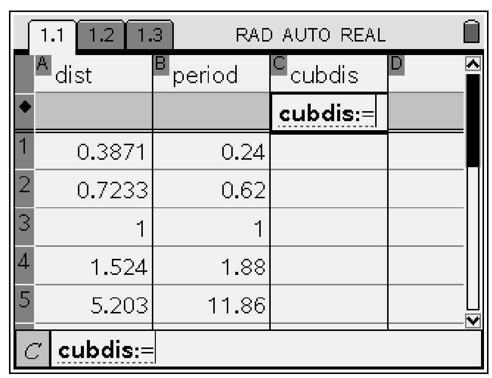
1. In physics, we often times transformation the data to look for a linear relationship.
   - If we can transform this data to show a linear function, then we are more certain of the relationship that exists.
2. If your equation is approximately, $T = d^{\frac{3}{2}}$, then one way to transform the data is to let $T^2 =$ ___________.
   - Explain how you can see this relationship from the regression equation.
3. Press (ctrl),and then use the NavPad to arrow left to access the data lists.
   - Arrow up to the white space next to "**C**" and type *cubdis* (Figure 22).



	A dist	B period	C cubdis	D
◆				
1	0.3871	0.24		
2	0.7233	0.62		
3	1	1		
4	1.524	1.88		
5	5.203	11.86		

Figure 22

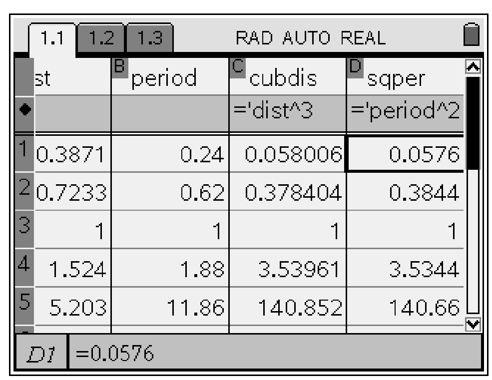
4. Move the cursor to the space just below the list name, and press (enter).
   - You will see **cubdis:=** appear (Figure 23).
   - This column is ready to be defined.



	A dist	B period	C cubdis	D
◆			cubdis:=	
1	0.3871	0.24		
2	0.7233	0.62		
3	1	1		
4	1.524	1.88		
5	5.203	11.86		

Figure 23

5. To define this list, press (sto→ var).
   - The variable menu allows you to link this list to another variable name.
   - Select **dist**, cube it, and press (enter).
   - Now repeat the procedure to square the period (Figure 24).



	st	B period	C cubdis	D sqper
◆			='dist^3	='period^2
1	0.3871	0.24	0.058006	0.0576
2	0.7233	0.62	0.378404	0.3844
3	1	1	1	1
4	1.524	1.88	3.53961	3.5344
5	5.203	11.86	140.852	140.66

Figure 24

The Data and Statistics Features

1. Press ctrl, and use the NavPad to move right to the graph.
2. Press ⌂ 5 for Home 5: Data & Statistics (Figure 25).

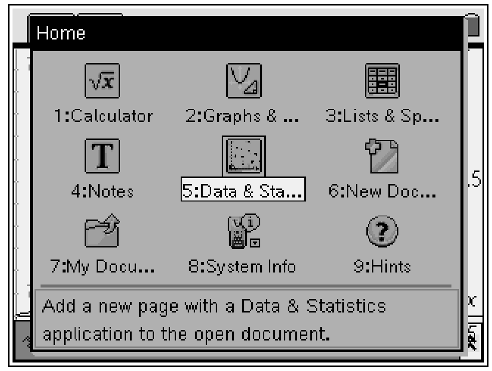


Figure 25

- The screen show random data points, not your data (Figure 26).

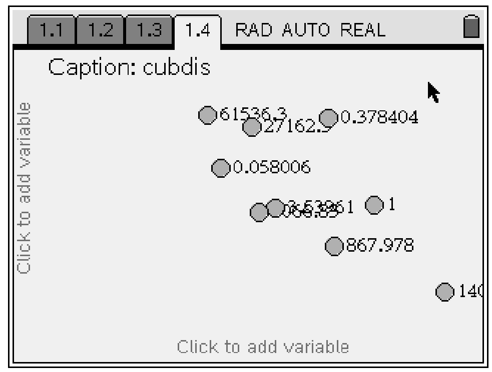


Figure 26

3. Use your NavPad to move the cursor to the horizontal axis.
   - Press , and choose "cubdis" (Figure 27).

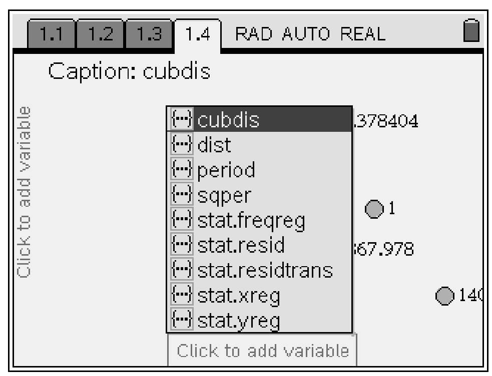


Figure 27

4. Now move the cursor to the vertical axis, and choose "sqper" (Figure 28).
5. What does the graph suggest?
   - What do you predict the equation of the line to be?

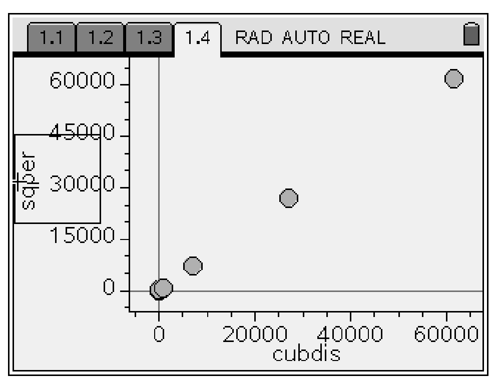


Figure 28

6. From the menu, choose 4:Analyze and then add a Movable line (Figure 29).

- Adjust it to fit the data.
- What is the slope?

7. Run a linear regression to compare the equation.

- Press menu 4 6 1 for Menu 4; Analyze, 6: Regression, 1: Show Linear.
- How does this equation compare with the movable line, and how does it compare with the power regression you performed on the original data?

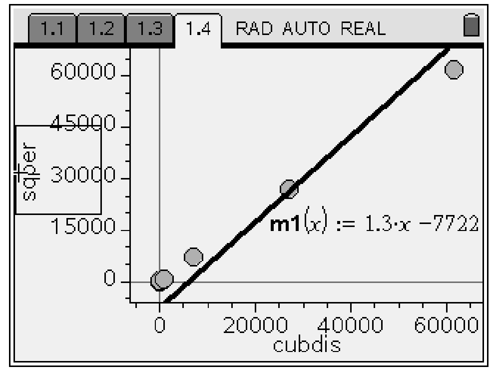


Figure 29

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Boyle's Law

Concepts

- Boyle's Law
- Volume
- Pressure

Materials

- TI-Nspire™ Math and Science Learning Technology
- EasyLink™
- *Boyle.tns* file

Overview

In this activity, we will use TI-Nspire™ math and science learning technology to explore Boyle's Law.

1. **Press** home.
2. **Press** 7 to select 7:My Documents.
3. **Select** Boyles law from the list, and press enter .
4. **Move** to page 1.2 by pressing ctrl ▶.
5. **Move** to page 1.3, grab point **P** by placing the arrow on the point (Figure 1).
   - Press and hold the click key.
6. **When** the hand closes, move your cursor up and down.
   - Watch what happens to the molecules in the box as it gets bigger and smaller.
7. **After** moving the box, move to the next page.
   - Answer the question on this page.
   - Continue through the activity, answer the questions as you go.
8. **Read** the instructions on page 1.7 (Figure 2).
9. **Move** to page 1.8.
   - Plug the pressure probe into an EasyLink™, and then to the hand held.

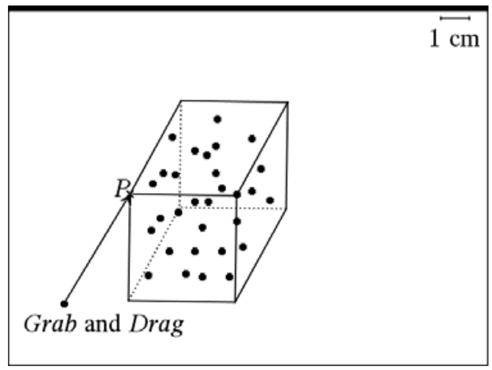

Figure 1

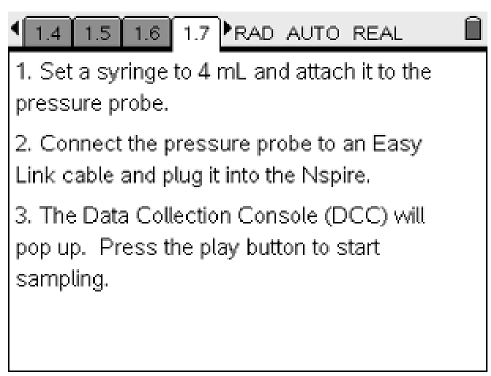

Figure 2

10. To start sampling, press the arrow keys until the start arrow is highlighted; and press enter (Figure 3).

- A collect button appears just below where the start button was located.
- Select this when you are ready to take a sample.
- To move between DCC, Data &Statistics and Notes page press ctrl tab.

11. Set the syringe to 4 mL, and connect it to the pressure probe.

12. Click on the collect button to take the first sample.

13. Enter the value of the volume of the syringe plus .5 mL to account for the tip.

- The first trial should be 4.5 mL (Figure 4).

14. Change the volume by 2 mL each time making sure to record the volume plus .5 mL.

- Take the pressure, and enter the following volumes 6, 8, 10, 12, 14 and 16.

15. After entering the last volume, arrow left to highlight the stop key; and press (Figure 5)

16. Answer the question on the following pages.

17. On page 1.12, read the instructions.

- Then move to page 1.13, and do the calculations (Figure 6).

18. Answer the questions on pages after the spreadsheet.

19. Calculate the average in the calculator split by typing **mean(ptimesv)** (Figure 7).

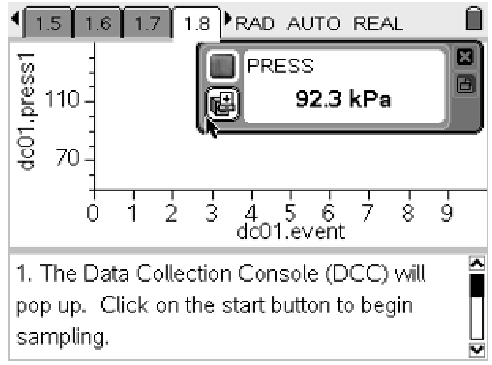

Figure 3

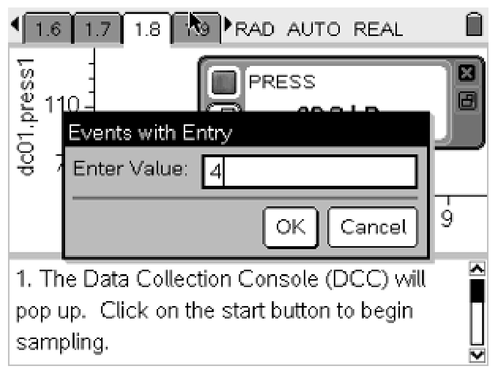

Figure 4

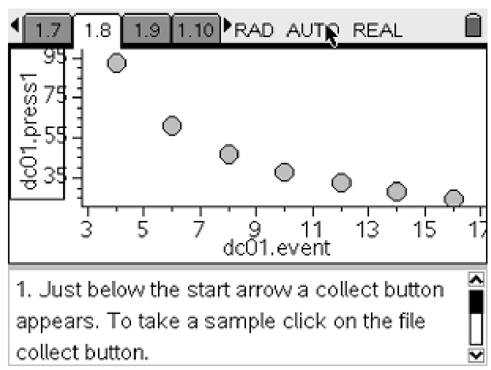

Figure 5

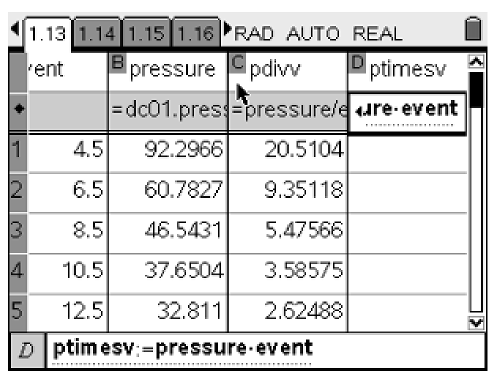

Figure 6

20. Move to page 1.17.

- Write the equation for the relationship between volume and pressure.

21. Read the directions on 1.18, and move back the next page.

- Press (menu).
- Press (4) to select Analyze.
- Press (4) to select plot function.
- Enter your average divided by x (Figure 8).

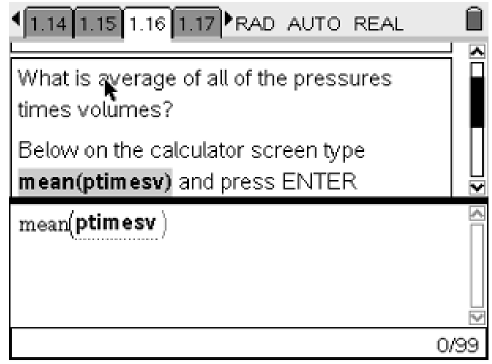

Figure 7

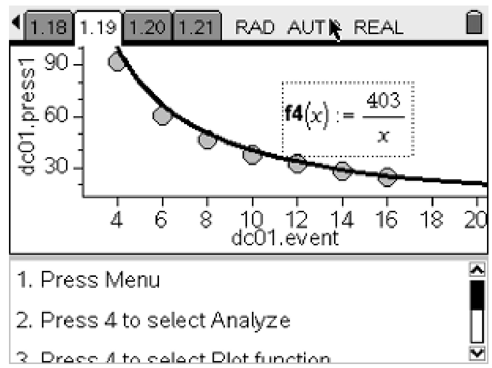

Figure 8

Questions

1.4 As the volume of the box increases what happens to the distance between gas molecules?

1.5 Pressure is caused by the number collisions gas molecules have with the container. When the volume increases what happens to the number of collisions.

1.6 As the volume increases what happens to the pressure?

1.9 Is the syringe easy or hard to push when the volume is low? Explain why this is.

1.10 As the pressure volume decreases what happens to the pressure?

1.11 Looking at the graph on page 1.8 is the pressure directly or inversely related (Figure 8)?

1.14 Is pressure divided by volume relatively constant?

1.15 Is pressure times volume relatively constant?

1.16 What is average of all of the pressures times volumes (Figure 7)?

1.19 Boyles law is the relationship between volume and pressure (Figure 8).

- Write the general form for the equation between volume and pressure.

Rolling a Ball on an Inclined Plane

Concepts

- Motion in one dimension
- Resolving vectors algebraically

Materials

- TI-Nspire™ math and science learning technology
- Vernier® CBR 2™ (Calculator-Based Ranger ™ 2) or Go!™ Motion motion sensor
- Pen or pencil
- Inclined plane at least 1 m long
- Small ball, such as a racquetball or small play ground ball
- Safety goggles
- Meter stick or tape measure
- *ballroll.tns* file

Overview

In this activity, students will collect distance and time data for a ball rolling up and down an incline. They will then find the best-fit parabola for the data, and construct a tangent to the parabola. They will capture data about the slope of the parabola at various points, and graph the results. They will then use those results to explore the relationship between the displacement and velocity functions and between the angle of the inclined plane and the acceleration of the ball.

Teacher Preparation

Before conducting this experiment, students should have been introduced to the equations relating displacement, velocity, and acceleration to time. They should also have learned how to use the sine and cosine functions to resolve vectors into perpendicular components.

- If time permits, you may wish to have students repeat the experiment and vary the angle of the inclined plane.
- The screenshots on the following pages demonstrate expected student results.
- Refer to the screenshots on page 11 for a preview of the student TI-Nspire™ document (.tns file).

Classroom Management

1. This activity is designed to be **student-centered**, with the teacher acting as a facilitator while students work cooperatively.

- Students should record their answers to the questions on a separate sheet of paper.

2. The ideas contained in the following pages are intended to provide a framework as to how the activity will progress.

- Suggestions are also provided to help ensure that the objectives for the activity are met.

3. In some cases, these instructions are specific to those students using TI-Nspire™ math and science learning handheld devices, but the activity can easily be done using TI-Nspire™ computer software.

Physics

The following questions will guide student exploration during this activity:

- What does a graph of displacement vs. time look like for a ball rolling up and down an inclined plane?
- How is the slope of a graph of displacement vs. time related to the equation describing velocity as a function of time?

Part 1: Collecting Displaced Data

1. Students will use a Vernier® CBR 2™ or Go!™Motion motion sensor to collect displacement data.

- When students reach page 1.3, they should connect the motion sensor to their handhelds or computers.
- This should activate the motion sensor, and a distance display should appear in the data collection box (Figure 1).
- The motion sensor should start clicking slowly, and the green light on the front should turn on.

2. After making sure all students are wearing safety goggles, give each student a racquetball and an inclined plane.

- The inclined plane can be of any angle and material, as long as the ball will roll up and down it smoothly. A piece of wood or stiff cardboard with one end placed on a stack of books will be fine.
- Instruct the students to set up their motion sensors so that the metal mesh on the sensor is facing down the ramp, parallel to its surface, as shown in Figure 2.

Q1. Predict the shape of the graph of displacement vs. time for a ball rolling up and then back down the ramp.

- Sketch it on a blank piece of paper.

Answer: Students' predictions will vary.

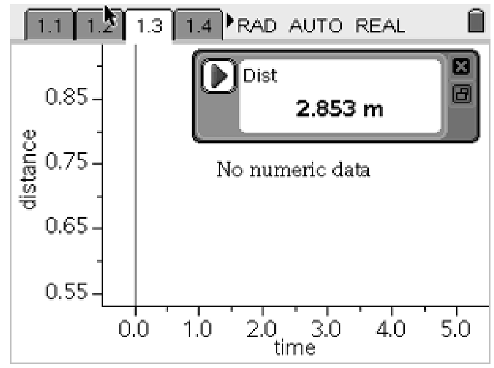

Figure 1

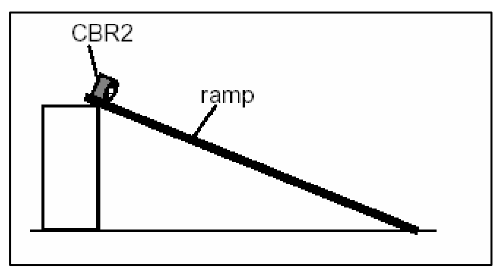

Figure 2

- Encourage students to discuss their answers, and think about other examples of motion they have studied.

Q2. Measure the height and length of your ramp.

- Use these data to calculate the angle between the ramp and the floor or table.
- Show your work.

Answer: Students' answers will vary.

- Make sure students have used the correct trigonometric function to calculate the angle.

Q3. During this activity, you will roll a ball up and down the ramp.

- Write the equations describing the ball's position and velocity along the ramp as a function of time.
- Assume that there is no friction between the ball and the ramp, and that the only acceleration is due to gravity.
- Use ***s*** for displacement, ***v*** for velocity, ***g*** for acceleration due to gravity, and ***t*** for time.
- **Hint**: What component of *g* is acting parallel to the ramp?
- Show your work.

Answer: For motion in one direction, the following general equations apply:

$s(t)= s_0 +v_i\, t + ½\, at^2$

$v(t)= v_0 + at$

The only acceleration is due to gravity, which acts at an angle è to the ramp.

- In this case, è is equal to the angle between the ramp and the floor.
- The component of g that is parallel to the ramp is therefore g·sin è.

- Making this substitution into the equations above yields the following:

  $s(t)= s_0 +v_i\, t + ½\, gt^2(\sin θ)$

  $v(t)= v_0 + gt(\sin θ)$

3. Have each group of students place their ball at the bottom of the ramp.

- Have them move the ball slowly up and down the ramp while observing the distance display.
- The display should change as the students move the ball.
- This indicates that the motion sensor is functioning correctly and is detecting the ball.
- Have students practice giving the ball a gentle push from the bottom of the ramp.
- The push should be enough to make the ball roll up to the top of the ramp, but not so hard that the ball hits the motion sensor.

Note: It is important that the ball roll up and down the ramp along a line that is as close to straight as is possible.

- Have students practice until they can roll the ball smoothly and evenly.
- You may want to add meter sticks along the sides of the ramp to keep the ball straight.

4. Next, students will collect distance data using the data collection software.

- The students should begin data collection a second or so before rolling the ball up the ramp.

- Once the ball has returned to the bottom of the ramp, students can cancel data collection by pressing the button in the data collection box.
- After students have collected their data, you can collect the balls.
- Then, students may remove their safety goggles and disconnect their motion sensors.

5. Next, students move to page 1.4, and create a scatter plot of the data.

- A sample data set for a ramp at an angle of approximately 10° is shown in Figure 3.
- Students should use the **Zoom-Data** command to zoom the graph so it shows only the parabolic section of the data set, as shown.

Q4. Describe the shape of the graph.

Answer: The data appear to lie along a parabola.

Q5. Does the graph match your prediction?

- If not, explain why you made the prediction you did.
- What assumptions did you make that were incorrect?

Answer: Students' answers will vary.

- Encourage metacognitive thinking to help students identify their errors in logic.

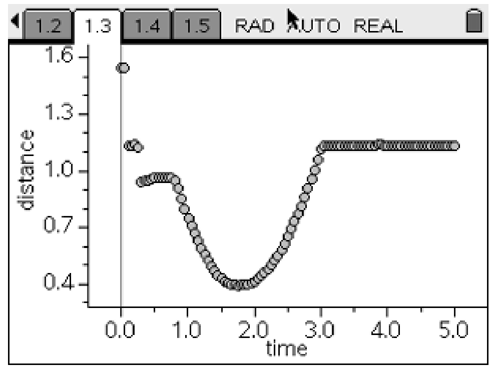

Figure 3

Part 2: Fitting a Curve to the Data

1. Next, students use the **Trace** tool to locate and mark the vertex of the parabola.

- **Note:** The data may flatten out at the bottom of the curve, as shown in Figure 4.
- In this case, students should mark the center of the lowest point.
- Then, students will attempt to fit a quadratic curve of the form to their data.

$$y = m(x - n)^2 + q$$

Q6. Use the coordinates of the vertex you marked to predict the values of n and q.

Answer: The variable **n** should be close to the *x*-coordinate or the time at the closest distance; the variable **q** should be close to the *y*-coordinate or closest distance.

2. After students have made their predictions, they change the plot to a **Function** plot, and enter their predicted equation into the function bar.

- Then they click on the function **f**1(x), and use the transformation tools to match function to the data.

Q7. Record the equation for the parabola that best fits your data.

Answer: Students' answers will vary.

Q8. How does the acceleration coefficient in the best fit equation compare with that in the equation for s(t) you wrote for question 3?

- Hint: Expand the equation for the best-fit parabola to write it in the form $ax^2 + bx + x$.

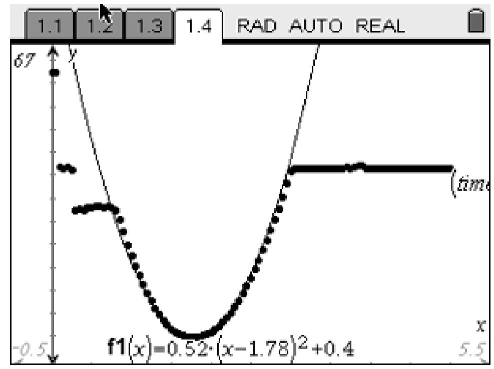

Figure 4

Answer: The acceleration coefficient of the best-fit equation should be similar to that in the equation students calculated in question 3.

- The match will not be exact, but the values should be somewhat similar.
- For the sample data set, the calculated acceleration coefficient is ½gsin(8.8°) = 0.75; the best-fit coefficient is 0.52.

Q9. Predict the shape of a graph of the velocity vs. time.

- Sketch it on a blank piece of paper.

Answer: Students' predictions will vary.

- Encourage students to discuss their answers.

Part 3: Advanced Topic—Relating Integral of Velocity to Position

1. On the next Graphs & Geometries application page, students will plot velocity vs. time (velocity vs. time)
See Figure 5.

- Then, they change to **Function** plot, and enter for **f2**(x)=x.
- Click on **f2**, and transform the line to match the velocity data (Figure 6).

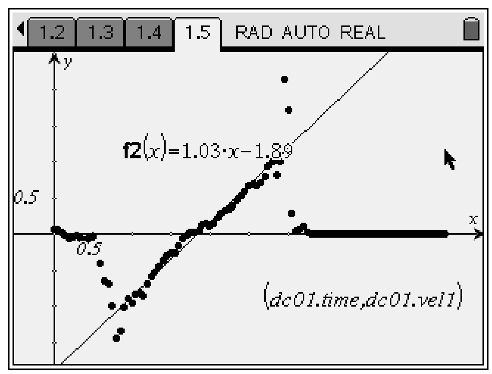

Figure 5

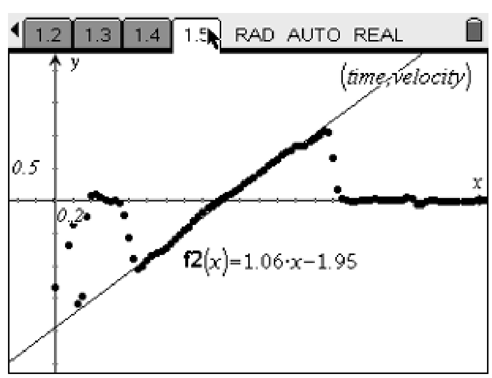

Figure 6

2. Construct a line perpendicular to the *x*-axis through the minimum velocity point that lies along the function.
   - Construct a second vertical line through the maximum Velocity point.
   - These two lines determine the region for calculating the area between the **f2** function and the x-axis.
3. Press (menu) (7) (5) for Menu 7:Measurement, 5:Integral.
   - Click on the function **f2**, and lower bound vertical line. Move the point a short distance along the x-axis.
   - You should see the value for the area. Assign the value of area to the variable **area**. Press (esc). Click on the value to make it grayed out, and press (sto▸ var) and Store Var and type area.
   - Press Menu (1) (7) for Menu 1:Actions, 7:Coordinates and Equations to get the coordinates of the movable integral point (Figure 7).
   - Assign the variable **tm** to the x-coordinate of this point.
   - Students will know the **tm** variable has been successfully assigned when x-coordinate becomes bold, as shown in Figure 8.
4. Move the integral point back to the left vertical line; the area value should be zero at this point.

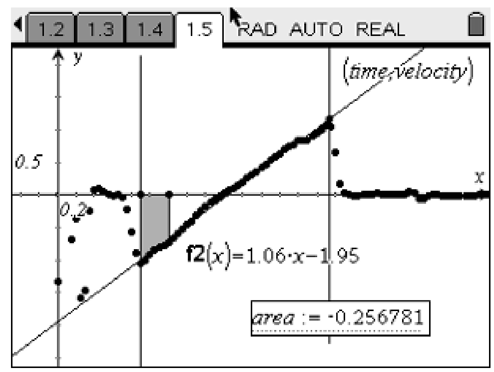

Figure 7

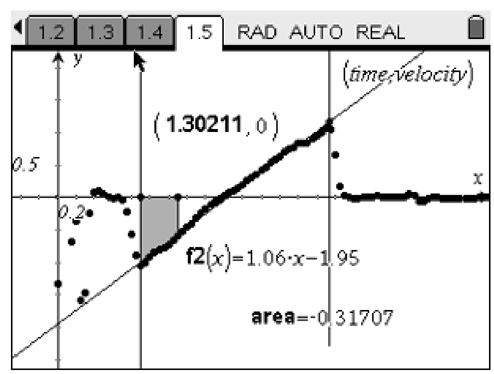

Figure 8

5. Next, students use the Lists & Spreadsheet application on page 1.6 to capture **tm** and **area** data (Figure 9).
   - Assign Column A to the variable **vtime** and Column B to the variable **varea**.
   - Set up the **automatic capture** command to record time and area data between the vertical lines. Press menu 2 1 to select 3:Data, 2:Capture and 1:Automatic. Type tm for the variable and press enter clear. Repeat for area.

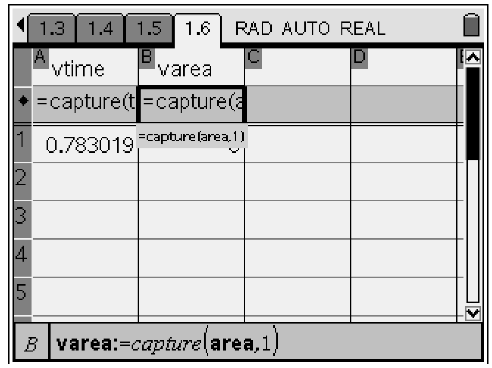

Figure 9

6. Go back to page 1.5.
   - Move the integral point from the left vertical line to the right vertical line (Figure 10).
   - The time and area data points will automatically be captured into the List & Spreadsheets application page.

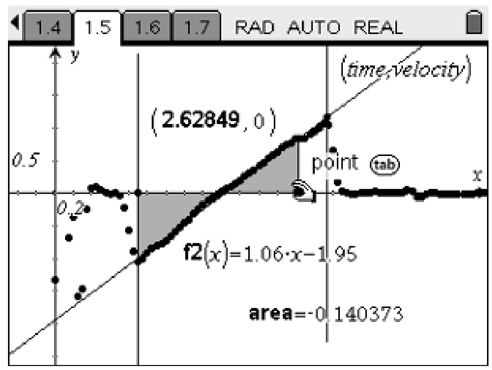

Figure 10

7. Page 1.7 plot of the position of the rolling ball to time, **distance** vs. **time**.
   - Add a second scatter plot of **varea** vs. **vtime** (Figure 11).
   - Adjust window with **Zoom Data**.

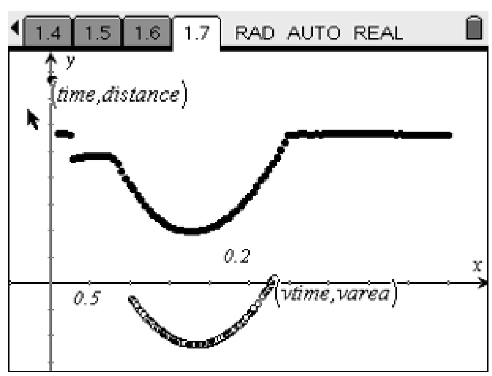

Figure 11

Q10. What is the relationship between the plot of **varea** vs. **vtime** and the experimental plot of position vs. time?

Answer: Except for a difference in vertical shift, which is related to the initial position of the ball, the two graphs give the same results for position vs. time of the rolling ball.

Student TI-Nspire™ File: *ballroll.tns*

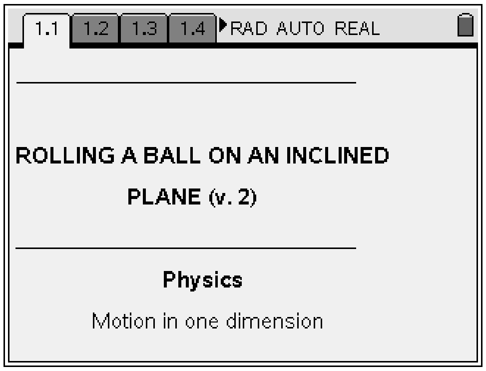

Figure 9

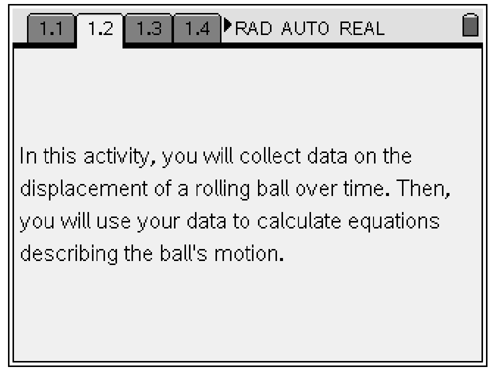

Figure 10

Figure 11

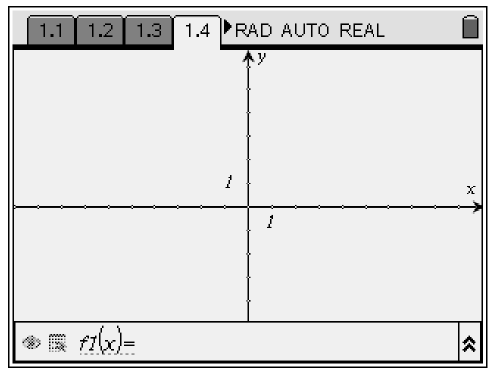

Figure 12

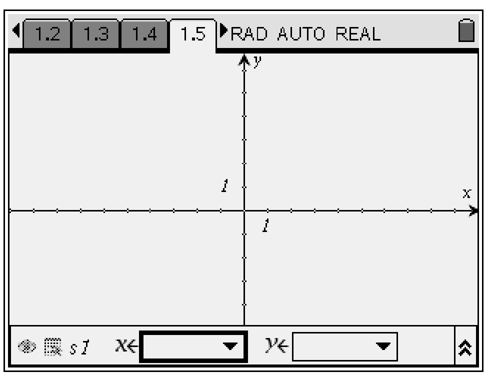

Figure 13

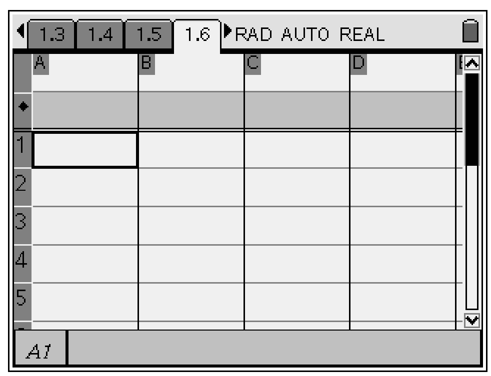

Figure 14

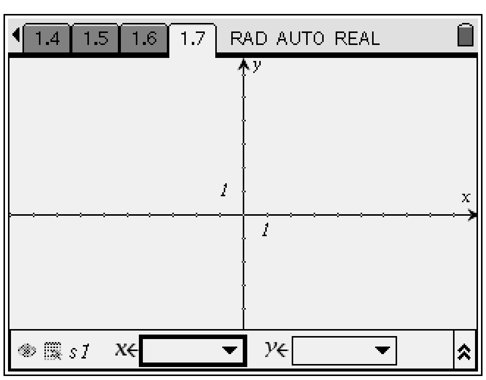

Figure 15

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Light, Intensity, and Distance

Concepts

- Light
- Intensity
- Distance

Materials

- TI-Nspire™ Math and Science Learning Technology
- *LightandDistance.tns* file

Overview

In this activity, we will explore the relationship between light, intensity, and distance.

Introduction

Open the *LightandDistance.tns* file.

1. Press ⌂.
2. Press 7 to select 7:My Documents
3. Find the Light and Distance file, highlight it, and press enter.
4. Once the document is open, move to the directions page by pressing ctrl ▶.
5. Follow the onscreen instructions to gather light data.
6. On page 1.4, to move between the graph, instructions, and the Data Collection Console, and then press ctrl tab .

Note: If the light gets to close to the paper the probe will max out (Figure 1).

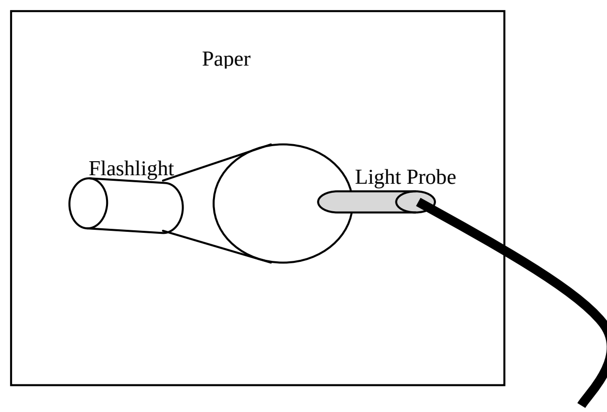


Figure 1

Distance	Intensity	Radius	Intensity*a	Intensity*d^2
30 cm				
40 cm				
50 cm				
60 cm				
70 cm				
80 cm				
90 cm				
			Average:	

Questions

1. What happens to the brightness (intensity) light on the wall as the light is moved away?

2. What happens to the size (area) of the circle that is produced by the light as it is moved away?

3. What is the relationship between intensity and area?

Note: By looking at the spreadsheet on page 1. 5 of the document you can compare values.

- Add a Data & Statistics application page, and set the x-axis to area and the y-axis to light.
  - a) As the area increases what happens to the intensity?
  - b) What mathematical relationship is this?

4. Add another Data & Statistics application page, and plot the inverse of the area (invarea) on the *x*-axis and intensity (dc01.light) on the *y*-axis.

- a) What does the graph look like?
- b) Does this confirm your thoughts in question 3?
- c) Calculate the linear regression for the data invarea and the intensity and record.

5. How is the distance from the wall and the area related?

- a) As you increase the distance what happens to the area?
- b) As you increase the distance what happens to the radius?
- c) How is the radius related to the area?
- d) What is the equation for radius and area?

6. Add another Data & Statistics application page.

- Plot the distance (dc01.event) and the radius on page, and calculate the regression equation.
  a) Regression equation
  b) What does x represent?
  c) What does y represent?
  d) Rewrite the equation in these terms

7. Since the radius is equal to the equation in problem 6, how could you substitute the distance in for radius to calculate area?

- Write the equation.

8. If the area is inversely related to the intensity, how would the distance from the wall be related to the intensity?

  a) What has to happen to the distance to get the area?
  b) If the area to intensity equation was changed to compare distance and intensity, what would this equation look like?
  c) If we consider all numbers in the equation to be a constant, what is the relationship between distance and intensity?

9. Add another Data & Statistics application page, plot distance (dc01.event) and intensity (dc01.light).

- Using the average of the intensity*distance^2 and designating it as k.

- Graph the function $f1(x):=k/x^2$ where f1(x) is the intensity, k is the average intensity*distance^2 and x is the distance.

10. Is the intensity directly or inversely related to area?

11. Why is the distance squared?

12. Is the intensity directly or inversely related to the square of the distance?

13. Write the equation relating Intensity I_1 and distance d_1 to Intensity two I_2 to distance two d_2.

Teacher's Notes

1. The experiment has shown that there is an inverse relationship between the area and the intensity.

$$I = k/A$$

2. Where **I** is the intensity of light, **k** is a constant, and **A** is the area of the circle of light produced.
3. Since area is determined by the radius of the circle, the above equation can be converted using the area equation.

$$A = \pi r^2$$

4. Because pi is a constant, it can be calculated into the constant **k** and a new $\mathbf{k_1}$ will be produced. This will convert radius into intensity.

$$K_1 = k/\pi$$

$$I = k_1/r^2$$

5. The radius then may be determined by the distance from the light and the angle at which the beam leaves the light.
6. Again the angle of beam will be constant and the $\tan\theta$ will be a constant.
7. At this point $\mathbf{K_1}$ can be converted to $\mathbf{k_2}$, and the relationship between distance and intensity can be generated.

$$r = d \tan\theta$$

$$r^2 = d^2 \tan^2\theta$$

$$k_2 = k_1/\tan^2\theta = k/(\pi \tan^2\theta)$$

$$I = k_2/d^2$$

8. **K_2** can be simply calculated by the product of the intensity and the distance squared.
9. The above explanation is to help understand that the light intensity is inversely related to the square of the distance from the source.

Simple Harmonic Motion

Concepts

- Simple harmonic motion
- Force, acceleration, and displacement

Materials

- TI-Nspire™CAS Math and Science Learning Technology
- Vernier® CBR 2™ (Calculator-Based Ranger ™ 2) or Go!™ Motion motion sensor
- Force Probe
- Pen or pencil
- Pendulum
- Balance
- Safety goggles
- Blank sheet of paper
- *SimpleHarmonicMotion.tns* file

Overview

In this activity, students collect data on the motion of a simple pendulum. They then graph the acceleration of the pendulum versus its displacement to show that the displacement of the pendulum is directly proportional to the force acting on it. They use this information to confirm that the motion of the pendulum fulfills the requirements of simple harmonic motion.

Teacher Preparation

Before carrying out this activity, review the concept of simple harmonic motion with students. Make sure they understand the requirements for motion to qualify as simple harmonic motion. A polystyrene fishing float and light-weight cord makes an excellent pendulum. The “bob” should have a mass of at least 40-50 g, and the string should be at least 100 cm long.

1. The force calculated in the activity takes into consideration only the horizontal acceleration of the pendulum.

  - To measure the entire net force acting on the pendulum, it would be necessary to include a second motion detector that measured the vertical motion.

- If the displacement of the pendulum is small relative to its length, the effect of ignoring the vertical motion is negligible.
- However, you should discuss this concept with students.

2. The screenshots on the following pages demonstrate expected student results.

- Refer to the screenshots on page 8 for a preview of the student TI-Nspire™ document (.tns file).

3. Students should use the force probe to determine the force of the pendulum. With this they can calculate the mass in kilograms.

Classroom Management

1. This activity is designed to be **student-centered**, with the teacher acting as a facilitator while students work cooperatively.

- Students should record their answers to the questions on a separate sheet of paper.

2. The ideas contained in the following pages are intended to provide a framework as to how the activity will progress.

- Suggestions are also provided to help ensure that the objectives for this activity are met.

3. In some cases, these instructions are specific to those students using TI-Nspire™ math and science handheld devices, but the activity can easily be done using TI-Nspire™ computer software.

The following questions will guide student exploration during this activity:

- Does the motion of a pendulum fulfill the requirements for simple harmonic motion?
- How can we make a mathematical model of simple harmonic motion?

Part 1: Collecting Displacement Data

1. Students will use a Vernier® CBR 2™ or Go!™Motion motion sensor to collect displacement data.

- When students reach page 1.3, they should connect the motion sensor to their handhelds or computers (Figure 1).
- This should activate the motion sensor, and a distance display should appear in the data collection box.
- The motion sensor should start clicking slowly, and the green light on the front should turn on.
- Change the length of the experimental to 10 s by selecting 1.**Experiment > 2. Set Up Collection > 1.Time Graph** (Figure 2).

2. After making sure all students are wearing safety goggles, pass out pendulums to the students.

- Students should set up their CBR 2™ sensors by opening up the light grey part of the sensor so it is perpendicular to the floor, as shown in Figure 3.
- The CBR 2™ should be about 50 cm from the resting point of the pendulum.
- Students should then practice pulling the pendulum back 20–30 cm and swinging the pendulum.

Figure 1

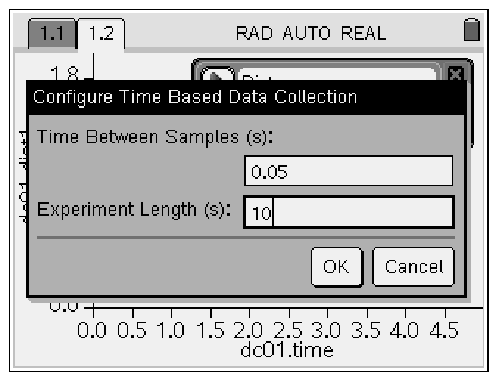


Figure 2

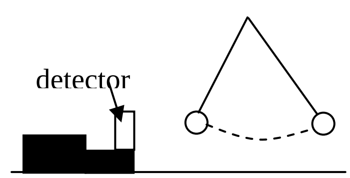


Figure 3

- The pendulum "bob" should remain in the field of the CBR 2™ as it oscillates and not hit the sensor as it swings.

3. When students are able to swing the pendulum correctly, they will begin data collection.

- They should pull the pendulum back 20–30 cm, release it, and then press ▶ on the screen.
- If the data are not smooth as shown in figure 4, or if there are gaps or large horizontal regions in the data set, have students repeat the data collection.
- The students should close the data collection box and disconnect the motion sensor after they have collected a "clean" data set like the one shown in Figure 4.

4. Next, students will advance to page 1.4, and make a scatter plot of displacement vs. time.

- They should adjust the scale of the graph so they can see all their data clearly.
- Then they change the graph type to **Function,** and enter 0.20*cos(x)+0.50 for **f1(x)** (Figure 5).
- They use the function transformation tools to fit the function to the data (Figure 6).

Q1. The cosine function is in the form a*cos(bx+c)+d.

- How does changing each of the parameters, a, b, c, and d, affect the function **f1(x)**?

Answer: "a" changes the amplitude; "b" changes the number of peaks which is the frequency in radians per second; "c" changes the locations of the peaks; and "d" moves the function up and down.

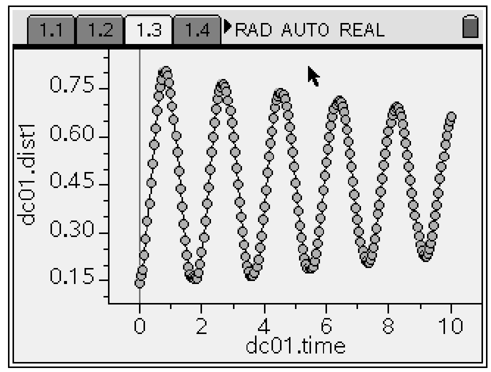


Figure 4

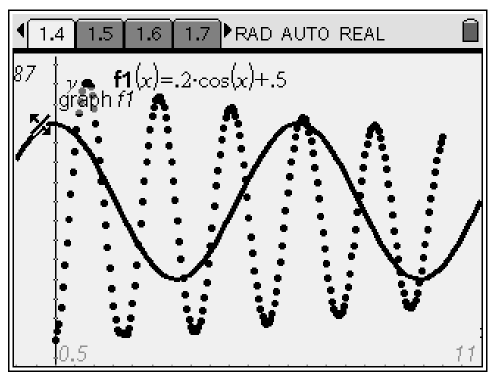


Figure 5

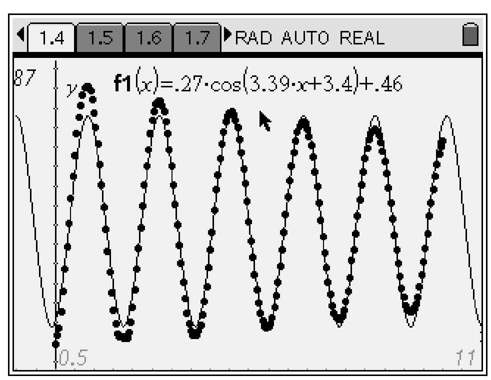


Figure 6

Q2. What is the frequency in cycles per second?

Answer: Divide the “b” parameter (the period of cosine) by 2π: 3.39/[2(3.1416)] = 0.54.

Q3. What is the relation between the value of “d” and the location of the sensor?

Answer: The value of “d” corresponds to the distance between the CBR 2™ and the rest position of the pendulum.

5. Next, students use **Trace** on the graph to locate the time (the x-value) of the maximum of the first peak (Note: m will appear when tracing at the peak) (Figure 7).
   - Enter the peak number and time in column A (**step)** and column B (**times) of the List and spreadsheet** (Figure 8).
   - Repeat for the other peaks.
6. On the following page students plot **times** vs. **step** and fit the data to a linear regression line (Figure 9).

Q4. What is the significance of the slope of this line?

Answer: The slope is the period in seconds of the pendulum.

Q5. What is the relationship between the frequency calculated for question 2 and the period?

Answer: *Frequency = 1/period: 0.54* s^{-1} = *1/1.85 s.*

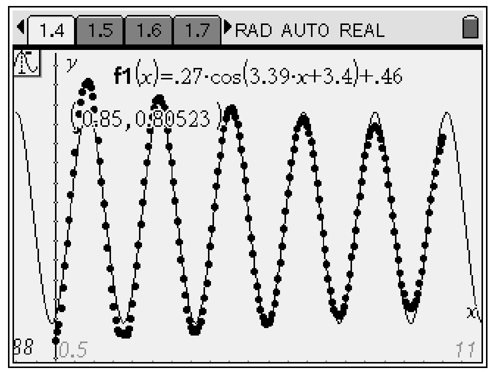


Figure 7

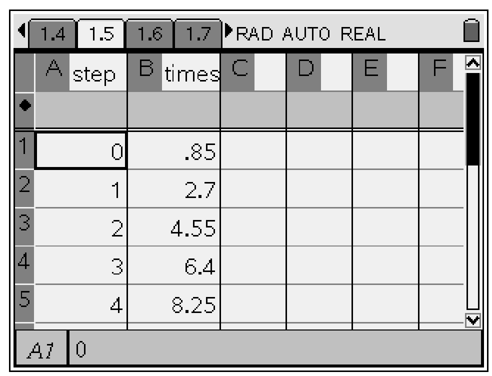


Figure 8

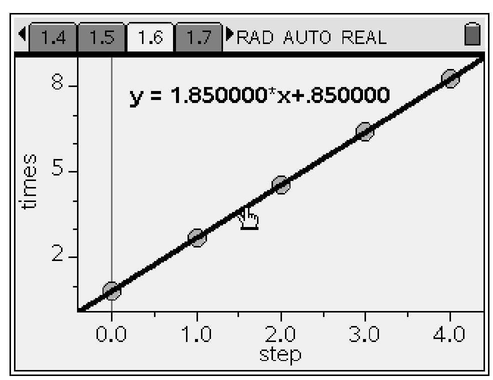


Figure 9

7. Students calculate the length of the pendulum, period = $(2\pi)\text{sqrt}(L/g)$, and measure the length of the pendulum (Figure 10).

Q6. How well does the calculated length of the pendulum compare to the measured length?

Answer: Student answers will vary. Generally the differences between the two values will be less than 10 percent.

Part 2: Force and Acceleration of the Pendulum

1. Next, students calculate the rest position using the **mean** function and the Calculator application page 1.8 (Figure 11).

Q7. What is the calculated rest position of your pendulum?

Answer: Student answers will vary. However, the calculated rest position should not be significantly different from the one determined graphically.

Q8. Compare the rest position you obtained graphically to the one you calculated.

- Comment on any differences.
- Which value is more accurate? Explain your answer.

Answer: Student answers will vary. The calculated value is probably more accurate, because the graphically determined value depends on the students' ability to identify the central point of the data.

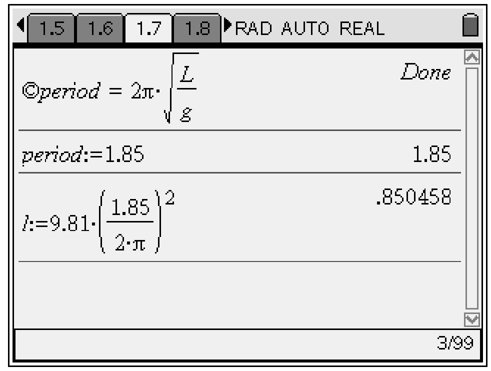


Figure 10

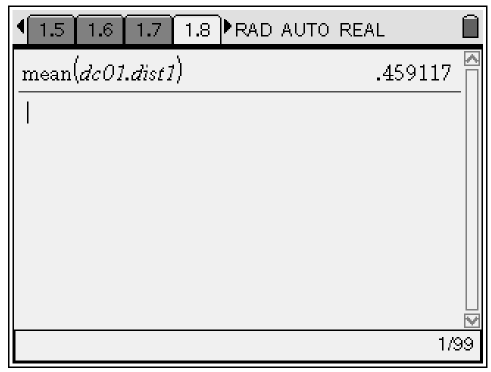


Figure 11

2. Next, students re-center their displacement values by subtracting the rest position from all the collected data points. Store these values in the table as zerodisp (Figure 12).

3. For a harmonic oscillator the restoring force should be –k*zerodisp = mass*acceleration.
    - Students plot acceleration vs. zerodisp (Figure 13).
    - The movable line tool can be used to produce a trend line for the data.

Q9. Does the acceleration data confirm that the pendulum is operating as a simple harmonic oscillator?

Answer: The displacement of the pendulum was directly proportional to the net force on the pendulum, which is a requirement for simple harmonic motion. Therefore, based on these data, the pendulum acted as a simple harmonic oscillator during this activity.

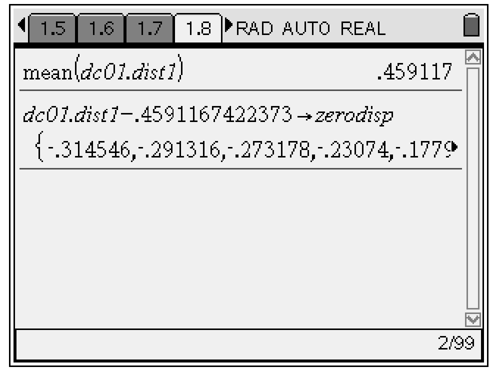


Figure 12

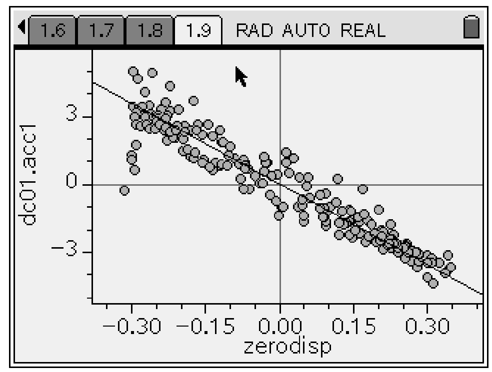


Figure 13

Student TI-Nspire™ File: *SimpleHarmonicMotion.tns*

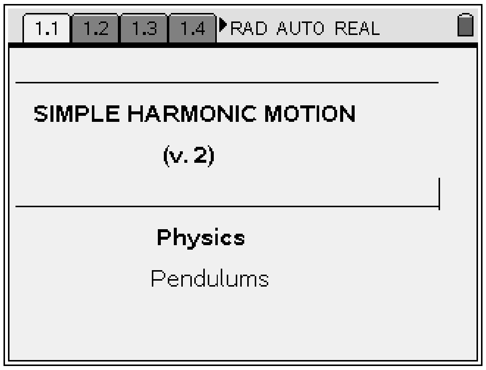


Figure 14

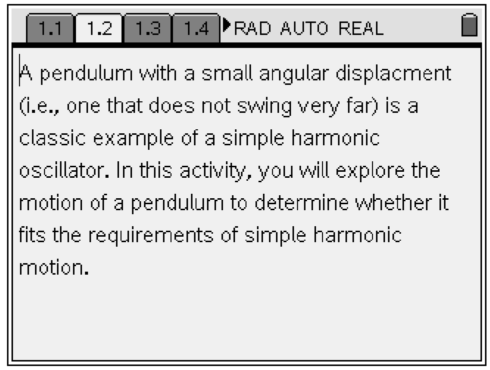


Figure 15

Figure 16

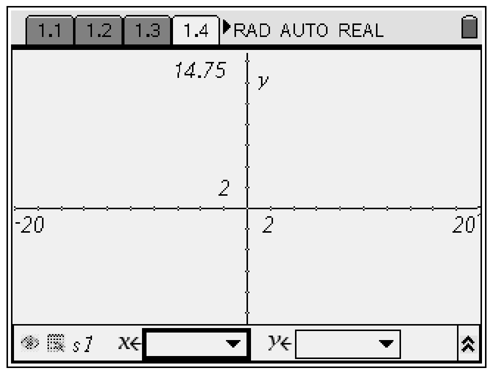


Figure 17

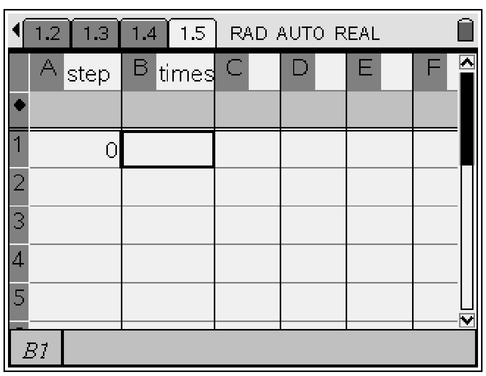


Figure 18

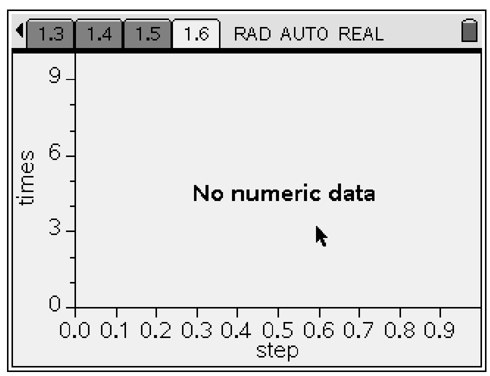


Figure 19

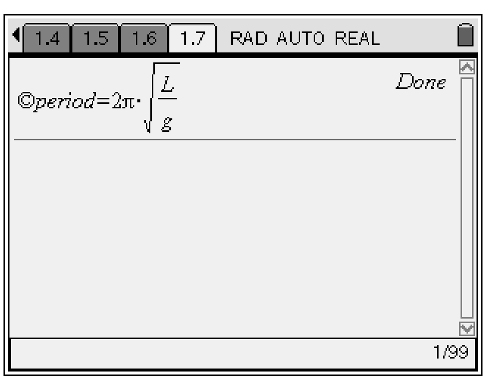


Figure 20

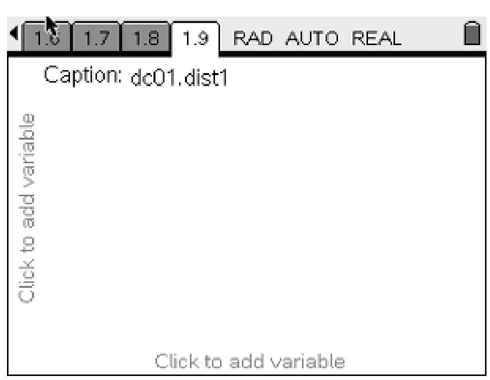


Figure 21

Air Resistance

Concepts

- Relationship between mass and air resistance
- Mathematical models of physical phenomena

Materials

- TI-Nspire™ Math and Science Learning Technology
- Five basket-style coffee filters
- Pen or pencil
- Vernier® CBR 2™ (Calculator-Based Ranger™ 2) or Go! ™Motion motion sensor
- Blank sheet of paper
- *AirResistance.tns* file

Overview

In this activity, students will collect data on the rate at which coffee filters fall. They will attempt to determine the terminal velocity for different numbers of falling filters. They will then explore the relationship between mass and terminal velocity.

Teacher Preparation

Students will probably be familiar with the idea that objects fall at different rates in air. Before beginning this activity, make sure students understand that these differences in falling rate are entirely due to the presence of air—some students may have the misconception that heavier objects fall more quickly than lighter objects.

1. The screenshots on the following pages demonstrate expected student results.

- Refer to the screenshots for a preview of the student TI-Nspire™ document (.tns file.)

Classroom Management

1. This activity is designed to be **student-centered**, with the teacher acting as a facilitator while students work cooperatively.

- Students should record their answers to the questions on a separate sheet of paper.

2. The ideas contained in the following pages are intended to provide a framework as to how the activity will progress.
   - Suggestions are also provided to help ensure that the objectives for this activity are met.
3. In some cases, these instructions are specific to those students using TI-Nspire™ math and science learning handheld devices, but the activity can easily be done using TI-Nspire™ computer software.

The following questions will guide student exploration during this activity:

- How does the mass of an object affect its terminal velocity?
- What is the best mathematical model for the drag force?

Part 1: Collecting Terminal Velocity Data

1. First, students will use a Vernier® CBR 2™ or Go!™Motion motion sensor to collect data on the velocity of a falling coffee filter.

- Students should clamp or hold their motion sensors as high above the ground as possible; a distance of at least 2 m is ideal.
- When students reach first Data & Statistics page, they should connect the motion sensor to their handhelds.
- Select the Data & Statistics application.
- This should activate the motion sensor, and a distance display should appear in the data collection box (Figure 1).

2. Select **Menu** > **1. Experiment** > **3. Set Up Collection** > **1. Time Graph** to specify the time between samples of 0.05 s and the experiment length of 3 s (Figure 2).

- Click **OK**.

3. One student from each group should now hold a single coffee filter below the wire mesh of the motion sensor, as shown in Figure 3.

- The coffee filter should be no less than 0.4 m from the motion sensor.
- Students can use the distance display on the handheld to determine the distance from the sensor to the filter.

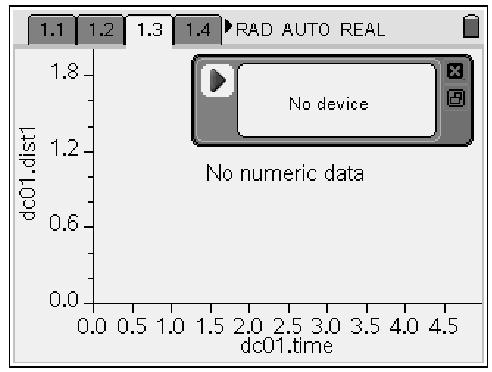


Figure 1

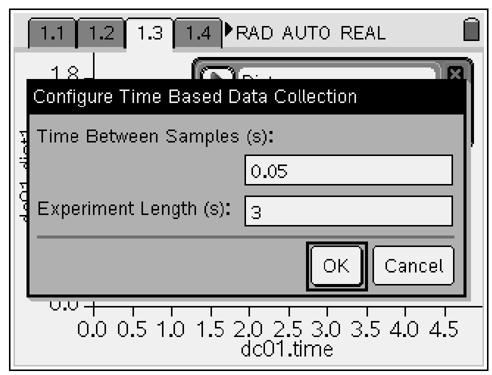


Figure 2

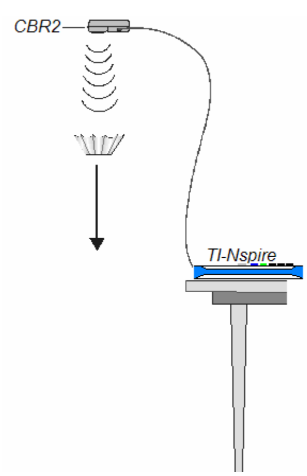


Figure 3

4. Next, students will begin data collection (press ▶); and release the filters.

- If time allows, have students practice releasing the filters several times.
- They should be able to release the filters so that they fall straight down with little sideways motion.
- Ideally, the students' motion graphs should resemble the one shown in Figure 4.

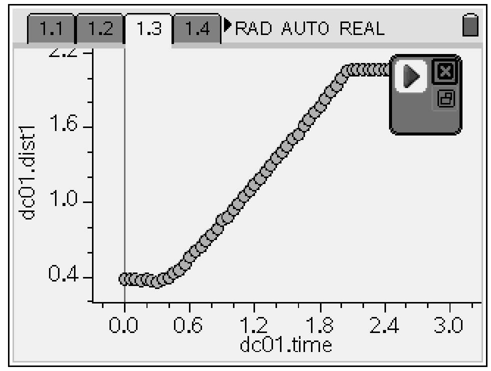


Figure 4

Q1. Describe the characteristics of the graph.

Answer: The graph should have a short region of curvature, followed by a long linear region.

- The linear region represents the time during which the filter was falling at constant velocity.

5. Next, students repeat the experiment using two coffee filters.

- Move to the next page which is labeled 2 Filters.
- Press ▶, and click the **Store** option (Figure 5).
- Click **OK** to save the first data set **dc01**.
- The data for two coffee filters will be found in **dc02**.
- Repeat this process for three, four, and five filters.
- If time is short, you may have each group carry out the experiment with a different number of coffee filters, and then pool the data using TI-Nspire™ Computer Link software.

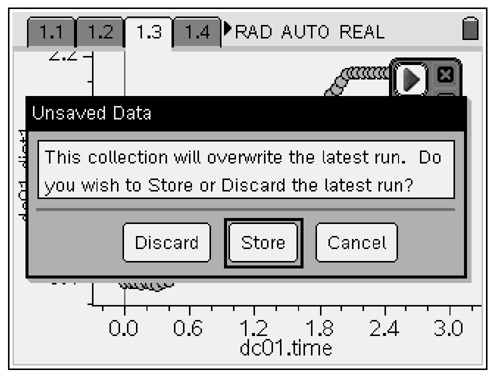


Figure 5

6. To graph each data set, students will plot **dc01.dist1** vs. **dc01.time**, **dc02.dist1** vs. **dc02.time**, etc. on subsequent Data & Statistics application pages 1.3 to 1.7.
   - On each of the graphs students should fit a straight line to the linear portions of their graphs.
   - They will use the **Movable Line** tool in the Data & Statistics application page to produce the line that best fits their data (Figure 6).
   - They should record the terminal velocity of the filter (the slope of the best-fit line) in Lists & Spreadsheet application page 1.8.

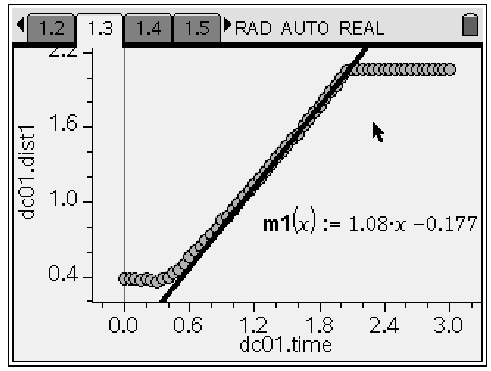


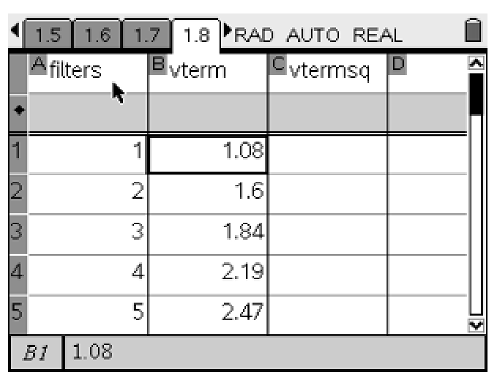
Figure 6

Q2. Why is using this movable line tool preferable to performing a linear regression in this case?

Answer: The data set contains regions at the beginning and the end that are nonlinear.

- A linear regression would attempt to fit a line to the entire data set, not just the linear portion.
- As an extension activity, you could have students "trim" the nonlinear data from their data sets; and then carry out a linear regression on the data.
- They can then compare the terminal velocities they determined using both methods, and discuss any differences.

7. Students should store their best-fit terminal velocities in Lists & Spreadsheet application page 1.8 (Figure 7).



1.5 | 1.6 | 1.7 | 1.8 | RAD AUTO REAL

	A filters	B vterm	C vtermsq	D
◆				
1	1	1.08		
2	2	1.6		
3	3	1.84		
4	4	2.19		
5	5	2.47		

B1 1.08

Figure 7

Part 2: Exploring the Relationships between Mass, Drag Force, and Terminal Velocity

1. First, students calculate the square of the terminal velocity for each of their trials in Lists & Spreadsheet application page 1.8 (Figure 8).

2. Next, students move to pages 1.9 and 1.10, which contains empty Data & Statistics application pages.

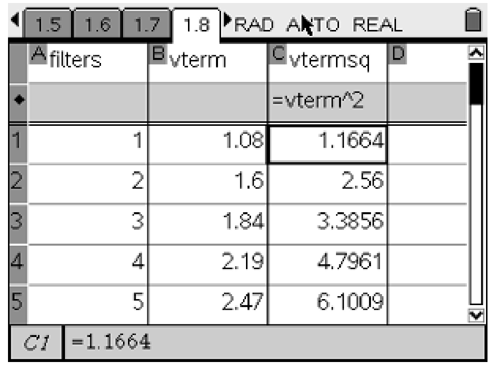
- They plot **vterm** vs. **mass** and **vtermsq** vs. **mass,** and use the **Movable Line** tool to determine which plot produces the best fit to the data (Figures 9 and 10).

Q3. From your graphs, which proportionality is consistent with your data; that is, which graph is closer to a straight line that goes through the origin?

- Why should the line go through the origin?

Answer: The graph of v_T^2 vs. mass is closer to a direct relationship than is the graph of v_T vs. mass.

- In particular, the v_T^2 vs. mass graph passes close to the origin, while the v_T vs. mass graph does not.
- The data therefore suggest that, for these coffee filters, mass is proportional to v_T^2.
- Note that both data sets are very close to linear.
- Students may have difficulty determining which data set gives the best fit to the linear relationship.



	A filters	B vterm	C vtermsq	D
◆			=vterm^2	
1	1	1.08	1.1664	
2	2	1.6	2.56	
3	3	1.84	3.3856	
4	4	2.19	4.7961	
5	5	2.47	6.1009	

C1 =1.1664

Figure 8

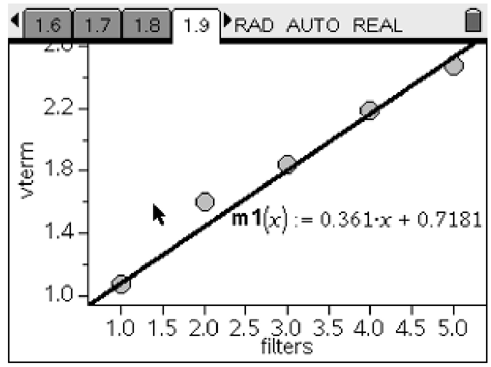


Figure 9

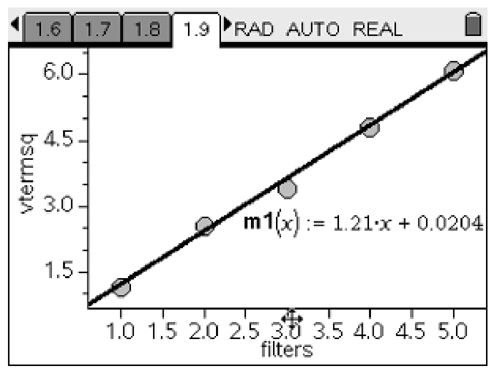


Figure 10

- Remind them that, for this relationship, it is important that the best-fit line pass through the origin; this will help them differentiate between the two lines.

Q4. From the choice of proportionalities in the previous step, which of the drag force relationships ($-bv$ or $-cv^2$) appears to model the real data better?

Answer: Since the v_T^2 vs. mass graph is close to a proportionality, it appears that the drag force is proportional to the square of the velocity, or $F_{drag} = -cv^2$.

Q5. If one filter falls in time t, how long would it take four filters to fall, assuming the filters are always moving at terminal velocity?

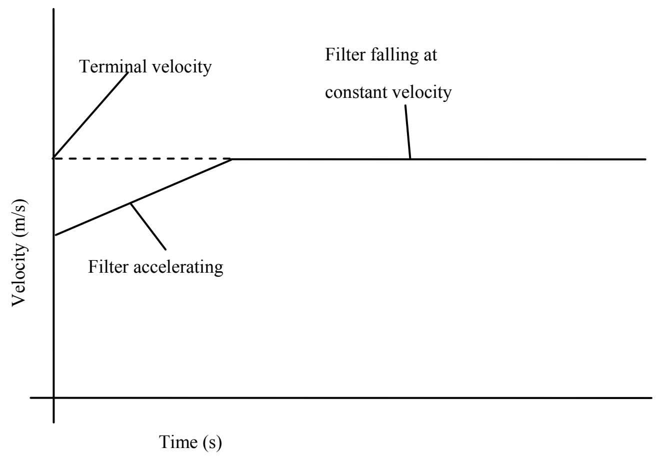
Answer: Since the graphs show that terminal velocity squared is directly proportional to mass, the terminal velocity of four filters is about twice as large as the terminal velocity of one filter.

- Therefore, if one filter falls in time t, four filters would fall in time 0.5t.

Q6. Make a sketch of velocity vs. time for a falling coffee filter. On the graph, label the following:

- region(s) in which the filter is accelerating (if any)
- region(s) in which the filter is falling at constant velocity
- the terminal velocity of the filter

Answer:



Q7. Describe the forces acting on the filter in each region.

- Explain how these forces produce the motion you have drawn.

Answer: At all times during the filter's fall, gravity is pulling it down, and air resistance (drag) is resisting its fall.

- During the time when the filter is accelerating, gravitational force is greater than drag force; so the net force on the filter is downward, and the filter accelerates.
- During the time when the filter is falling at constant velocity, gravitational force is equal to drag force.
- The net force on the filter is therefore zero, and the filter does not accelerate.

Q8. Why does a coffee filter reach terminal velocity after falling less than 1 m, but a basketball or other heavier object must fall farther before reaching terminal velocity?

Answer: Drag force is proportional to velocity (or velocity squared), so it increases as long as the falling object is accelerating.

- The object stops accelerating when the drag force equals the weight of the object.
- Therefore, heavier objects must fall a greater distance (a longer time) to reach a high enough velocity for the drag force to balance their weight.

Student TI-Nspire™ File: *AirResistance.tns*

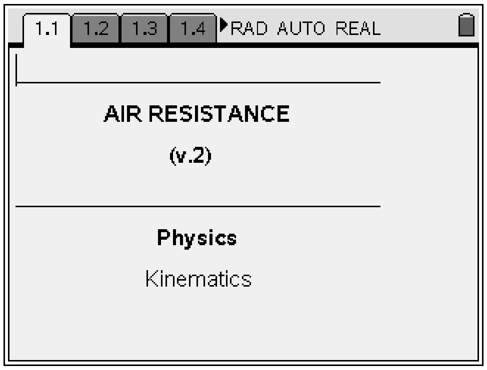


Figure 11

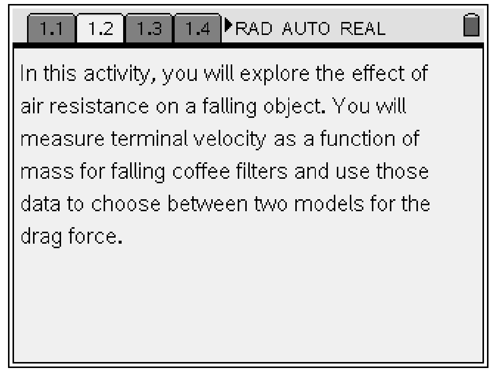


Figure 12

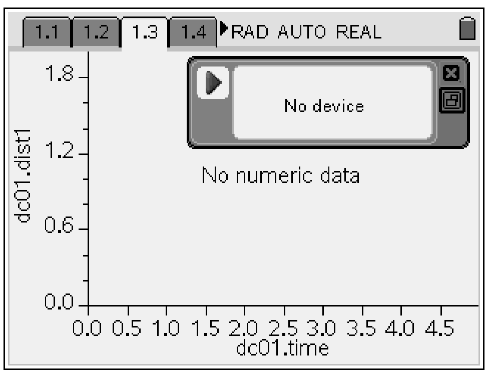


Figure 13

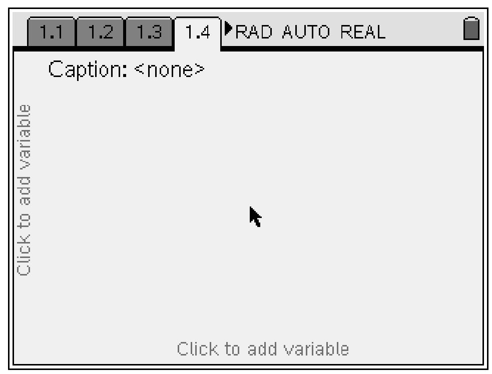


Figure 14

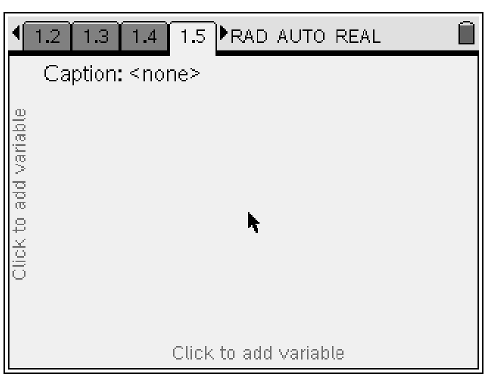


Figure 15

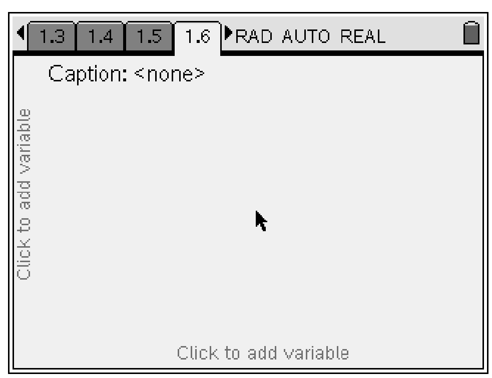


Figure 16

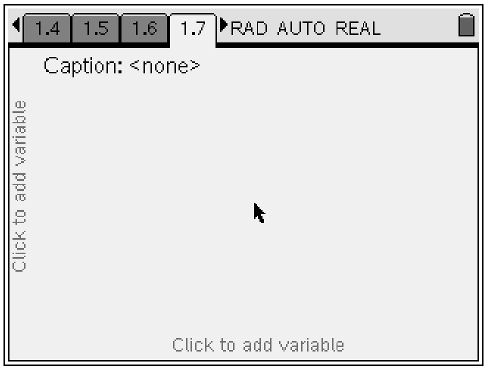


Figure 17

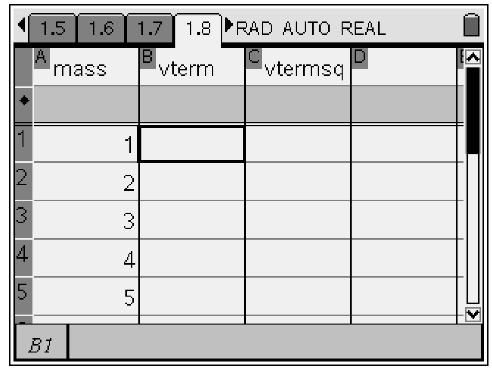


Figure 18

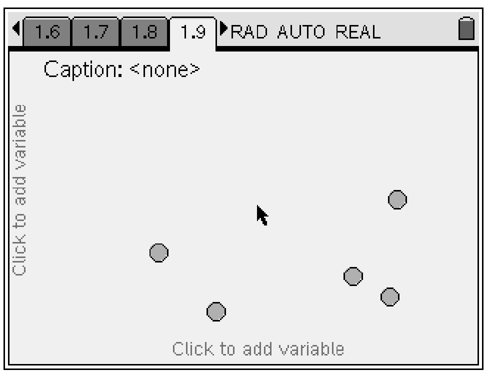


Figure 19

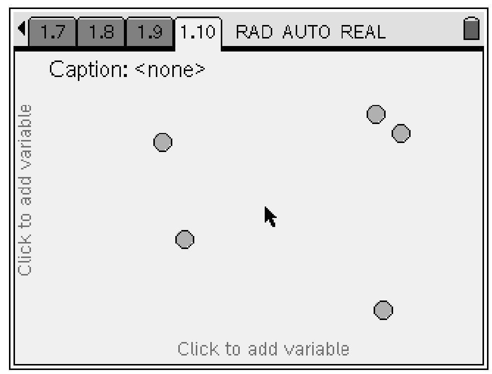


Figure 20

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Forces on Point Charges

Concepts

- Use Coulomb's law to solve problems of force between electric charges
- Solve problems involving point charges, and force using vector addition

Materials

- TI-Nspire™ Math and Science Learning Technology
- Pen or pencil
- *PointCharges.tns* file

Overview

In this activity, students explore interaction between charged point particles. They first explore graphical vector addition, and then use vector addition rules to explore the net forces on charged particles. The pre-constructed templates used in this activity include charged particles, represented by points, and forces of interactions between the particles, represented by vectors. When students change magnitudes of charges or distances between the particles, the vectors representing the forces of interaction change correspondingly.

Teacher Preparation

Before carrying out this activity, you should review Coulomb's Law with students. Students should also be comfortable with the concept of a vector and the rules of vector addition before beginning this activity.

1. The screenshots on the following pages demonstrate expected student results.

- Refer to the screenshots on final page for a preview of the student TI-Nspire™ document (.tns file).

Classroom Management

1. This activity is designed to be **student-centered**, with the teacher acting as a facilitator while students work cooperatively.
    - Students should record their answers to the questions on notebook paper.
    - If time allows, a whole-class discussion of the activity would be useful for the students.
2. The ideas contained in the following pages are intended to provide a framework as to how the activity will progress.
    - Suggestions are also provided to help ensure that the objectives for this activity are met.
3. In some cases, these instructions are specific to those students using TI-Nspire™ math and science learning handheld devices, but the activity can easily be done using TI-Nspire™ computer software.

The following questions will guide student exploration during this activity:

- How do the positions and charges of particles affect the forces acting on the particles?
- How can a pre-constructed scenario be used to solve electrostatics problems?

The purpose of this activity is to provide students with an opportunity to explore the interactions between charged particles when three or more particles are present in the system. TI-Nspire™ technology provides students with dynamic environment for explorations and tools for graphical and numerical analysis for variety of situations.

This activity consists of three parts. The pre-constructed templates could also be used by the students in and out of class to verify solutions of various problems dealing with point charges and forces between them.

Part 1: Graphical Vector Addition

1. In this part of the activity, students use TI-Nspire™ features to practice graphical methods of vector addition.

- First, students should open the file **PointCharges.tns**; and read the first two pages.
- They should then proceed to page 1.3, which contains an empty Graphs & Geometry application page.
- Students should change the page to the Plane Geometry view by pressing menu 2 2 for Menu 2: View, 2: Plane Geometry View, and hide the scale menu 2 7 for Menu 2: View, 7: Hide Scale.

It is recommended that students change the document settings for the angle measurements to be in degrees for this activity.

- To change the document settings, students should press ctrl to enter the page sorter view, press menu 5 for Menu 5: Document Settings (Figure 1).

2. Next, students use the **Vector** tool by pressing menu 6 8, for Menu 6: Points & Lines, 8: Vectors, to draw two coterminal vectors, AB and AC, on page 1.3 (Figure 2). (Note: when placing a point type the letter to label it.)

3. Next, students should use the **Translation** tool by pressing menu A 3 for Menu A: Transformations, 3: Translation to translate vector AB to point C.

- Click on Vector AB and then click on point C to translate the endpoint.
- They should label the image of point *B* as *D* using the **Text** tool.

4. Next, students construct vector AD, the sum of AB and AC (Figure 3).

- They then find the magnitude and direction of vector AD using the **Length** and **Slope** measurement tools from the **Measurement** menu.

Q1. How does the length of vector AD compare to the lengths of vectors AB and AC when AB and AC are collinear?

Answer: When AB and AC are parallel, AD is equal to the sum of the lengths of AB and AC.

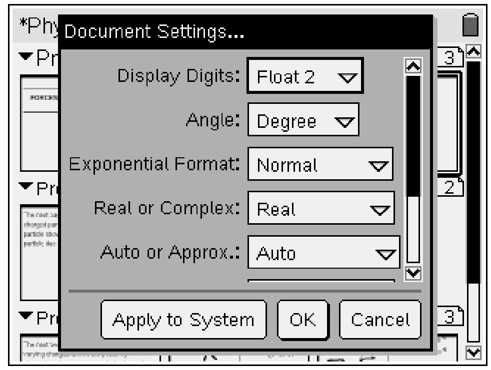

Figure 1

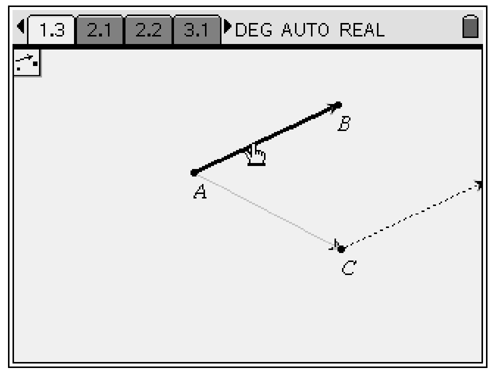

Figure 2

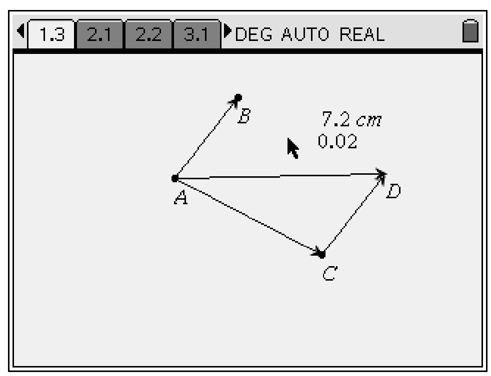

Figure 3

Q2. How does the direction (slope) of vector AD compare to the slopes of vectors AB and AC?

Answer: The slope of AD is partway between the slopes of AB and AC.

- Students can use the **Slope** measurement tool to observe the direction of the resultant vector, or they can find the angle of the vector to the horizontal.
- Since the slope of the line is equal to the inverse tangent of the angle, students can use the **Text** and **Calculate** tools to find the angle of the resultant vector.
- The instructions for using this method are given below:
  - Choose the Text tool by pressing menu 1 6 for Menu 1: Tools, 6: Text.
  - Click on a blank space anywhere on the screen. The text box will open in the edit mode.
  - Enter the expression $\tan^{-1}(s)$ and press enter.
  - Choose the Calculate tool by pressing menu 1 8 for Menu 1: Tools, 8: Calculate.
  - Click on the expression.
  - When Select s? appears on the screen, move the cursor, and click on the value of the slope (Figure 4).
  - Press enter, and the value of the slope should appear (Figure 5).
  - Drag the expression to the desired location, and press enter again.

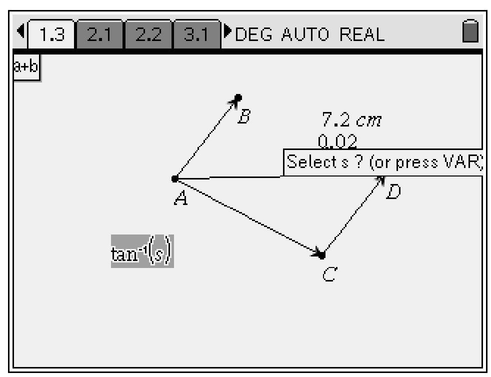

Figure 4

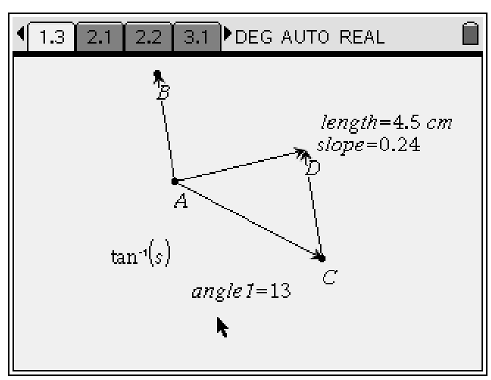

Figure 5

Remind students that, when the angle is in the second or third quadrant, they need to add 180º to the angle calculated using the inverse tangent since the computer only reports a reference angle.

5. Next, students explore the addition of three coterminal vectors.

- They first construct vector AE, which should point in a direction different from those of vectors AB, AC, or AD (Figure 6).
- Then use the **Translation** tool to add vector AE to vector AD.
- They label the endpoint of this vector *F* and construct vector AF, which is the sum of vectors AB, AC, and AE.

6. Next, students explore how vector AF changes as they move points A, B, C, and E around the screen (Figure 7).

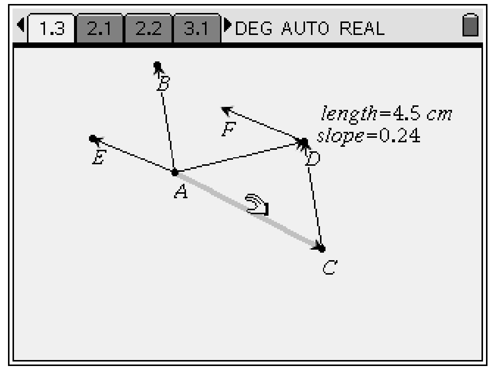

Figure 6

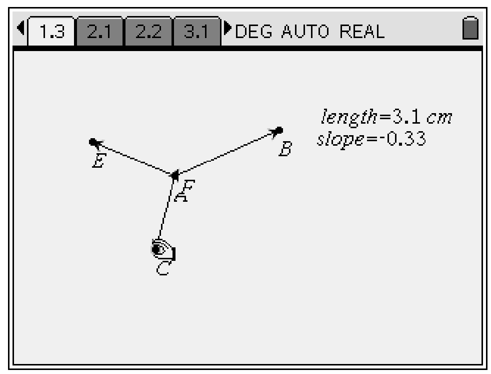

Figure 7

Q3. Describe the lengths and positions of vectors AB, AC, and AE required to make AF become as small as possible.

Answer: There are several combinations of vectors AB, AC, and AE that will make vector AF extremely small.

- Encourage students to compare their results with one another and discuss the different solutions.

Part 2: Interaction between Three Similarly Charged Particles

Next, students should move to page 2.1; and read the text there.

- Page 2.2 shows three positively charged particles (Figure 8).
- The vectors attached to each point show the forces on each particle that are produced by the other two particles.
- Students can drag each point and thus vary the distances between the particles.
- The numbers on the right-hand side of the screen give the magnitudes of the charges on each particle.
- Students can click on these labels and vary the magnitudes of the charges.
- The charges are given in units of coulombs (C), and distances are in meters.

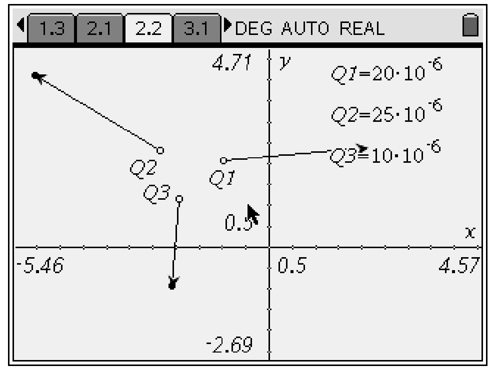

Figure 8

1. Next, students should find the net force on each particle using head-to-tail vector addition.

- They should then hide the individual force vectors on each particle, so that only the net forces are visible.

2. Next, students should vary the locations and charges of points *Q1*, *Q2*, and *Q3;* and observe the results (Figure 9).

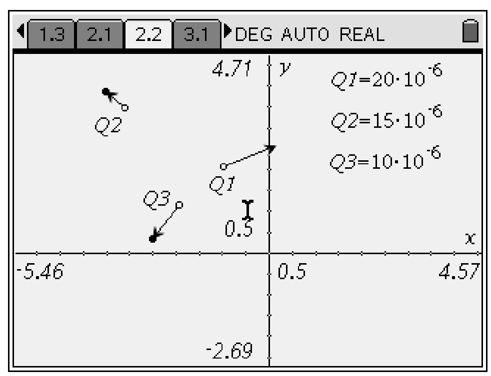

Figure 9

Q4. Describe the changes you observed in the net force on each particle when you vary magnitudes and positions of the particles.

- Did you notice any patterns?

Answer: Due to the open-ended nature of the question, many different responses are possible.

- However, students should note that the net forces on the particles decrease as the particles get farther apart and increase as they get closer together.
- Students should also note that the net forces on the particles are directly related to the charges on the particles (i.e., the larger the charges, the larger the net forces).

Q5. What happens to the net force on a particle when its charge is much larger or smaller than the charges on the other two particles?

Answer: When the charge on one particle is much larger than the charges on the other particles, the net forces on all three particles increase significantly (Figures 10 and 11).

- As the charge on one particle approaches zero, the net force on that particle also approaches zero.
- Note that students may think that the force on a particle can never go to zero if the other two particles have non-zero charge.
- Guide them to understand that electric forces affect only charged particles, so a particle with a zero charge will experience no net force.

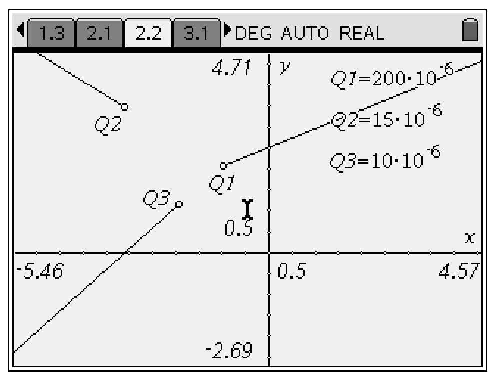

Figure 10

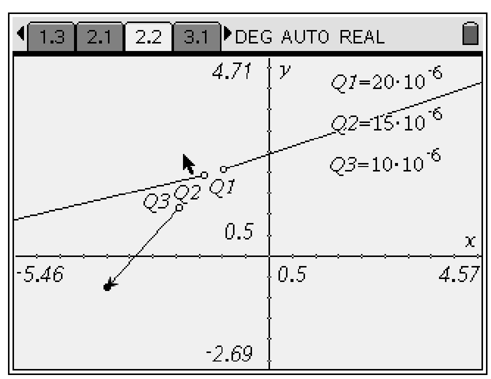

Figure 11

Q6. What happens to the net force when one of the particles moves very close to one of the other particles?

Answer: The forces on the particles become very large.

Q7. What happens to the net force on a particle when it is moved far away from the other two particles?

Answer: The forces on the particles become very small.

Q8. What happens to the net force on a particle when it is located exactly between two equally charged particles?

Answer: The net force on the central particle is zero, because the vectors acting on the central particle have equal magnitudes, but opposite directions.

Q9. Three particles, each with a charge of +11 μC, are located at the corners of an equilateral triangle with sides of length 1 m.

- Calculate the magnitude and direction of the net force on each particle.
- Hint: Use the template on page 2.2, adjust the scale if needed, and reproduce the conditions of the problem. Then calculate magnitude and angle for each net force (Figure 12).

Answer: Students can use the template on page 2.2 to solve this problem graphically.

- They can place the three charges, for example, on the points $(-1, 0)$, $(0, 0)$, and $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

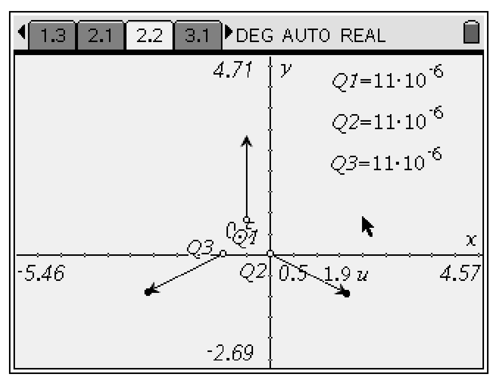

Figure 12

- They can then display the coordinates of points *Q1*, *Q2*, and *Q3* using the **Coordinates and Equations** tool by pressing menu 1 7 for Menu 1:Actions, 7: Coordinates and Equations.
- They can use the **Length** and **Slope Measurement** tools to determine the magnitudes and directions of the resulting forces.
- They should find that the magnitude of the force on each particle is 1.9 N and the directions are as follows:
  - For (0, 0), θ = –30°; for $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, θ = 90°; and for (–1, 0), θ = 210°.
  - Students will probably need to adjust the scale of the graph by pressing menu 4 1 for Menu 4: Window, 1: Window Settings to see the net forces clearly.
  - Students should also calculate the magnitudes and directions analytically, using Coulomb's law and vector addition rules.
  - Coulomb's law yields the net force on each particle, as shown on the next page (make sure students convert μC to C):

$$F_c = 2k\frac{q^2}{r^2}\cos(30°)$$

$$F_c = (2)\left(9\times10^9\,\frac{N\cdot m^2}{C^2}\right)\left(\frac{11\times10^{-6}C}{1m}\right)(0.866)$$

$$F_c = 1.9N$$

- Students can use the law of cosines to determine the direction of each vector.

Problem 3: Interaction between Particles of Opposite Charge

In this part of the activity, students explore the interactions between positively and negatively charged particles.

- Note that in each template the sign of the charge cannot be changed.
- Students should read page 3.1 before moving on to the simulations.

1. Students should first move to page 3.2, which shows two positively charged particles and one negatively charged particle (Figure 13).

- They should use tail-to-head vector addition to find the net force on each particle, and then hide the individual force vectors on each particle so only the net forces are visible.
- They should change the positions and charges of the particles, and observe the results.

2. Next, students should move to page 3.3, which shows two positively charged particles and two negatively charged particles (Figure 14).

- They should again find the net force on each particle, and hide the component forces.

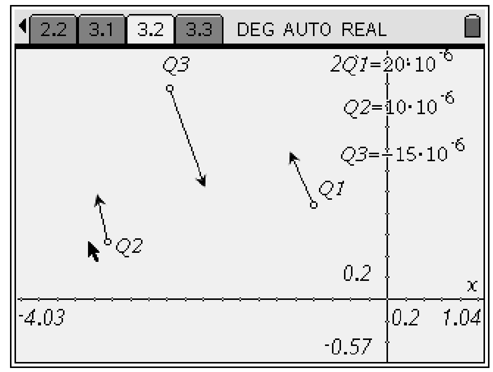

Figure 13

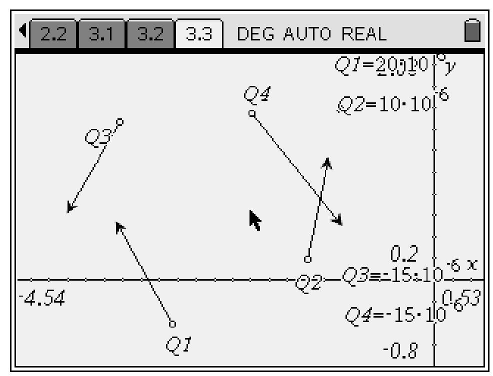

Figure 14

Q10. What happens to the net forces when oppositely charged particles move very close to one another?

Answer: The magnitudes of forces on the oppositely charged particles increase significantly and approach infinity.

- Their directions become opposite.
- At the same time, the net force on the third particle approaches zero (Figure 15).

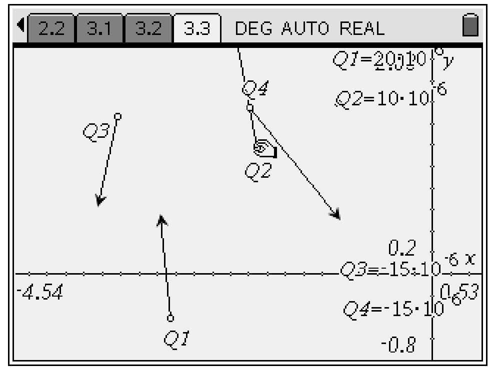

Figure 15

Q11. What happens to the net force on a negatively charged particle when it is located exactly between two particles of equal positive charge?

Answer: When a negatively charged particle is located exactly between two equally positively charged particles, the net force on the negatively charged particle is zero, because the two force vectors acting on the central particle have equal magnitudes, but opposite directions.

Q12. Explore other symmetrical arrangements of the particles, and describe any patterns that you observe.

Answer: Encourage students to explore different symmetrical arrangements of particles, and discuss their results with the class.

- Encourage them to predict the net forces resulting from various symmetrical arrangements, and then use the templates on pages 3.2 and 3.3 to test their predictions (Figure 16).

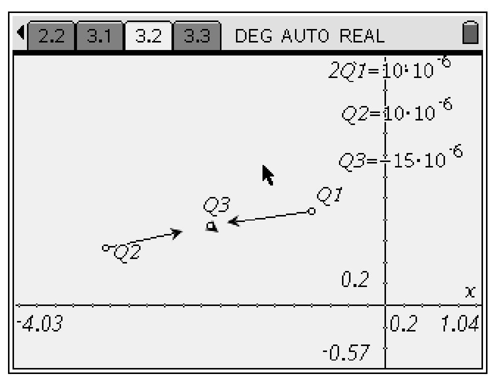

Figure 16

Q13. A particle of charge +100 μC is located at (–2, 0), and a particle of charge +200 μC is located at (2,0) (Figure 17).

- Where should a negatively charged particle with charge –50 μC be placed so that the net force on this particle has a magnitude of 2 N and is directed at –135°?
- Hint: You can see the coordinates of a point by pressing menu 1 7 Menu 1: Actions, 7: Coordinates and Equations, and then clicking on the point.

Answer: Two possible coordinates are (7, 5.6) and (-0.385, 0.082).

- This particular problem is too advanced for analytical solution.
- It is sufficient if students verify their graphical solution by calculations.
- In order to do that they will need to measure the magnitudes and angles of all forces, and show that the net force in the system is zero.

Q14. A right isosceles triangle is formed by the charges Q_1 = +150 μC, Q_2 > 0, and Q_3 = –120 μC located at points (0,10), (0, 0), and (–10, 0), respectively (Figure 18).

- The fourth charge Q_4 = –145 μC is located at the midpoint of the hypotenuse.
- The net force on charge Q_4 is 10 N in the positive *x*-direction.
- What is the charge on Q_2?

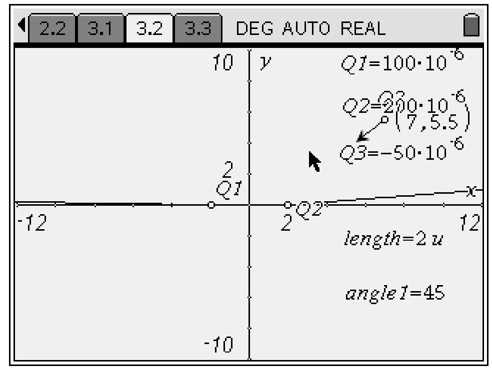

Figure 17

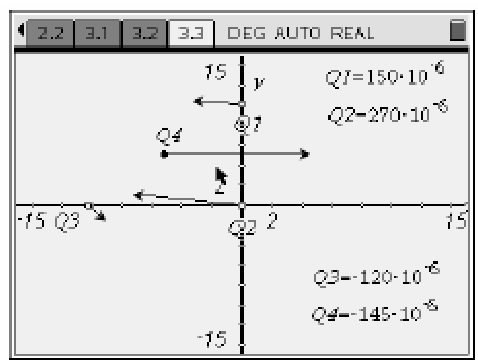

Figure 18

Answer: The charge on particle Q_2 should be approximately +270 μC.

- Once again, students should verify that the net force in the system is zero after they find their solution.
- If you wish, you may assign additional problems or explorations to student pairs or groups for homework or independent projects.

Student TI-Nspire™ File: *PointCharges.tns*

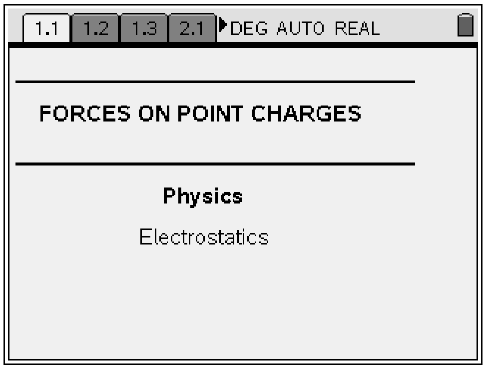

Figure 19

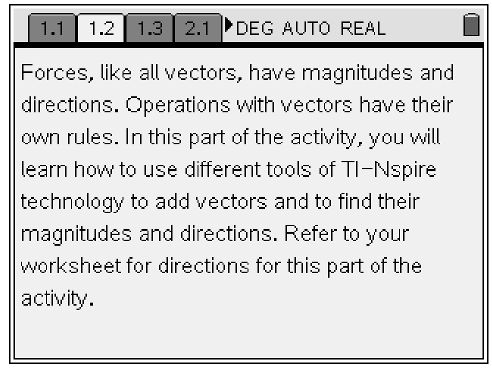

Figure 20

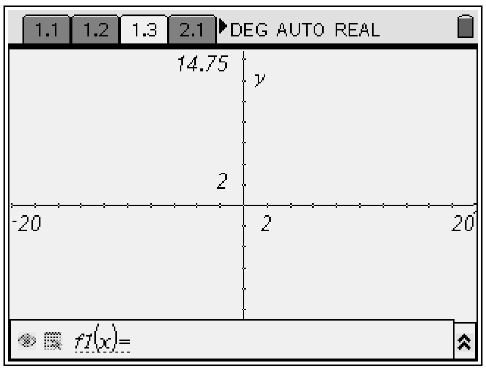

Figure 21

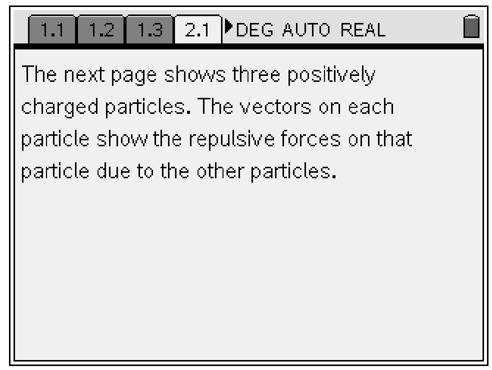

Figure 22

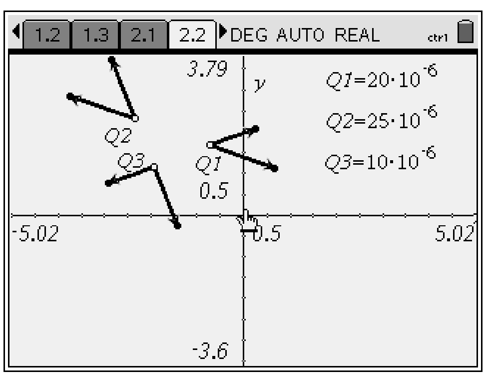

Figure 23

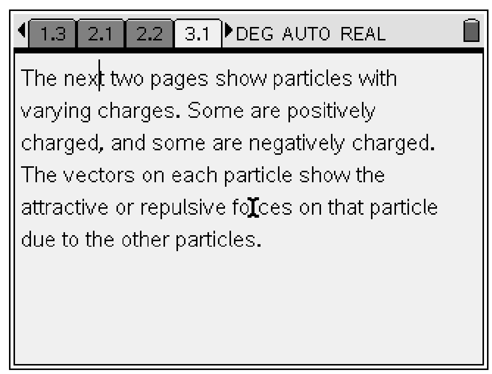

Figure 24

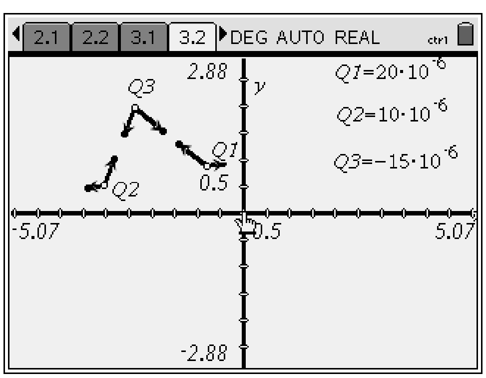

Figure 25

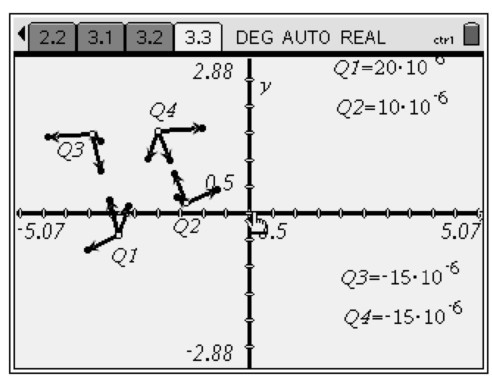

Figure 26

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Resistance-Capacitance Time Constant

Concepts

- The collection of voltage vs. time data
- The rate of discharge of capacitors
- Exponential functions

Materials

- TI-Nspire™ CAS Math and Science Technology
- Vernier® Voltage Probe
- Pen or pencil
- 10μF capacitor
- 22 and 47 kΩ resistors
- 9 V battery
- hook-up wire with alligator clips
- safety goggles
- blank sheet of paper
- *RCtimeconstant.tns* file

Overview

In this activity, students will explore the time constant for the rate of discharging and charging a capacitor in a circuit containing a capacitor and a resistor.

Teacher Preparation

Before carrying out this activity, review electrical units such as ohms, volts, coulombs, and farads with students. Review the concept of a capacitor with them.

- The screenshots on the following pages demonstrate expected student results. Refer to the screenshots on the last pages for a preview of the student TI-Nspire™ document (.tns file.)

Classroom Management

1. This activity is designed to be **teacher-led** with students following along on their handhelds.

- You may use the following pages to present the material to the class and encourage discussion.

- Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.

2. Students may answer the questions posed in the .tns file using the Notes application or on a separate sheet of paper.

3. In some cases, these instructions are specific to those students using TI-Nspire™ math and science learning handheld devices, but the activity can easily be done using TI-Nspire™ computer software.

The following questions will guide student exploration in this activity:

- How fast do capacitors discharge?
- How does resistance affect the rate of discharge of a capacitor?
- What is the relationship between the rate of discharging and the rate of charging a capacitor?

Students will collect voltage vs. time data for discharging and charging a capacitor in a circuit containing a resistor and a capacitor.

- In a second experiment they will use a different resistor.
- Using the experimental data, students will determine the time constant of a *RC* circuit.

Part 1: Discharging a Capacitor

1. Students should open the file ***RCtimeconstant.tns,*** and read the first two pages.

- Page 1.3 gives a schematic diagram of the RC (resistor-capacitor) circuit (Figure 1).

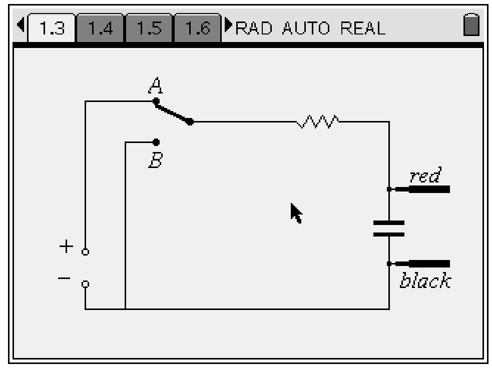


Figure 1

Q1. With the switch in position A is the capacitor being charged or discharged?

Answer: *charged*

Q2. Predict the effect of resistance on the rate of discharge of a capacitor?

Answer: Most students should be able to predict that increasing the resistance will cause the capacitor to discharge (and charge) more slowly.

Q3. Predict the graph of voltage vs. time for a discharging capacitor.

Answer: Many students will draw a straight line with a negative slope.

2. Next, students should follow the instructions on page 1.4 for setting up the experiment (Figures 2 and 3).

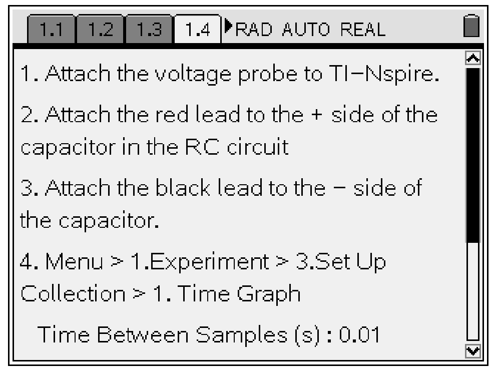


Figure 2

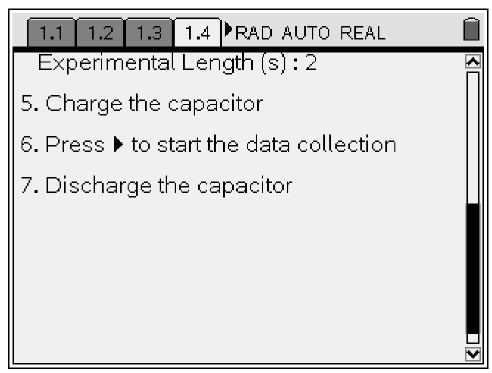


Figure 3

Q4. Describe the relationship of voltage and time.

Answer: After an initial period voltage decreases rapidly with time.

- As time continues the rate of decrease slows down as voltage goes to zero (Figure 4).

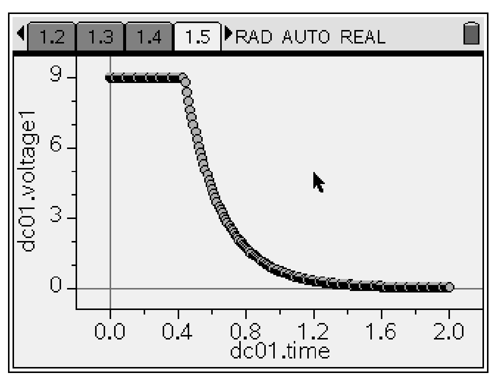


Figure 4

Q5. How well did your predicted graph of voltage vs. time match the experimental graph?

Answer: Student answers will vary.

3. To find the regression equation to fit the discharge data, the students must remove the initial region of constant voltage.

- Students should click on the last data point in the horizontal region and note the time (the value of x) (Figure 5).

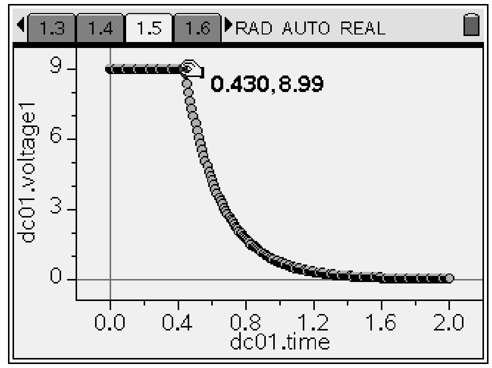


Figure 5

4. On page 1.6, students should enter **dc01.time** and **dc01.voltage** in the formula rows of column A (**time**) and column B (**voltage**).

- See Figure 6.

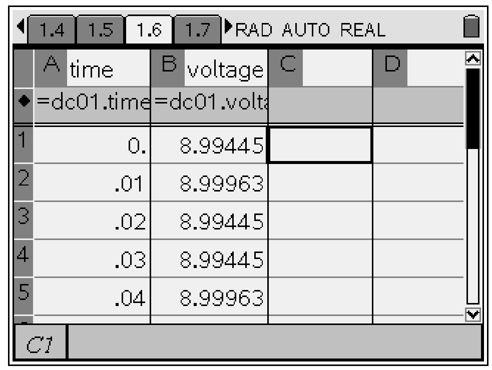


Figure 6

5. Arrow down column A to the row containing the initial discharge time found in step 3 (Figure 7).

- Holding down CAPS, press the right arrow on the NavPad Cursor Control pad, then hold down the down arrow on the NavPad until all the data is highlighted (Figure 8).
- The CAPS key can now be released.
  Press ctrl-c to copy the highlighted data.

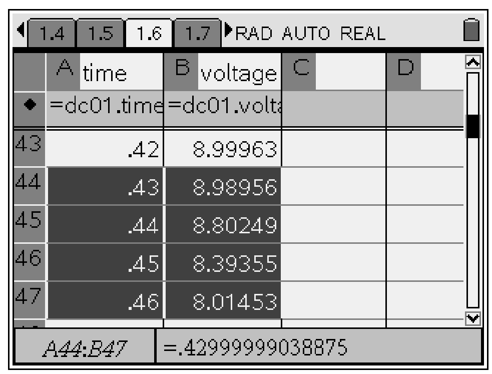


Figure 7

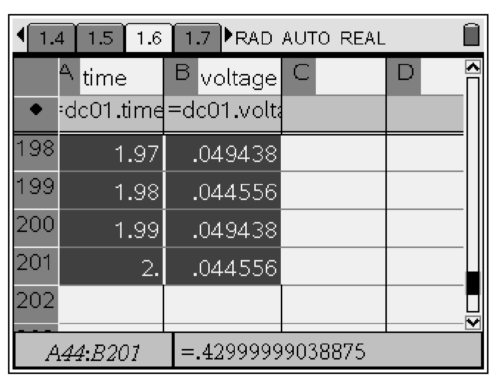


Figure 8

6. Move to the first data cell of column C.

- Press ctrl-v to paste the copied data.

  Arrow up, and name column C as **time1** and column D as **volt1** (Figure 9)**.**

7. Arrow over to column E (Figure 10).

- Name this column **time11**.
- In the formula row of column E enter **time1 – c[1]** to re-zero the time.
- The times in column E (**time11)** correspond to the discharge times.

8. Page 1.8 is a Graphs & Geometries application page.

- Students do a scatter plot **volt1** vs. **time11**.
- Next they enter the experimental values of the initial voltage, **v0**, the resistance, **r1**, and the capacitance, **c1** (Figure 11).

Q6. How well does the equation V(t) = V0*exp[-t/(R*C)] fit the graph of **volt1** vs. **time11**?

Answer: Student answers will vary.

Q7. R*C is known as the time constant. Show that multiplying the units ohms times farads gives a unit of time.

Answer: ohm x farad =

$$\left(\frac{\text{volt}}{\text{amp}}\right) x \left(\frac{\text{coulomb}}{\text{volt}}\right) = \left(\frac{\text{coulomb}}{\text{amp}}\right) = \left(\frac{\text{coulomb}}{\text{coulomb/sec}}\right) = \text{sec}$$

Q8. Calculate the time constant for your experiment.

Answer: *$22 \times 10^3\ \Omega \times 10 \times 10^{-6}\ F = 0.22\ s$*

Q9. When time is equal to the time constant, R*C, the voltage, V, is what fraction of V0?

Answer: *$V/V0 = e^{-1} = 0.368$.*

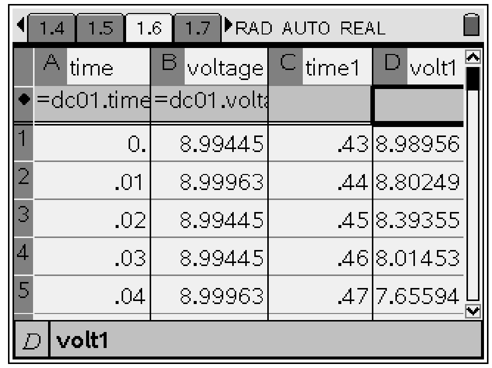


Figure 9

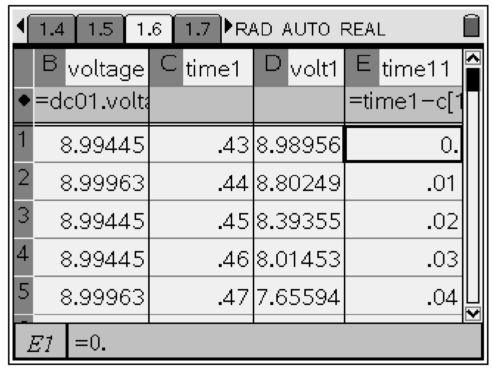


Figure 10

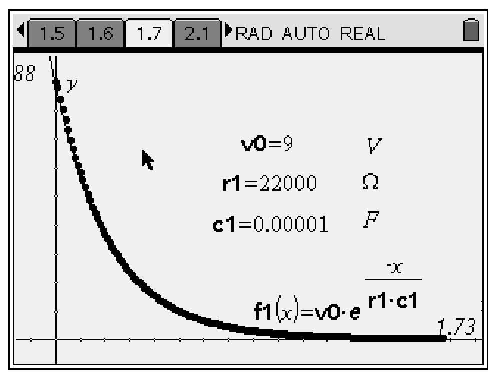


Figure 11

Part 2: Charging a Capacitor

1. Students repeat the experiment, this time collecting data for charging a capacitor (Figure 12).

Q10. Predict the graph of voltage vs. time for charging a capacitor.

Answer: Many students will be able to predict a rapid initial increase in voltage followed by a slow increase as the circuit reaches the maximum voltage.

Q11. Compare the experimental graph to your prediction.

Answer: Student answers will vary.

2. Next, students enter the data into columns A and B in a Lists & Spreadsheets application page.

- Using the same method as given above for the discharge experiment, copy and paste the part of the data corresponding to charging the capacitor in columns C and D.
- Re-zero the time data, as before, in column E (Figure 13).

Q12. For discharging a capacitor, V(t) = V0*exp[-t/(R*C)].

- Predict the equation for charging a capacitor.

Answer: V(t) = V0{1 – exp[-t/(R*C)]}

3. Use a Graphs & Geometries application page to plot **volt1** vs. **time11**.

- Edit the values of **v0**, **r1**, and **c1** to show that the function **f1** fits the experimental data (Figure 14).

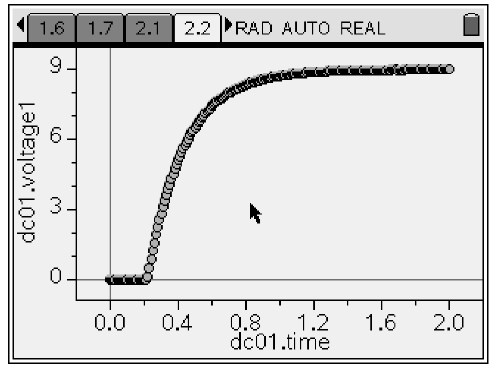
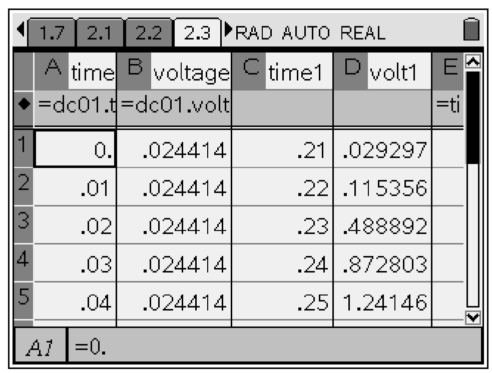


Figure 12



	A time	B voltage	C time1	D volt1	E
◆	=dc01.t	=dc01.volt			=ti
1	0.	.024414	.21	.029297	
2	.01	.024414	.22	.115356	
3	.02	.024414	.23	.488892	
4	.03	.024414	.24	.872803	
5	.04	.024414	.25	1.24146	

A1 =0.

Figure 13

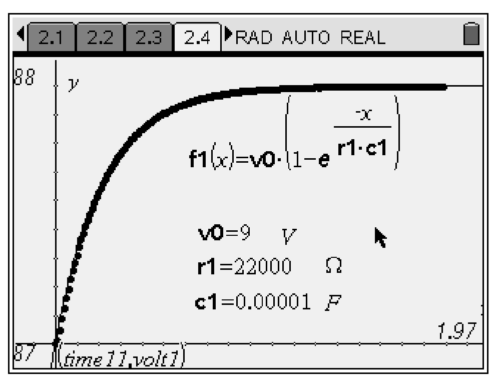


Figure 14

Q13. Does your experimental data confirm that the time constant for discharging and charging a capacitor are the same?

Answer: Yes

Part 3: Changing the Resistance (or Capacitance) of the Circuit

1. Repeat the experiment using a different resistor (or capacitor).

Q14. If you increase the resistance, should the time constant increase or decrease?

Answer: Increase, since the time constant = R*C.

Q15. If you increase the resistance, should the rate of discharge increase or decrease?

Answer: Decrease.

Student TI-Nspire™ File: *RCtimeconstant.tns*

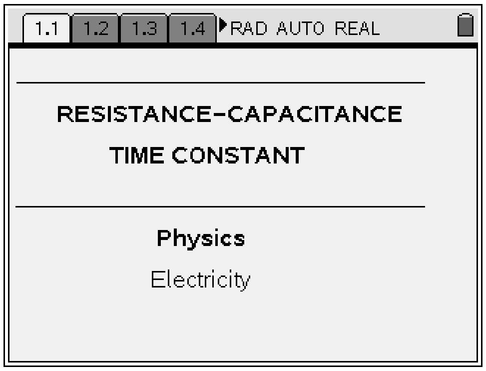


Figure 15

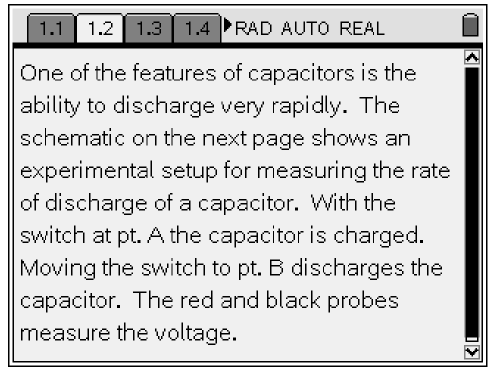


Figure 16

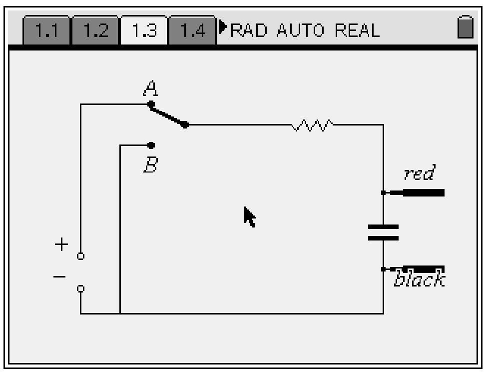


Figure 17

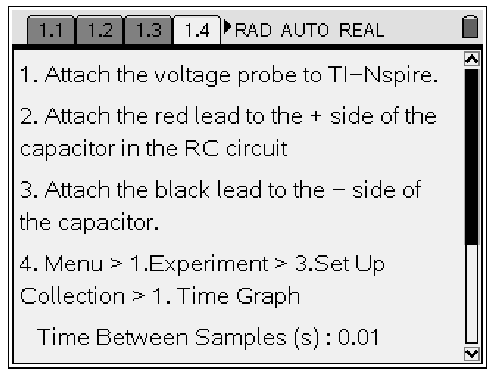


Figure 18

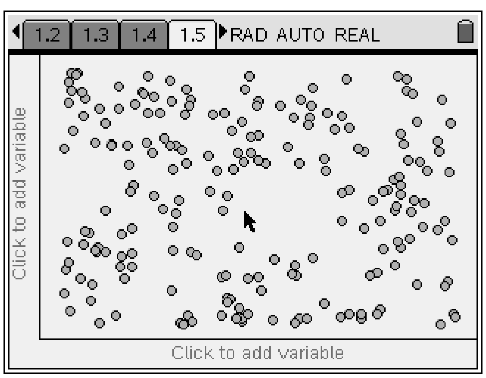


Figure 19

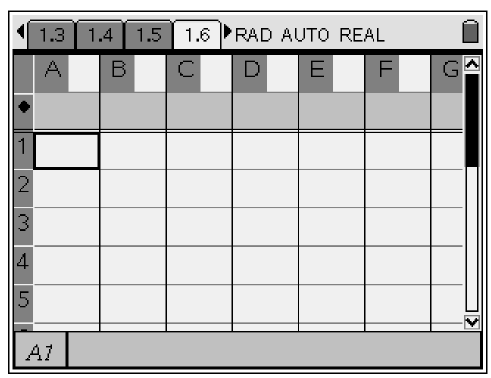


Figure 20

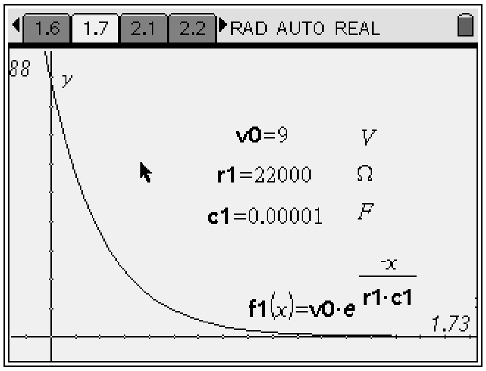


Figure 21

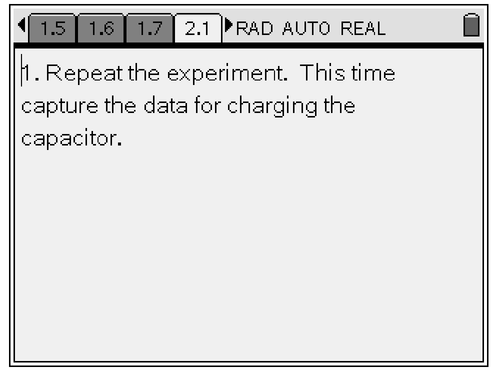


Figure 22

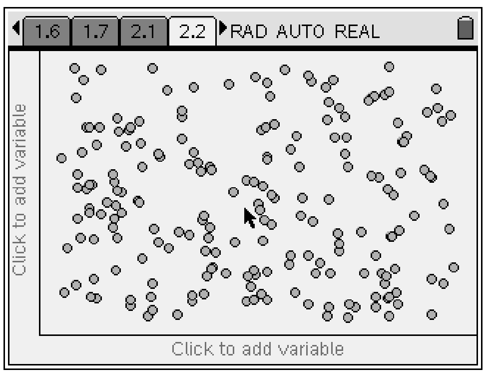


Figure 23

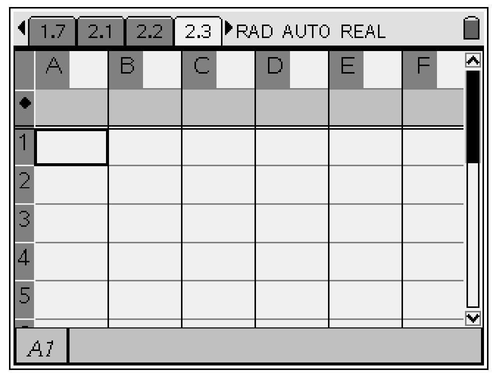


Figure 24

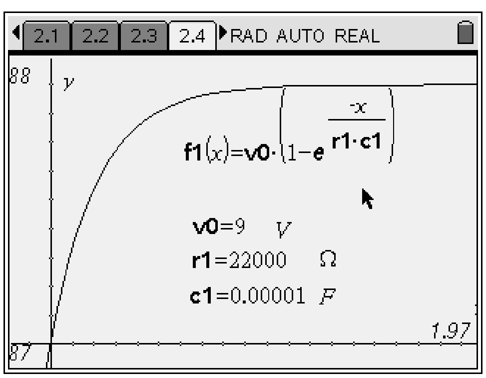


Figure 25

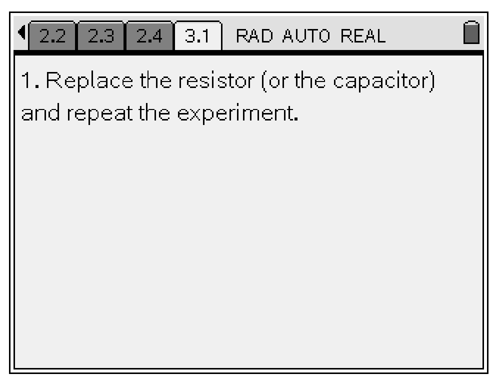


Figure 26

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The Tale of the Tape

Student Worksheet

Name ________________________

Class ________________________

In 1997, Mark McGwire hit a monstrous home run off pitcher Randy Johnson. The ball was hit from a height of about 3 feet, and it landed in the stands 440 feet from home place, 60 feet above the playing surface. At the time, it was claimed that the ball would have traveled 538 feet if the path were unobstructed.

Your problem is to investigate the path of the ball in three ways and to determine how far you think the ball might have traveled if not stopped by the stands.

Before you begin the investigation, you might like to try websites such as www.mlb.com which has several videos of home runs archived. The site www.HitTrackerOnline.com has archived video from long home run hits from the past few years.

Approach 1 – Using a Parabola

In the first approach to this problem, you will use a moveable parabola to try to physically fit a curve to the two points. At one point, you will be asked to determine how close your curve fits the two points.

1. Enter the function evaluations below.

f(0) = ________________ and f(440) = __________________

2. What is the meaning of the *x*-intercept in the context of this problem?

Approach 2 – The Regression Equation

3. How far did the ball travel in this solution?

Approach 3 – The CAS solution

4. In this problem, two different velocities are considered. What is the difference between them?

After completing this investigation and looking at the video from the websites:

5. Do you think that 500 feet home runs are possible?

6. If so, how many players are capable of hitting such a home run?

7. In the 1950s, Mickey Mantle was the big hitter for the New York Yankees. Try to find information about how far some of his home runs traveled.

Lesson Plan Notes for the Tale of the Tape

Course(s): Algebra 1 and Calculus.

Topic(s): Scatter plots, functions, graphing, equations, recursive sequences, derivatives, and local maximums.

Related NCTM Standards:

Algebra Standards:

1. Students should generalize patterns using explicitly defined and recursively defined functions.
2. Students should use symbolic algebra to represent and explain mathematical relationships.
3. Students should identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
4. Students should draw reasonable conclusions about a situation being modeled.
5. Students should approximate and interpret rates of change from graphical and numerical data.
6. Students should analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
7. Students should judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

8. Students should draw reasonable conclusions about a situation being modeled.

Suggested Total Time: One hour.

Specific Pre-requisite knowledge:

Students should be familiar with the following:

- Using spreadsheet formulae
- Recursive sequences
- Constructing a scatter plot
- Finding the function of best fit
- Derivatives of functions

Materials Required/Classroom Set-up/Preparation:

The following equipment will be needed:

- Each student should have access to a TI-Nspire™ math and science learning handheld with the file "TaleofTape.tns" pre-loaded.
- If the CAS solution is to be explored, then students should have access to a TI-Nspire™ CAS math and science learning handheld.

Lesson Notes:

The step-by-step solutions present three approaches for solving this problem. The first two could be done by an Algebra 1 student while the CAS solution is for Calculus students only.

1. If CAS is not an option, an alternative could be to find the point of intersection with the graphs and the x-axis.

2. As students are working through the solutions, you may want to keep the following issues in mind:

- It is likely that every student or group will have a different solution for the moveable parabola. This should not concern them. Stress that they are not trying to get any particular solution (which you might demonstrate) and that almost every solution has validity. The only solutions that should be excluded are ones where the maximum point occurs after the point (440, 60).
- Anticipate some problems as students move through the regression and scatterplots. There is likely to be some confusion as to when they should press enter to open a field and when to press ▼ on the NavPad cursor control.
- Encourage students to try different values for the coordinates of the third point in the second approach.
- The CAS model is the most straightforward, although younger students would have no appreciation for the mathematics involved in this solution.

3. Students will need to be kept on task. Each model might take some time, and they might become discouraged.

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The Tale of the Tape

Materials

- TI-Nspire™ Math and Science Learning Handheld
- *TaleofTape.tns*

Overview

In this activity, students will model the path of a baseball and solve problems related to the distance the ball travels.

Introduction

In 1997, Mark McGwire hit a home run off of Randy Johnson. The claim was made that the ball would have traveled 538 feet if it hadn't landed in the stands. The ball was hit from a height of about 3 feet, and it landed in the stands 440 feet from home plate, 60 feet above the playing surface.

We will create a model for this situation and determine how far this ball might have traveled. There are three different approaches to the problem:

1. Using the movable parabola feature.
2. Using a regression equation.
3. Using the CAS. In this approach, we add in another piece of information and make use of some physics formulae.

Approach 1: The Movable Parabola

1. Open the *TaleofTape.tns* document.
2. Read the first three pages of the document.
3. On the Lists & Spreadsheet application page (page 1.4), name column A `dis` and name column B `ht`.
4. Enter 0 into cell A1, and 440 into cell A2.
5. Enter 3 into cell B1 and 60 into cell B2 (Figure 1).
6. On the Graphs & Geometry application page (page 1.5), press menu 3 4 for Menu 3:Graph Type, 4:Scatter Plot.
7. Choose `dis` for the x-entry and `ht` for the y-entry.
8. You will need to adjust the window by pressing menu 4 1 for Menu 4:Window, 1:Window Settings.
9. Complete the dialog box using appropriate values, and press enter (Figure 2).
10. Press ctrl ▲ to display the page sorter.
11. Highlight page 1.5 (Figure 3).
12. Press ctrl C to copy the page.
13. Move to highlight page 1.6.
14. Press ctrl V to insert a copy of page 1.5 after page 1.6.
15. With page 1.7 highlighted, press enter to open page 1.7.
16. On page 1.7, press menu 3 1 for Menu 3:Graph Type, 1:Function.

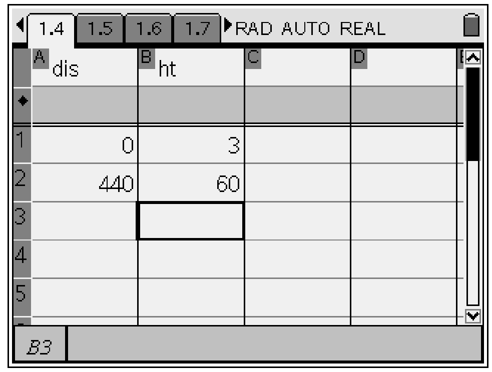


Figure 1

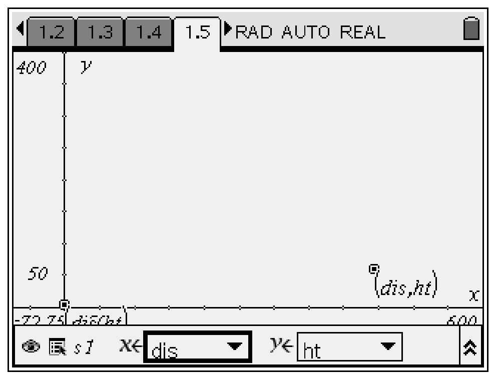


Figure 2

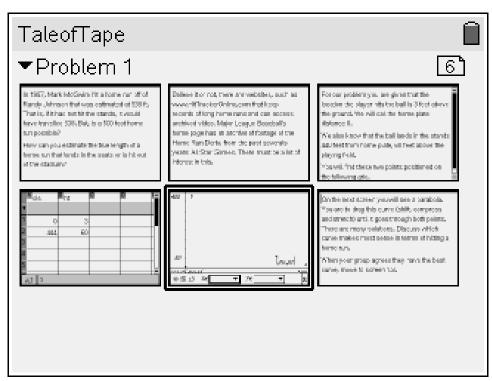


Figure 3

17. For the equation, enter a function that opens downward.
18. Discuss the rationale for this beginning equation with the students.
    - The function in Figure 4 started as `f1(x) = -Ø.ØØ1(x-25Ø)²+3ØØ`.
    - This is an arbitrary starting function.
19. Move your cursor to one of the ends of the parabola, and note the symbol and the words `graph f1` appear (Figure 5).
20. Press ctrl to grab the parabola.
    - You will be able to bend the graph in both directions.
21. When you are satisfied with the shape of the parabola, press enter or .
22. Move the cursor close to the vertex, and note that a different symbol appears along with the words `graph f1` (Figure 6).
23. Press ctrl to grab the parabola, and use the arrow keys to translate the parabola.
24. Move the parabola to a position where it appears to pass through the two points.
    - There are an infinite number of graphs that are possible solutions. One graph is shown in Figure 7.

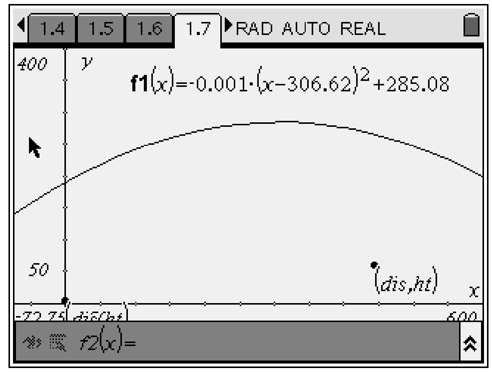


Figure 4

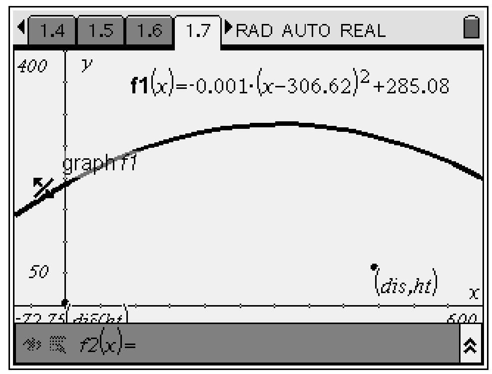


Figure 5

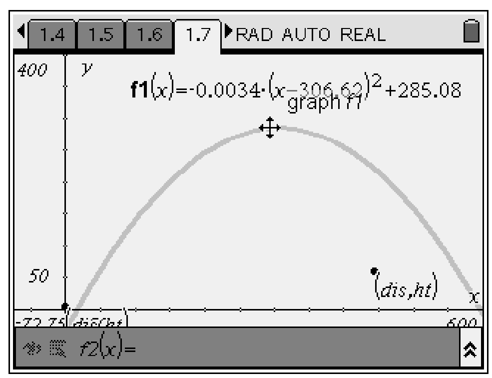


Figure 6

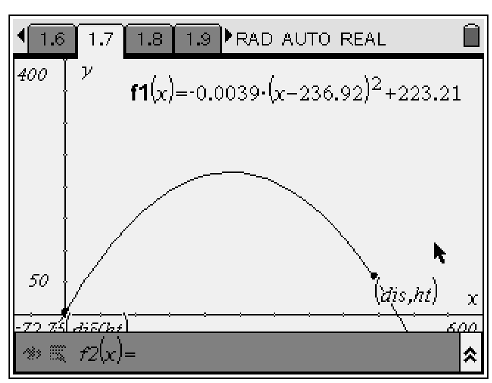


Figure 7

25. To test whether the function graphed on page 1.7 passes through the two points, use the Calculator application page (1.9), and find the actual values for f1(0) and f1(440) (Figure 8).

- Comments may be added to a Calculator page by pressing (menu) (1) (6) for Menu 1:Actions, 6:Insert Comment.
- If these function values are close to 3 and 60, respectively, you should continue.
- If not, go back to the Graphs & Geometry application page, and change the function again.

Note: If using a CAS device, add another Calculator application page, and solve for the x-intercepts of f1(x) (Figure 9).

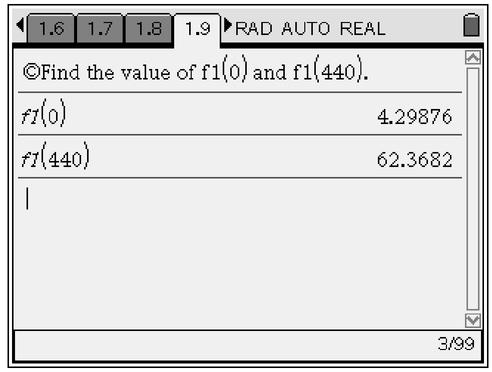


Figure 8

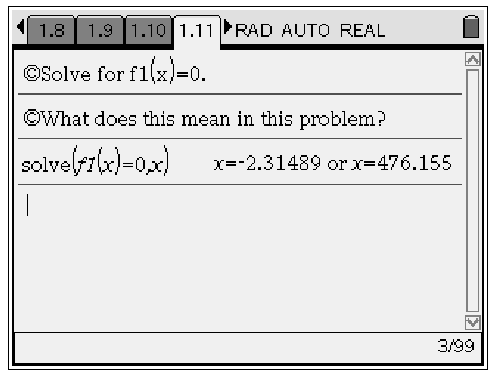


Figure 9

Approach 2: Using a Regression Equation

Note: Pages 2.1 and 2.2 in the .tns document are Notes pages. Page 2.3 is a copy of the Spreadsheet application on page 1.4 with the screen split vertically and a Graphs & Geometry application added.

The first two rows of the spreadsheet are the same as before, using the two points provided in the problem. The third point, (220, 100), is completely arbitrary (Figure 10).

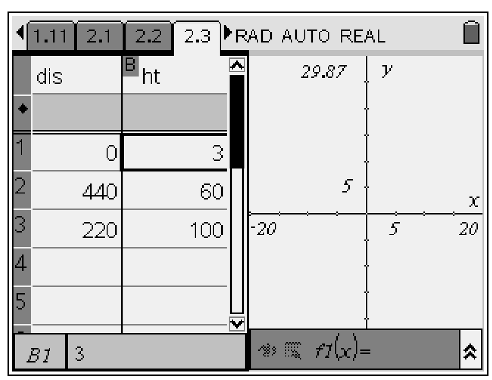


Figure 10

1. While in the Lists & Spreadsheet work area, press (menu) (4) (1) (6) for Menu 4:Statistics, 1:Stat Calculations, 6:Quadratic Regression.

2. Press tab to move between fields.
3. Complete the top part of the dialog box as shown in Figure 11.
4. As you move down the Quadratic Regression dialog, fill the 1st Result Column entry field with `d[]`.

Note: The [] comes from typing [and]—it is not a special symbol.

5. Press tab to highlight the OK button at the bottom of the dialog box, and press enter.
   - Press the left arrow key to see the results in columns D and E (Figure 12).
6. Press ctrl tab to move to the Graphs & Geometry work area.
7. Press menu 3 4 for Menu 3:Graph Type, 4:Scatter Plot.
8. At the bottom of the work area, for the x entry, choose `dis`, and for the y entry, choose `ht` (Figure 13).
9. Press menu 4 9 for Menu 4:Window, 9:Zoom-Data (Figure 14).
10. Press menu 3 1 for Menu 3:Graph Type, 1:Function.

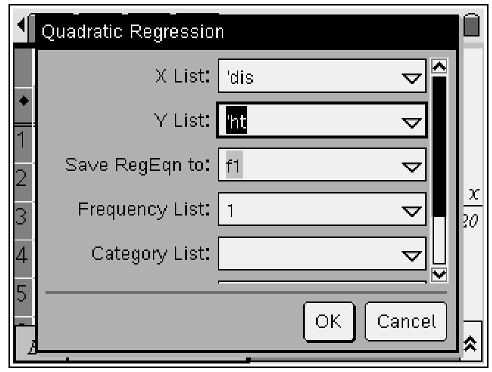


Figure 11

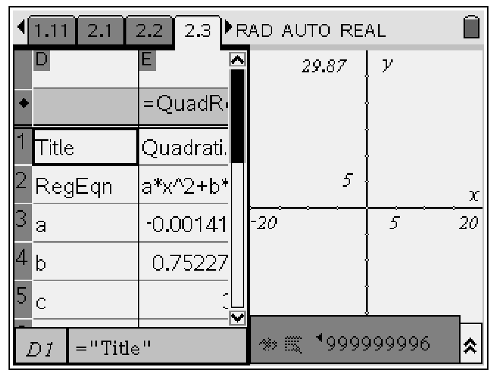


Figure 12

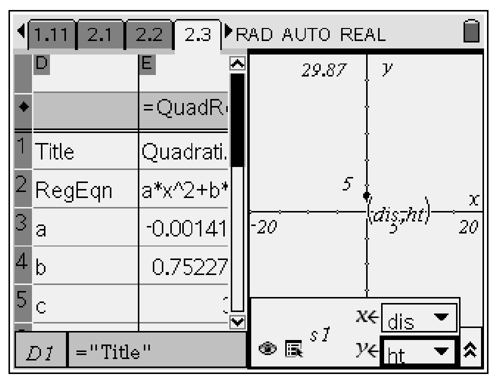


Figure 13

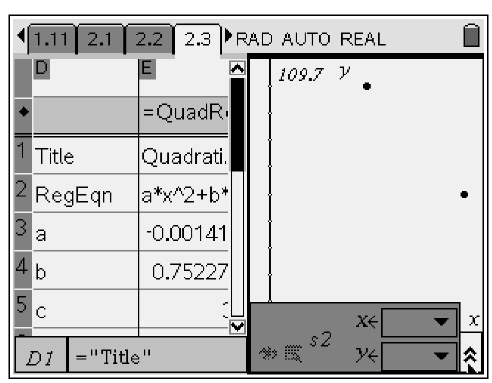


Figure 14

11. Use the NavPad cursor controls to move up to f1(x), the regression equation, in the entry line, and then press enter (Figure 15).

Note: If using a CAS device, insert a Calculator application page after the Notes page (page 2.4), and solve for the x-intercept(s) of f1(x) (Figure 16).

An interesting aside is shown in this approach when you go back to the Lists & Spreadsheet application page (page 2.3) and change the height for the third point.

As soon as you press enter, you will see the point change in the Graphs & Geometry application page (Figure 17). The regression equation will also be updated.

However, the last solution for the x-intercepts will need to be re-calculated.

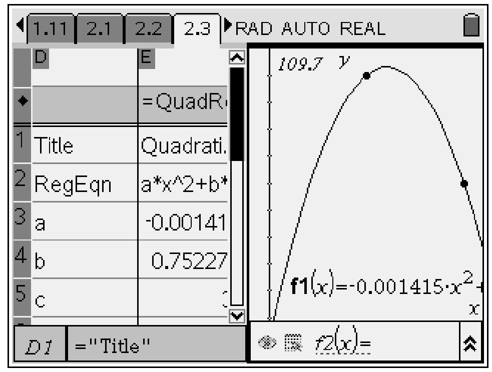


Figure 15

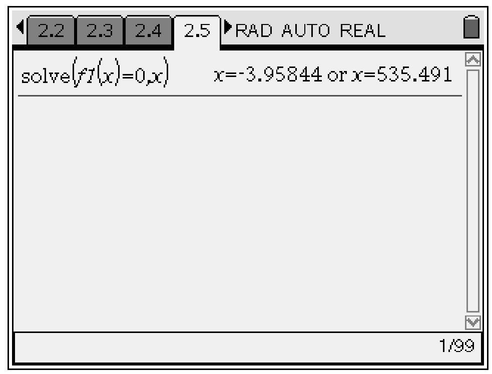


Figure 16

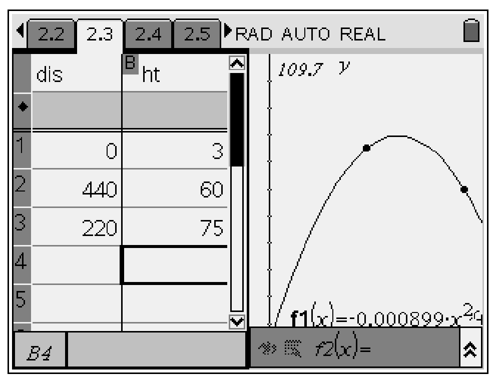


Figure 17

Approach 3: Using CAS

The following illustrates potential solutions as students insert pages and work through the investigation. The page numbering will change as students insert pages.

Note: Additional information - the ball took 5.1 seconds to travel from home plate to the stands.

1. On page 3.2, students are instructed to insert a Calculator application page, and enter the calculation for the forward velocity.
2. In necessary, press ctrl enter for an approximate answer.
3. Store this value to variable `vf`.
   - Press ctrl sto→ var for store (Figure 18).
4. On page 3.5, students will be instructed to insert another Calculator application page.
5. Press menu 1 1 for Menu 1:Actions, 1:Define.
   - Define the function h(t) as shown in Figure 19.
   - Use `v` as the coefficient of the second term, and be sure to enter a multiplication symbol in between `v` and `t`.
6. In the second line, press menu 3 1 for Menu 3:Algebra, 1:Solve.
7. On the third line, re-define the function by copying the command from the first line, pressing the | key, and copying the result from the second line.
8. Enter h(t) and press enter; the updated function will be displayed (Figure 19).

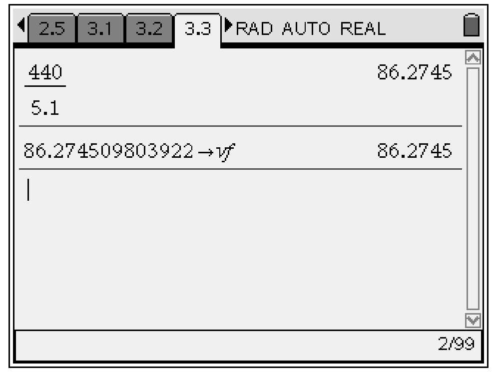


Figure 18

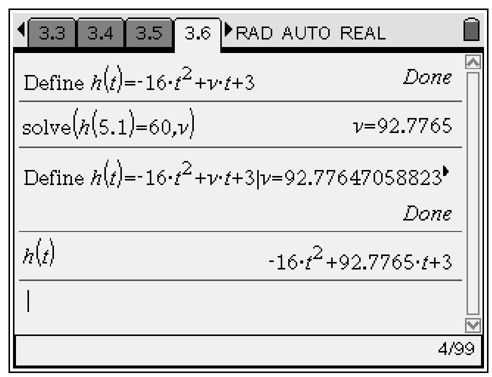


Figure 19

9. On page 3.7, students will be instructed to insert another Calculator application page.

10. On the first line, solve for the *x*-intercept of the function (Figure 20).

- Since the first answer is negative, that solution should be discarded.

11. On the next line, press V F .

12. To get the constant, move up to the previous line, and highlight the results.

13. Press enter to paste the results, and use the clear key to edit out all of the variables and the negative decimal answer.

14. Press enter (Figure 20).

15. On page 3.9, students will be instructed to insert a new Calculator application page.

16. Define dh(t) to be the derivative of h(t) with respect to t (Figure 21).

- The second line is not necessary, but it does show the derivative function.

17. In the third line, solve for the zero of the derivative function.

18. In the fourth line, note the use of the | key between the name of the function and the result (Figure 21).

Note: This rich problem could alternatively be solved using parametric equations and graphing them.

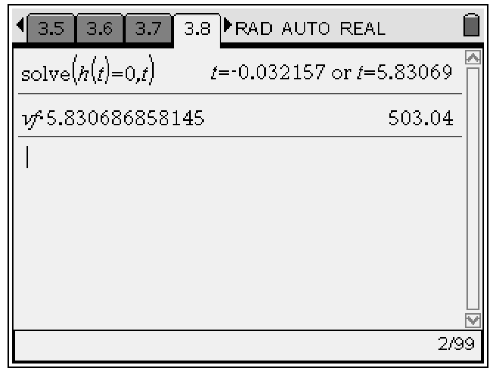


Figure 20

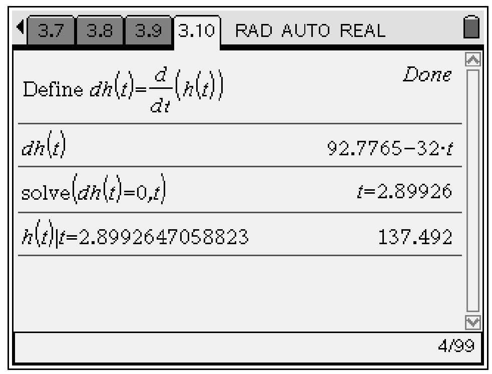


Figure 21

Force of Tension and Free Body Diagrams

Concepts

- Force of mass
- Force of gravity
- Tension

Materials

- TI-Nspire™ Math and Science Learning Technology
- Force probe
- Weights
- Cart, Pulley
- *Force.tns* file

Overview

In this activity, students will evaluate the force of gravity on a sliding mass; the force of the mass hanging from a pulley; and the normal force on a sliding mass. In addition, they will also evaluate the force of tension between the sliding mass and the hanging mass.

Problem 1

1. Open the *force.tns* file.

- Press [home] [7] for Home 7:My Documents.
- Select *force.tns*.
- On page 1.1, read the introduction.
- Move to page 1.2 for instructions.
- On page 1.3, change the values of mass1 and mass2 (Figure 1).
- Observe the Fg1, Fg2 and T.

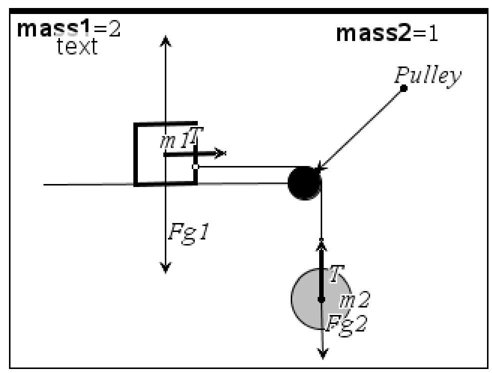


Figure 1

Questions

1. What happens to Fg1 when m1 is increased?

2. What happens to Fg2 when m2 is increased?

3. What happens to T when mass1 increases?

4. What happens to T when mass1 increases?

5. Assuming the surface is frictionless the force of tension will equal what force?

6. What happens to the force vectors as the block slides along the surface?

Problem 1 continued

1. Move to page 1.4, and read the notes.

2. On page 1.5, connect the force probe to the TI-Nspire™ math and science learning handheld.

3. Turn the switch to 10 N on the force probe.

4. Hang the mass from the force probe.

- Record the force in the data table as force of gravity 2.

5. Hang the cart from the force probe.

- Record it as force gravity 1.

6. Hang the Force probe from the table and record.

7. Connect a force probe to the frictionless cart.

8. Connect a string to the front of the force probe over a pulley and to a mass.

9. Pull the cart back so the weight hangs just below the pulley.

10. Click on the start button, and release the cart.

11. Determine the average sliding or rolling force.

12. Create a moveable line that travels through the forces while the cart is rolling.
    - Press menu 4 2 for Menu 4:Analyze, 2: Add moveable line.
    - Arrow near the line when the double arrow appears, press and hold until the hand closes.
    - Then use the arrow keys to move the line to fit the sliding data.
    - Record the y-intercept as the average force.
13. Repeat the process by hanging a larger mass from the pulley?

Questions

1. What happens to the force reading while the cart is rolling across the ramp?

2. What is the average force while the cart is rolling?

3. How does this average force relate to the force of the hanging mass?

4. The force probe is connected to the string over the pulley and to the hanging mass.
    - In a free body diagram what is the force that the probe is feeling described as?

5. By changing the hanging mass, did this have an effect on the force the probe felt while being pulled across the ramp?

6. The average Force of the cart sliding across the table is called the Net force.

- What is the difference in the net force and the force of the hanging mass?
- What makes these two forces different?

Data Table

	Small mass	Large mass
Force gravity cart		
Force probe gravity		
Force of Cart and probe (F_{g1})		
Mass of cart and probe		
Force gravity 2 (F_{g2})		
Mass of hanging mass		
Tension (F_T)		
Acceleration of Cart and probe		

1. Draw the free body diagram for the Free body diagram for the cart and mass on the pulley.

Problem 2: Force Equals Mass Times Acceleration

1. Move to page 2.2
2. Measure the force of the cart and motion detector.
   - Record them in the data table.
3. Disconnect the force probe, and connect the motion detector.
4. Change the time setting for the motion detector to 0.05 seconds for 1 second.
   - Press menu.
   - Press 1 to select Experiment.
   - Press 3 to select Set Up Collection.
   - Press 1 to select Time Graph
   - Leave the first setting at 0.05 seconds
   - Press tab to move to experiment length, and enter 1.
   - Press tab to highlight OK, and press to save settings.
5. Connect the motion detector to the cart.
6. Connect the string to the cart.
7. Place the first mass on the end of the string, and hang it over the pulley.
8. Pull the cart away from the pulley so the hanging mass is just below the pulley.
9. Click on the sample button, and then release the cart.

Questions

1. When the cart is moving across the ramp what is the shape of the distance time graph.

2. Change the plot to velocity and time.

- For the region of the graph where the cart is moving what shape does the velocity time graph have?

3. What are the units for the slope in the distance time graph?

4. What does the slope represent in the distance time graph?

5. Is the slope at each point along the distance time graph the same?

6. What are the units for the slope in the velocity time graph?

7. What does the slope represent in the velocity time graph?

8. Is the slope at each point along the velocity time graph the same?

Problem 2 continued

- Velocity is a change in distance over change in time.
- Acceleration is a change in velocity over the change in time.

1. Change the graph to velocity and time.

- Move the mouse to the y-axis where it says dc01.distance, and press [click].
- Arrow down to velocity, and press [click] to select velocity.

2. Create a line of fit for the section where the cart is moving.

- Press [menu] [4] [2] for Menu 4:Analyze, 2: Add moveable line Move the cursor onto the line.
- There are two possible cursors that will appear.
    - o If it is a 4 arrow cursor, this will move the line without changing the slope.
    - o If the cursor is two arrows in a circle, this will change the slope.
- Grab the line with either cursor by pressing and holding [click] while one of the cursors is on.

Note: you may need to use both types of cursor to get the line on the set of linear points. To release the line, press [esc].

3. Once the line is on the linear set of points where the cart was moving, record the slope in the data table as acceleration.

	Trial 1	Trial 2
Force or motion detector		
Force of cart		
Force of cart and detector		
Force of hanging mass		
Acceleration		
Mass of Cart and detector		
Net force		

Questions

1. Calculate the mass of the cart and detector assuming the acceleration of gravity is 9.81 m/s^2 using the equation F=m*a, where F is force, m is mass and a is acceleration.

2. Calculate the net force on the cart and motion detector using the measured acceleration and mass of cart and detector. Using $F_{net} = m*a$.

3. The Net force is the forward force of the cart along the table or tension force being applied by the string.

- Is this force equivalent to the force of the hanging mass?
- If it isn't what would explain the reason for it not being equivalent?
- What would cause the net force to be less than the hanging force?

4. Draw the free body diagram and label all forces.

Focusing on Light

Concepts

- Difference between semicircular mirrors and parabolic mirrors
- Reflection of light

Materials

- TI-Nspire™ Math and Science Learning Technology
- pen or pencil
- blank sheet of paper
- *Foucsinglight.tns* file

Overview

In this activity, students explore the reflection of light by parabolic and semicircular mirrors. They begin by exploring reflection using a series of flat mirrors that are attached to one another to create a flexible mirror that can simulate a curved mirror. They then explore reflection by a true parabolic mirror and by a semicircular mirror. They use their observations to differentiate between curved and parabolic reflectors.

Teacher Preparation

This activity assumes students already have a reasonable understanding of plane mirrors. This task requires no formal preparation but may be enhanced by including physical examples of parabolic mirrors. However, the focus of the activity should be on the presence or absence of a focal point, rather than the appearance of an image in the mirrors used.

1. A useful demonstration, to include at the conclusion of this activity, is to have students position a series of plane mirrors in a light box to create a 'focal point.'
   - Students should note that the 'focal point' is more of a region, because the plane mirrors can only approximate a parabola.
2. The screenshots on pages 3–7 demonstrate expected student results.
   - Refer to the screenshots on pages 9 and 10 for a preview of the student TI-Nspire™ document (.tns file.)

3. To download the .tns file, go to http://education.ti.com/exchange and enter "8738" in the search box.

Classroom Management

1. This activity is designed to be **teacher-led** with students following along on their handhelds.
   - You may use the following pages to present the material to the class and encourage discussion.
   - Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.
2. Students may answer the questions posed in the .tns file using the Notes application or on a separate sheet of paper.
3. In some cases, these instructions are specific to those students using TI-Nspire™ math and science learning handheld devices, but the activity can easily be done using TI-Nspire™ computer software.

The following questions will guide student exploration in this activity:

- How are parabolic mirrors different from semicircular mirrors?
- Do all curved mirrors have a focal point?

Students will carry out the activity using simulated mirrors. They will first explore reflection by a series of plane mirrors. Then, they will compare reflection from a parabolic mirror to reflection from a semi-circular mirror.

Problem 1: Reflection from a Parabolic Mirror

1. Students should open the file **Focusinglight.tns,** and read the first two pages.

- Page 1.3 shows a group of five plane mirrors reflecting a series of light rays (Figure 1).
- The dotted lines in the image are incident light rays; they cannot be adjusted.
- Students should adjust these mirrors so that the reflected light passes through the single point marked A.
- It may take students several tries to align the rays correctly.
- They should then answer question 1 on page 1.4.

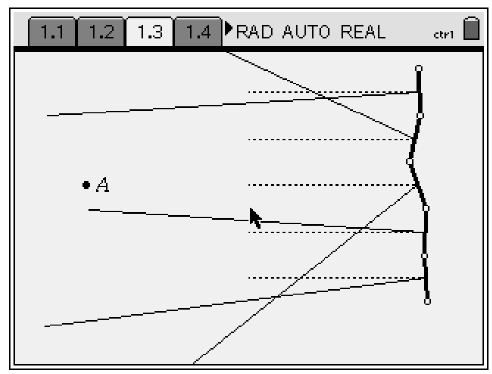


Figure 1

Q1. Describe the general shape formed by the plane mirrors when the reflected light passes through point A.

Answer: The mirrors form a parabola with a gentle curvature, as shown in Figure 2.

- “Curved” or “concave” are also acceptable answers.
- Encourage students to examine the way the light rays reflect off each individual mirror.
- They should notice that the reflections off of each plane mirror follow the same rules of reflection they are familiar with.

2. Page 1.5 shows a flexible parabolic mirror (Figure 3).

- The mirror will retain its parabolic shape even if students make it wider or narrower.
- The incident light ray is shown as a dotted line.
- The reflected ray is a thin solid line.
- A tangent to the parabolic mirror has been drawn in and is representative of a flat mirror at point P on the curve.
- Students should drag point P around the mirror, and observe how the reflected light ray changes.
- They should then answer question 2 on page 1.6.

Q2. Describe what happens to the reflected ray as you move point P along the parabolic mirror.

Answer: The reflected ray seems to rotate around a single point (Figure 4).

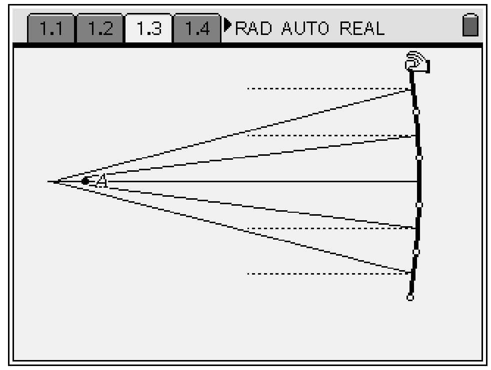


Figure 2

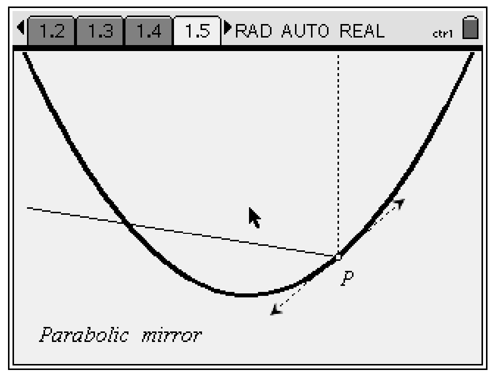


Figure 3

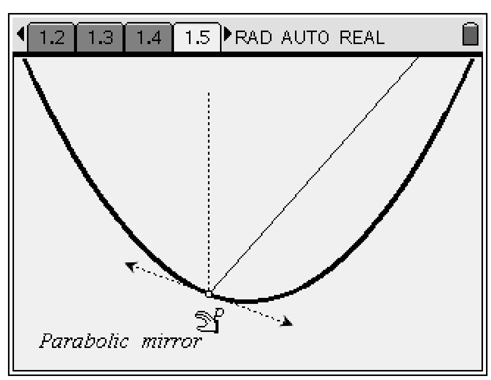


Figure 4

3. Page 1.7 contains an image of a second parabolic mirror with a number of parallel incident light rays striking the mirror in different locations (Figure 5).
   - The multiple incident rays help identify important characteristics associated with reflected rays.
   - Have students drag points **P**, **Q**, and **R** along the mirror and observe how the reflected rays move.
   - They should then answer questions 3-5 on pages 1.8 and 1.9.

Q3. Describe what happens to the reflected rays as you move points P, Q, and R along the parabolic mirror.

Answer: The reflected rays rotate around a single point, which is the point at which all three rays intersect.

Q4. Adjust the width of the parabola.

- How does this affect the reflected rays as you move the three points?

Answer: Regardless of the width of the parabola, the reflected rays always converge on a single point.

Q5. Based on your observations, make a general statement about how parabolic mirrors reflect incident light rays that are parallel to the parabola's line of symmetry.

Answer: A parabolic mirror reflects incident light rays to a single point.

- Explain to students that this point is called the "focal point" of the mirror.

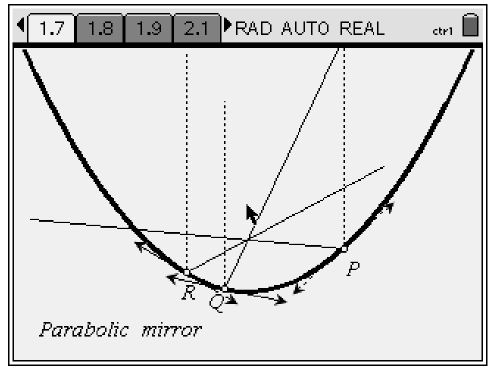


Figure 5

- Encourage students to manipulate the simulation to explore the relationship between the curvature or width of the mirror and the location of the focal point.

Problems 2: Reflection from a Semicircular Mirror

1. Page 2.1 shows a semicircular mirror (Figure 6).

- The incident light ray appears dotted.
- The incident light ray is shown as a dotted line.
- The reflected ray is a thin solid line.
- A tangent to the parabolic mirror has been drawn in and is representative of a flat mirror at point **P** on the curve.
- Have students drag point **P** around the mirror, and watch how the reflected light ray changes.

2. Page 2.2 contains an image of a second semicircular mirror with three incident light rays and the corresponding reflected rays (Figure 7).

- Have students drag points **P**, **Q**, and **R** along the mirror and observe the results.
- They should then answer questions 6 and 7 on pages 2.3 and 2.4.

Q6. Based on your observations, make a general statement about how semicircular mirrors reflect incident light rays that are parallel to the mirror's line of symmetry.

Answer: The reflected rays do not intersect at a single point.

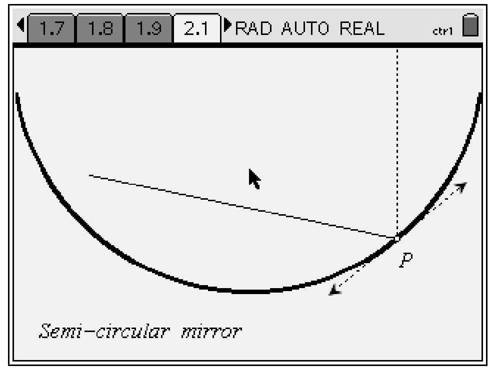


Figure 6

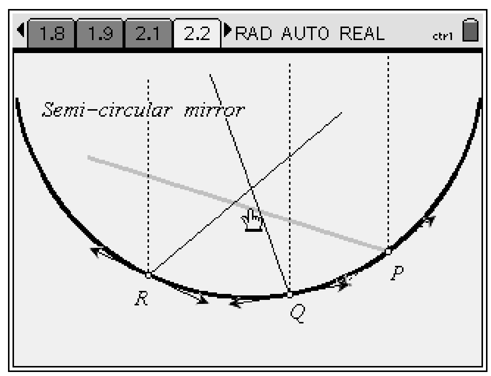


Figure 7

Q7. In a reflecting telescope, a curved mirror reflects incident light rays toward a single point in the eyepiece.

- Which would be better to use in such a telescope, a parabolic mirror or a semicircular mirror?
- Explain your answer.

Answer: A parabolic mirror would be best. Semicircular mirrors do not reflect light toward a single point, but parabolic mirrors do.

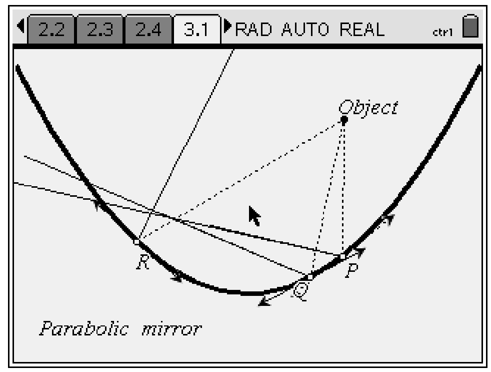


Figure 8

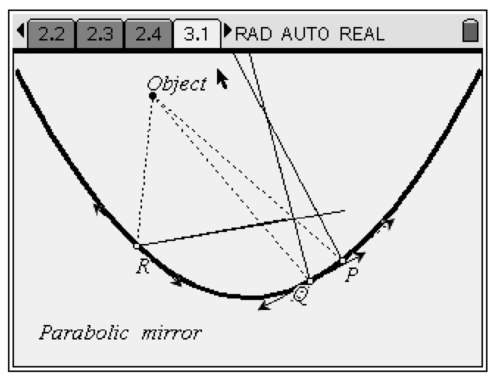


Figure 9

Problem 3: Reflection of Non-Partial Incident Rays

1. Page 3.1 shows a parabolic mirror reflecting incident light rays originating from a single **object** (Figure 8).
   - The incident rays are not parallel to one another.
   - Have students drag points **P**, **Q**, and **R** around the parabolic mirror, and observe the results.
2. Next, have students move the **object** around the screen, and observe the results (Figure 9).
   - Then, have students answer question 8 on page 3.2.

Q8. Describe how the parabolic mirror reflects light coming from a single object that produces incident rays that are not parallel.

Answer: The reflected rays do not converge on a central point.

- Encourage students to discuss the significance of this.
- They should realize that it is impossible to use a parabolic mirror to produce a real image of an object placed within or close to the curvature of the mirror.
- As an extension activity, you can challenge students to move points **P**, **Q**, and **R** so that the three reflected rays meet at a single point.
- You can also ask students to see if they can place the object in a position so that all the reflected rays emerge parallel.

Student TI-Nspire™ File: *Foucsinglight.tns*

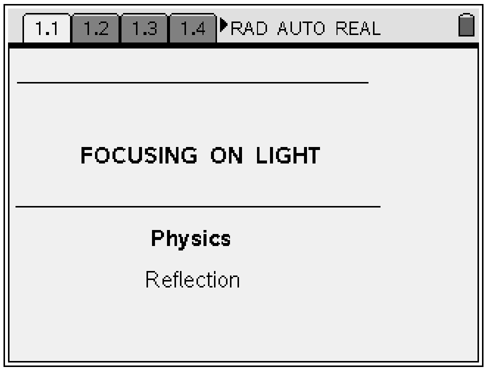


Figure 10

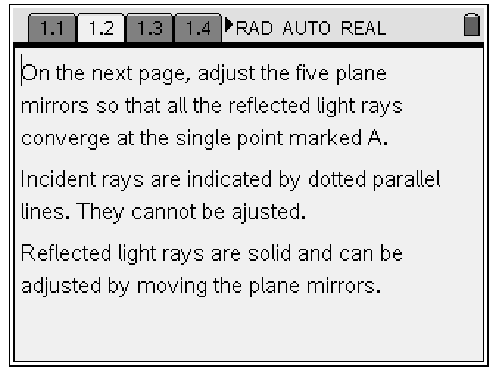


Figure 11

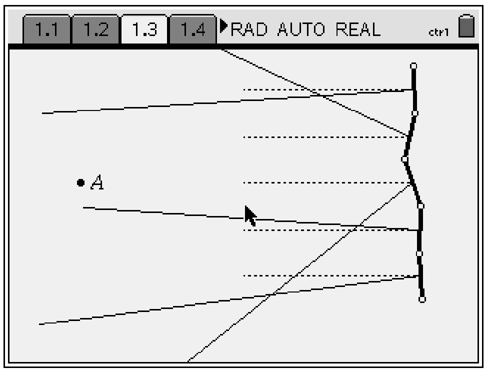


Figure 12

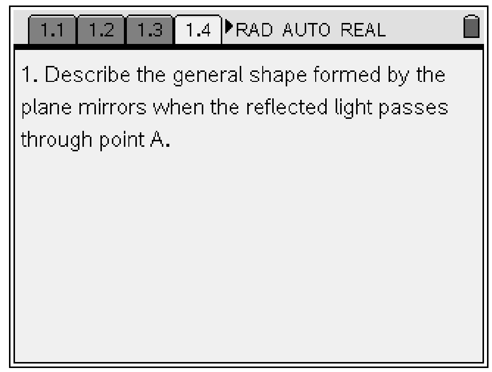


Figure 13

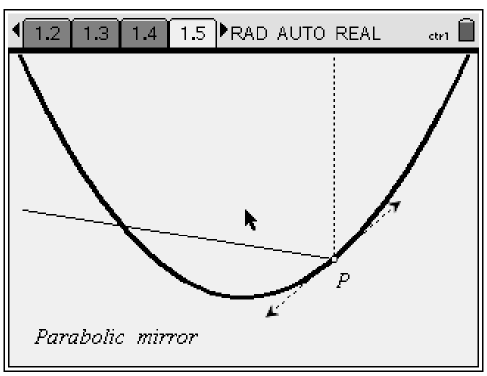


Figure 14

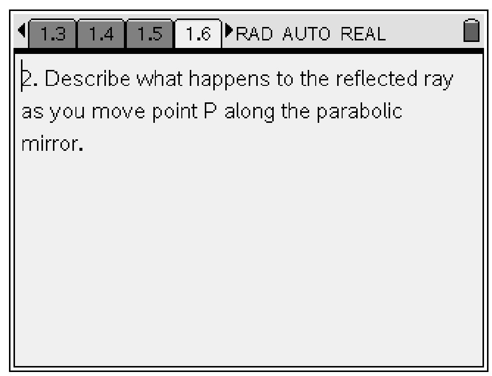


Figure 15

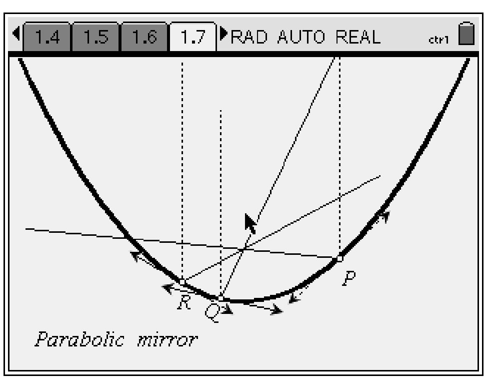


Figure 16

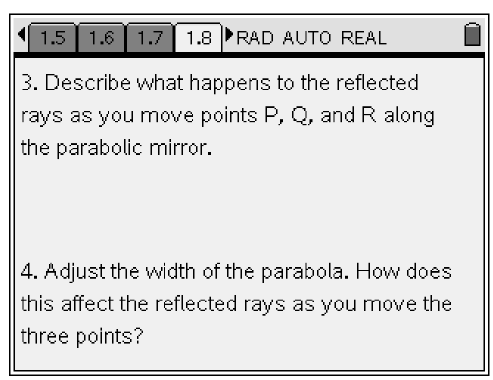


Figure 17

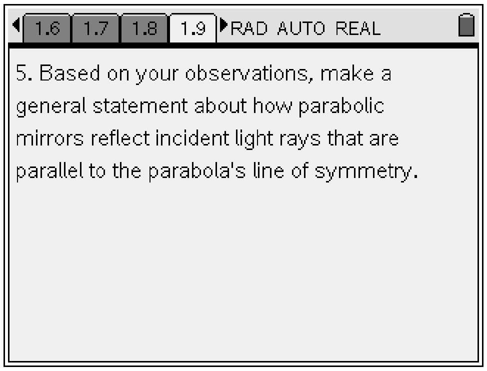


Figure 18

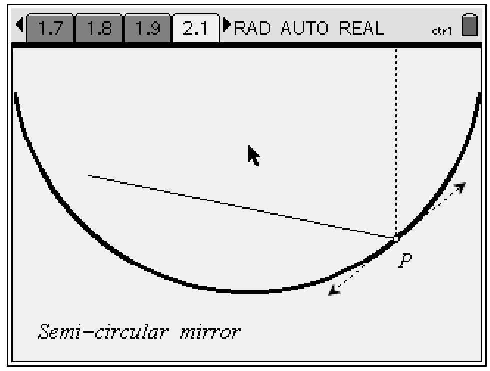


Figure 19

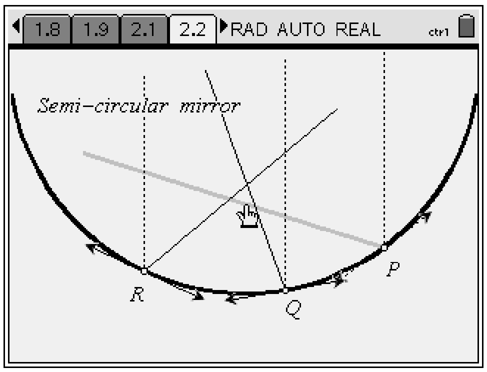


Figure 20

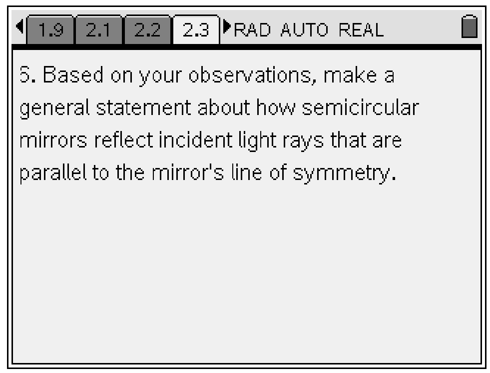


Figure 21

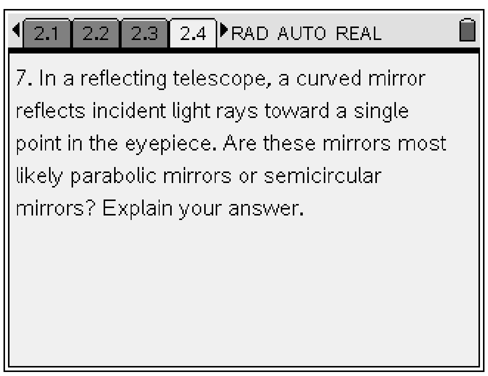


Figure 22

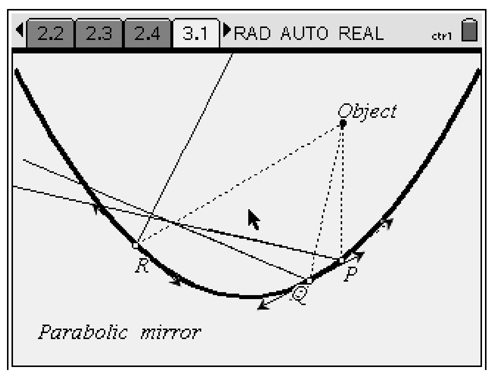


Figure 23

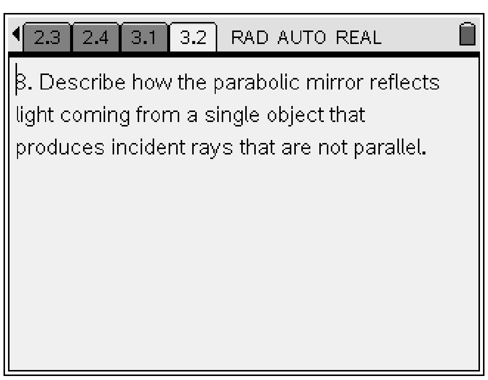


Figure 24

Nuclear Binding Energy

Concepts

- Mass loss and binding energy
- Ratio of neutrons and protons in stable nuclei
- Fission and fusion of nuclei

Materials

- TI-Nspire™ Math and Science Learning Technology
- Pen or pencil
- Blank sheet of paper
- *Nuclearbinding.tns* file

Overview

In this activity, students will explore the energy that results from the strong force in nuclei. The activity begins the unexpected result that the mass of nucleus is less than the masses of the protons and neutrons that compose the nucleus. Students use the equation $E = mc^2$ to calculate the energy equivalent of mass. In part two of the activity, students analyze a graph to determine the number of neutrons and protons in stable nuclei. The activity ends with an investigation of how fission and fusion of nuclei can be accounted for in terms of the binding energy per nucleon.

Teacher Preparation

Before carrying out this activity, review the relationships between the numbers of protons and neutrons and atomic number and mass number. Go over nuclear symbols: for example, He-4 and ${}_{2}^{4}He$. Distinguish between mass number, atomic mass, and nucleus mass (all the masses in this activity are for the nuclei, not the atoms). Review the atomic mass unit and Einstein's equation $E=mc^2$.

The screenshots on pages 3–9 demonstrate expected student results.

- Refer to the screenshots on pages 11-14 for a preview of the student TI-Nspire™ document (.tns file.)

Classroom Management

1. This activity is designed to be **teacher-led** with students following along on their handhelds.
    - You may use the following pages to present the material to the class, and encourage discussion.
    - Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.

Students may answer the questions posed in the .tns file using the Notes application or on a separate sheet of paper.

In some cases, these instructions are specific to those students using TI-Nspire™ math and science learning handheld devices, but the activity can easily be done using TI-Nspire™ computer software.

The following questions will guide student exploration in this activity:

- Is mass conserved in nuclear reactions?
- What is the ratio of neutrons to protons in stable nuclei?
- How is it that both of these processes, fusion (combining nuclei) and fission (splitting nuclei), release energy?

Students will calculate the mass loss in the formation of nuclei, and relate this mass loss to nuclear binding energy.

- Students will construct graphs of neutrons vs. protons for stable nuclei and binding energy per nucleon vs. mass number.
- Students will then calculate the mass loss and energy released in nuclear fission and fusion reactions.

Part 1: Mass Loss

1. Students should open the file *nuclearbinding.tns,* and read the first two pages.

- Page 1.3 illustrates that the He-4 nucleus has less mass than the neutrons and protons that compose the nucleus (Figure 1).

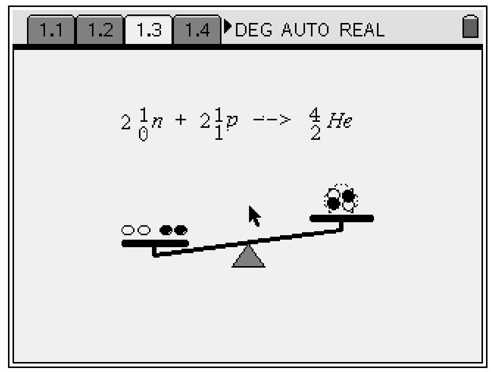


Figure 1

Q1. What is surprising about the mass of He-4 compared to the masses of the neutrons and protons that compose the nucleus?

Answer: The mass of the nucleus is less than the masses of the neutrons and protons.

Q2. Is mass conserved in this nuclear reaction?

Answer: No. Mass is lost.

2. Next, students should read the text on page 1.5; and then examine the data on page 1.6 (Figure 2).

- This gives the mass in amu of a neutron, a proton, and a He-4 nucleus.
- After students have examined the data on page 1.6, they should answer question 3.

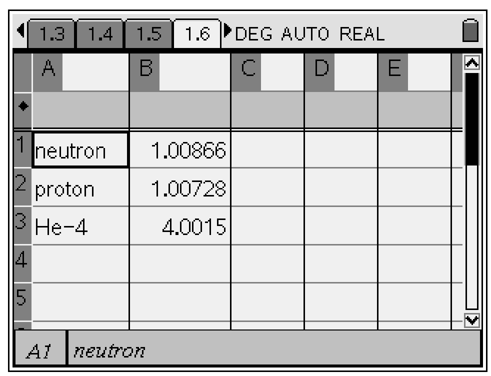


Figure 2

Q3. How much mass is lost in the formation of a He-4 nucleus from 2 neutrons and 2 protons?

Answer: To answer this question, subtract the actual mass of the He-4 nucleus from the total mass of two protons and two neutrons, as shown below: mass loss = 2(1.00866) + 2(1.00728) – 4.0015 = 0.03038 amu.

3. Next, students should read the text on page 1.8; and then move on to Calculator application page 1.9 (Figure 3).

- The calculator application page should already be displaying the values of the various constants.
- Students should use the equation $E = mc^2$ to calculate the energy equivalent of 1 amu in MeV.
- Then, they should answer questions 4 and 5.
- Finally, they should read the text on page 1.11.

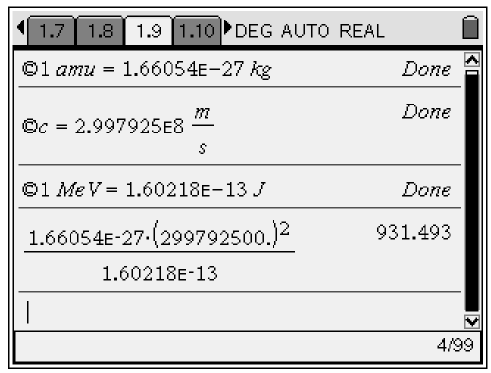


Figure 3

Q4. What is the energy equivalent in MeV of 1 amu?

Answer: 931.49 MeV

Q5. The amount of energy released in the formation of a He-4 nucleus is 28.3 MeV.

- Does this value agree with the mass loss you calculated in question 3?

Answer: All the "lost" mass is converted into energy, 0.03038 amu (931.49 MeV/1 amu) = 28.3 MeV; yes, the values agree.

Part 2: Neutrons and Protons in Stable Nuclei

1. Students are shown the symbols used to indicate the atomic number and mass number of nuclei (Figure 4).

- The mass number is equal to the number of neutrons and protons in the nucleus.
- The atomic number is equal to the number of protons.

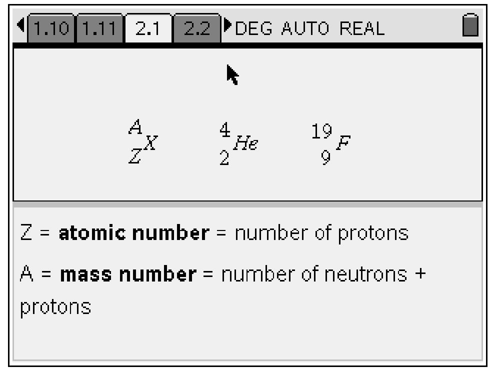


Figure 4

Q6. What is the mass number of F-19?

Answer: 19

Q7. How many neutrons are in F-19?

Answer: 10

Q8. Write an equation for the number of neutrons in terms of A and Z.

Answer: number of neutrons = A – Z

2. Next, students should read the text on page 2.3; and then move to page 2.4 (Figure 5).

- Lists & Spreadsheet application page 2.4 contains data on the number of protons and neutrons in 81 assorted stable nuclei.
- Students should enter the equation they derived in question 8 into column D of the spreadsheet.

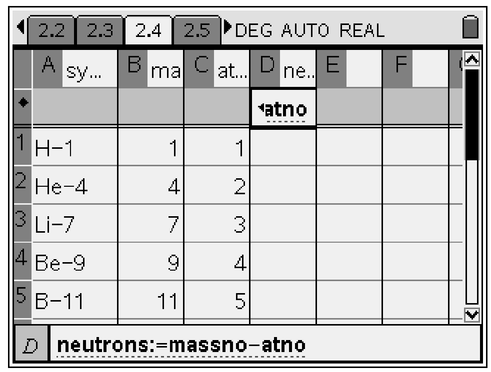


Figure 5

3. Next, students should read the text on page 2.5; and then make a plot of **neutrons** vs. **atno** on Data & Statistics application page 2.6 (Figure 6).

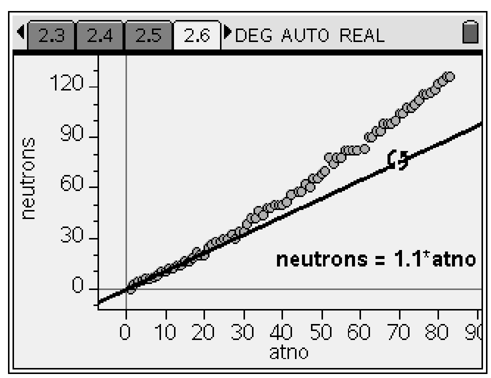


Figure 6

- Then, students should use the **Movable Line** tool by pressing menu 3 2 Menu 3:Actions, 2:Add Moveable Line (Figure 7).
- This will determine the ratio of neutrons to protons for stable nuclei in two regions: (1) light nuclei ($Z < 20$) and (2) heavy nuclei ($Z > 60$).
- Students should then answer questions 9 and 10 using this information.

Q9. For the stable nuclei what can you say about the ratio of neutrons to protons a Z increases?

Answer: As Z increases, the ratio of neutrons to protons increases.

Q10. Explain this trend by taking into account the strong nuclear force and the electric force.

Answer: As Z increases the number of protons increase.

- The increase in number of protons produces greater electrical forces of repulsion.
- The additional neutrons increase the strong nuclear forces which counteract the increased electrical repulsion of the protons.
- As the number of protons increases, a higher ratio of neutrons to protons is needed to offset the electrostatic repulsion between the protons.
- This is why the slope of the line increases with atomic number.

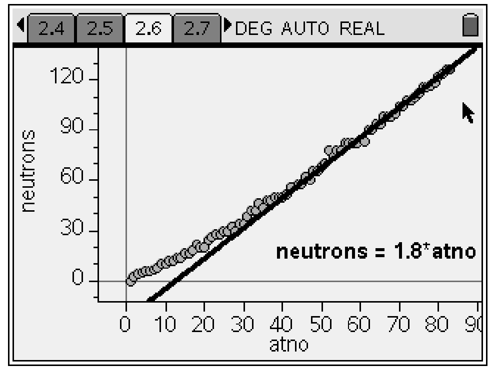


Figure 7

Part 3: Fusion and Fission

1. Next, students should read the text on page 3.1 before moving to page 3.2.
   - Lists & Spreadsheet application page 3.2 is similar to the one on page 2.4, but it includes adioactive nuclei in addition to stable nuclei (Figure 8).
   - Students should enter equations into the spreadsheet to calculate the mass loss and binding energy for all the listed nuclei.
   - In column F, students should enter the equation **massloss** = **atno** · 1.00728 + **neutrons** · 1.00866 – **mass** (Figure 9).
   - In column G, students should enter **ebind1** = **massloss** · 931.49.
2. Next, students should make a scatter plot of **ebind1** vs. **massno** on Data & Statistics application page 3.3 (Figure 10).
   - After examining the graph, students should answer questions 11 and 12.

Q11. Make a general statement about the relationship between binding energy and mass number.

Answer: The binding energy increases as mass number increases.

Q12. What is the relationship between number of nucleons and mass number?

Answer: The number of nucleons equals the mass number.

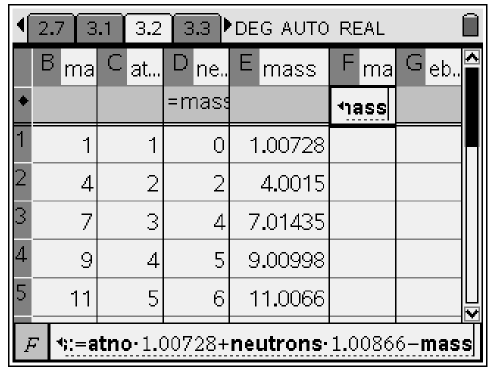


Figure 8

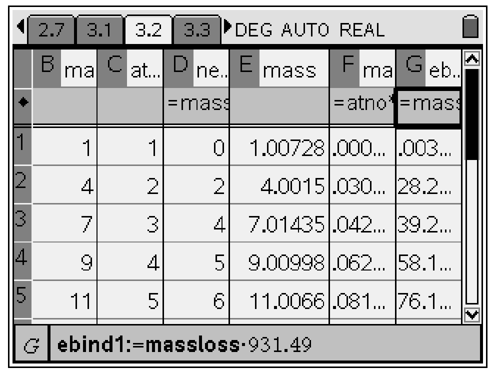


Figure 9

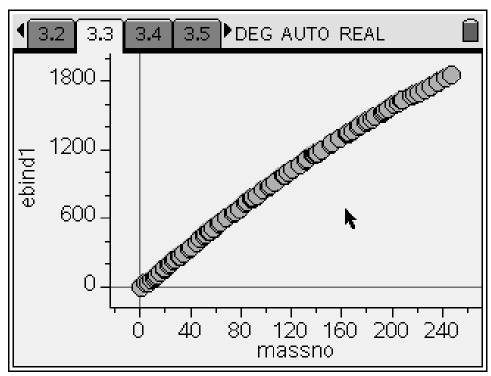


Figure 10

3. Next, students should go back to Lists & Spreadsheet application page 3.2.

- They should enter the equation for the binding energy per nucleon, **ebind2** = **ebind1**/**massno**, into the spreadsheet in column H.
- Students should then use the Data & Statistics application on page 3.5 to make a scatter plot of **ebind2** vs. **massno** (Figure 11).
- Students should use this scatter plot to answer questions 13–16.

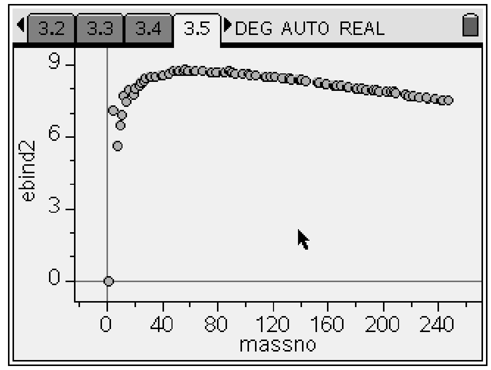


Figure 11

Q13. Which nucleus has the greatest binding energy per nucleon?

Answer: Fe-56

Q14. Which light nucleus has an unusually high binding energy per nucleon?

Answer: He-4

Q15. Make a general statement about the binding energy per nucleon and mass number for light nuclei.

Answer: The binding energy per nucleon increases with mass number for light nuclei.

Q16. Make a general statement about the binding energy per nucleon and mass number for heavy nuclei.

Answer: The binding energy per nucleon decreases with mass number for heavy nuclei.

4. Next, students should move to page 3.8; and read the text there.

- They should then examine the information on page 3.9, which shows a nuclear fission reaction for the fission of U-235 (Figure 12).
- Students should use the information on page 3.9 to answer questions 17–19.

Q17. Is mass gained or lost in this U-235 fission reaction?

- Hint: the mass of a neutron is 1.00866 amu.

Answer: The mass is lost.

- The mass of one neutron plus one U-235 nucleus is greater than the mass of three neutrons plus one Ba-140 nucleus plus one Kr-93 nucleus.

Q18. What is the change in mass and energy in MeV for the U-235 fission reaction?

Answer: Mass loss = 235.8174 = 0.1847 amu 236.0021, and Energy = 0.1847*931.49 = 172.0 MeV

Q19. Is energy required or released in the fission reaction?

Answer: Because mass is lost, energy must be released.

5. Next, students should move to page 3.11 which shows a nuclear fusion reaction (Figure 13).

- They should use the information on page 3.11 to answer questions 20–23.

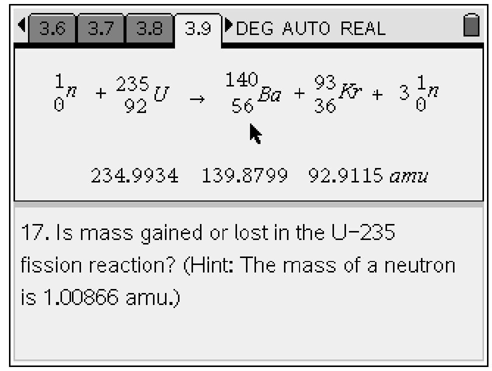


Figure 12

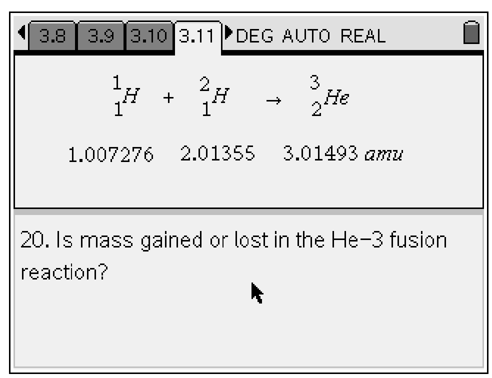


Figure 13

Q20. Is mass gained or lost in the He-3 fusion reaction?

Answer: Mass is lost.

- The mass of one He-3 nucleus is less than the total mass of one H-1 nucleus and one H-2 nucleus.

Q21. What is the change in mass and energy in MeV for the He-3 fusion reaction?

Answer: Mass loss = 3.01493 = 0.0059 amũ3.02083 and Energy= 0.0059*931.49 = 5.5 MeV

Q22. Is energy required or released in the fission reaction?

Answer: Because mass is lost, energy must be released.

Q23. Would energy be released in the fusion of heavy nuclei? Explain.

Answer: No, energy would not be released.

- Fusing heavy nuclei would produce a combined nucleus with a lower binding energy per nucleon.
- Therefore, the process would require energy.
- The mass would be greater than the sum of the masses of the original nuclei.

Student TI-Nspire™ File: *nuclearbinding.tns*

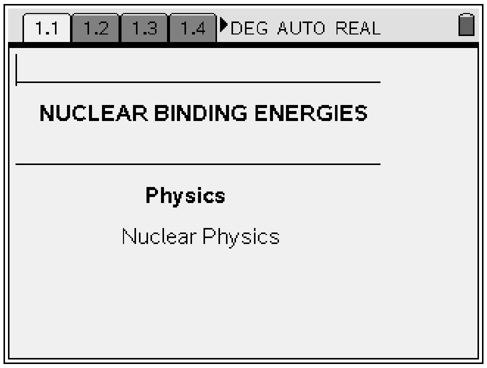


Figure 14

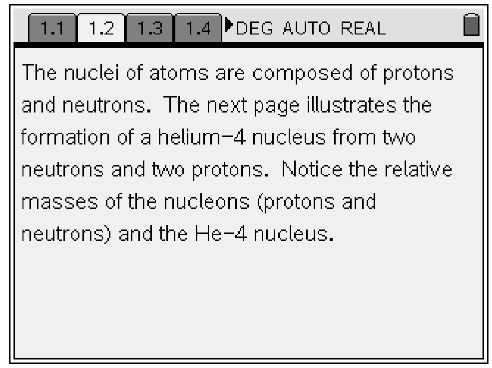


Figure 15

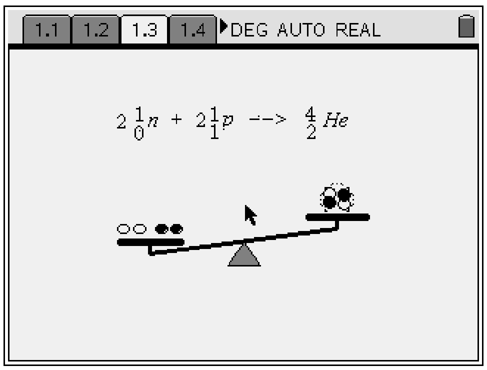


Figure 16

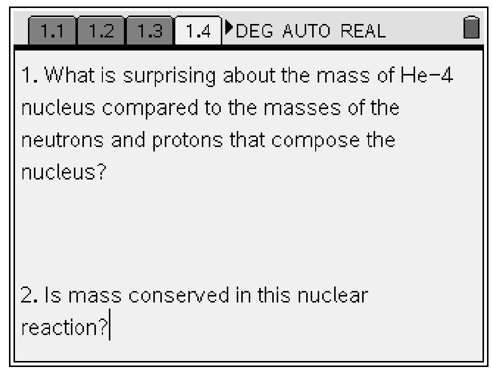


Figure 17

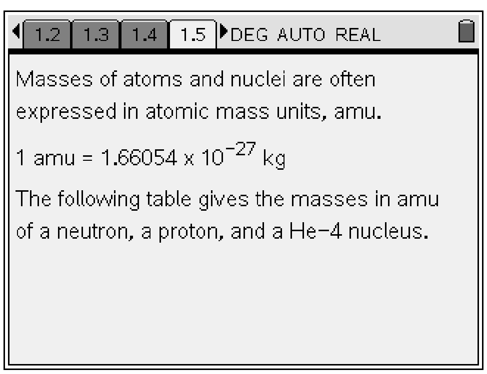


Figure 18

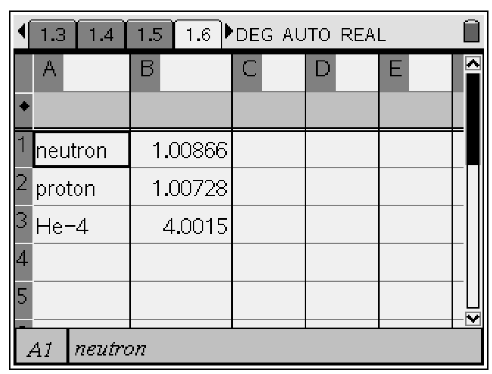


Figure 19

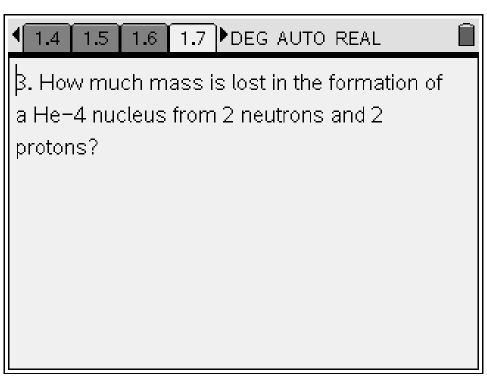


Figure 20

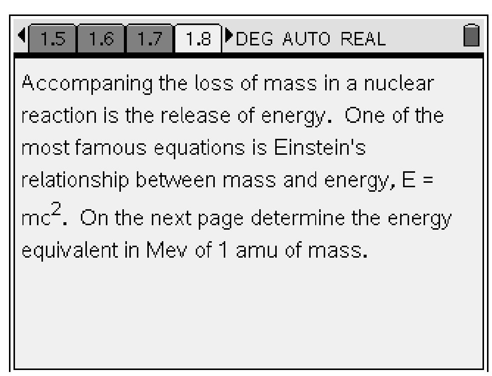


Figure 21

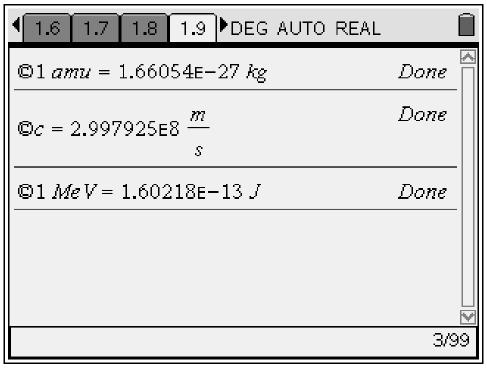


Figure 22

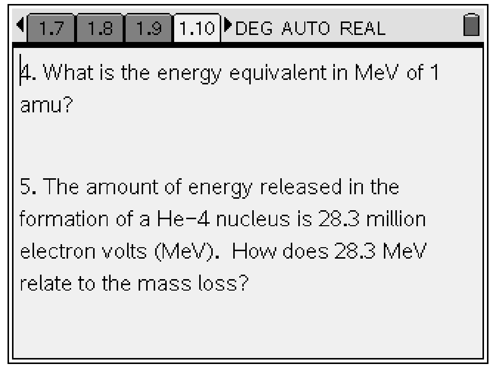


Figure 23

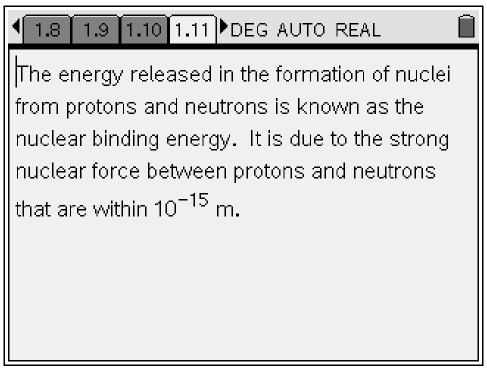


Figure 24

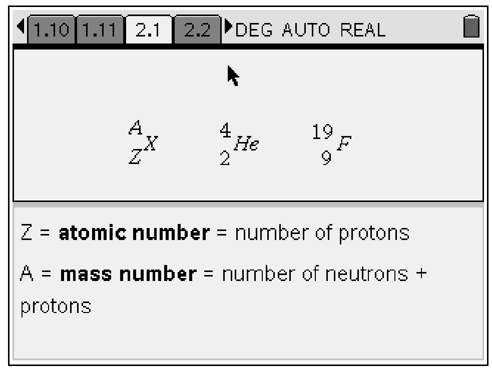


Figure 25

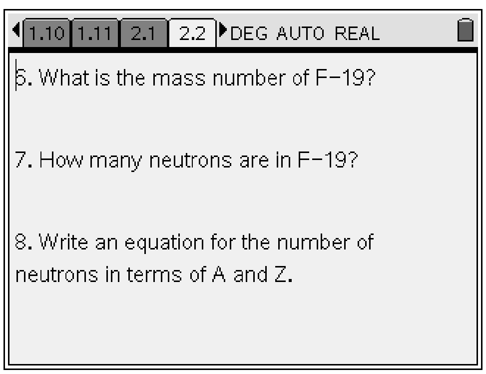


Figure 26

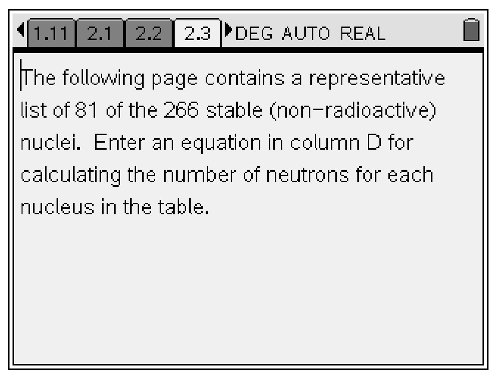


Figure 27

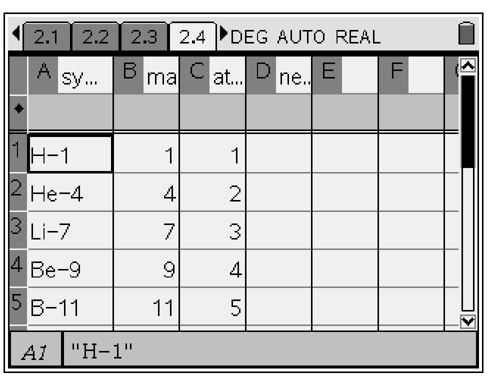


Figure 28

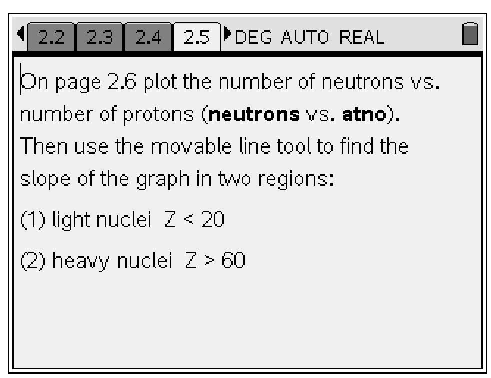


Figure 29

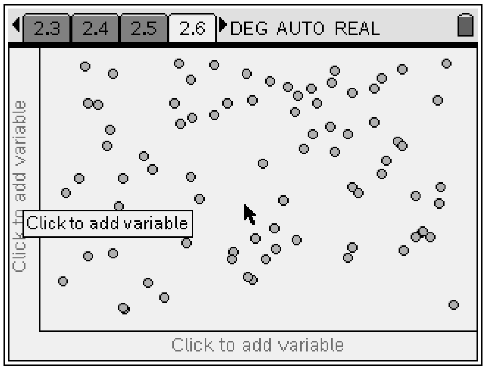


Figure 30

2.4 2.5 2.6 2.7 DEG AUTO REAL

9. For the stable nuclei, what can you say about the ratio of neutrons to protons as Z increases?

10. Explain this trend by taking account of the strong nuclear force and the electric force.

Figure 31

2.5 2.6 2.7 3.1 DEG AUTO REAL

The spreadsheet on the next page lists the nuclear masses for a number of nuclei. This list includes some radioactive nuclei with Z > 83. In column F write an equation to calculate the mass loss of each nucleus in the table. Recall the mass loss of He-4 is 0.0304 amu. In column G write an equation for binding energy in Mev. Recall 1 amu = 931.49 MeV.

Figure 32

2.7 3.1 3.2 3.3 DEG AUTO REAL

	A sy...	B ma	C at...	D ne...	E mass	F m
♦				=mass		
1	H-1	1	1	0	1.00728	
2	He-4	4	2	2	4.0015	
3	Li-7	7	3	4	7.01435	
4	Be-9	9	4	5	9.00998	
5	B-11	11	5	6	11.0066	

A1 "H-1"

Figure 33

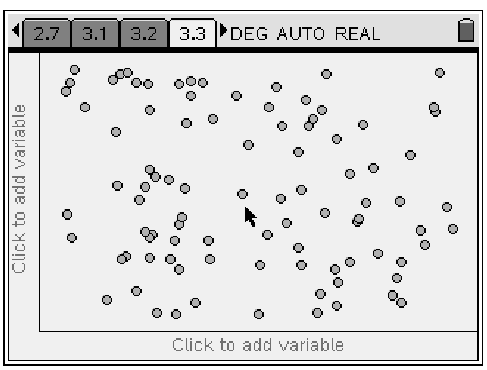


Figure 34

3.1 3.2 3.3 3.4 DEG AUTO REAL

11. Make a general statement about the relationship between binding energy and mass number.

12. A nucleon is a proton or neutron. What is the relationship between number of nucleons and mass number?

Figure 35

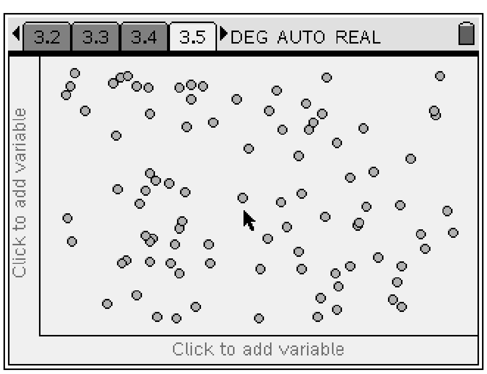


Figure 36

3.3 3.4 3.5 3.6 DEG AUTO REAL

13. Which nucleus has the greatest binding energy per nucleon?

14. Which light nucleus has an unusually high binding energy per nucleon?

Figure 37

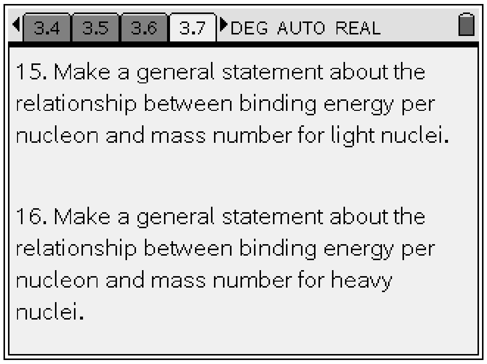


Figure 38

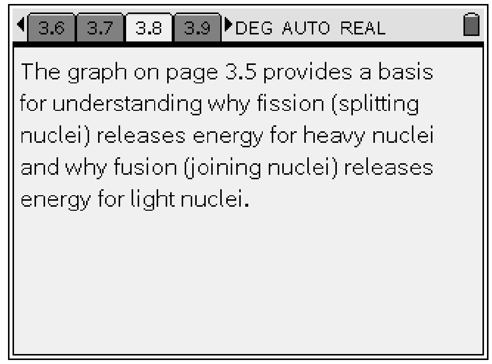


Figure 39

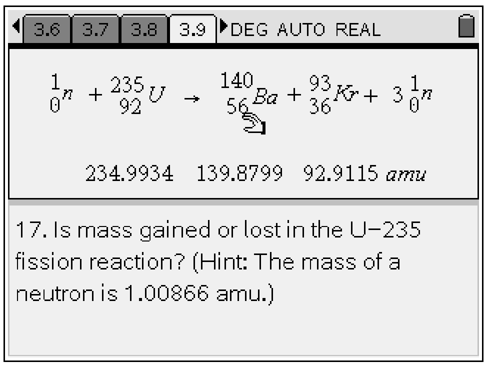


Figure 40

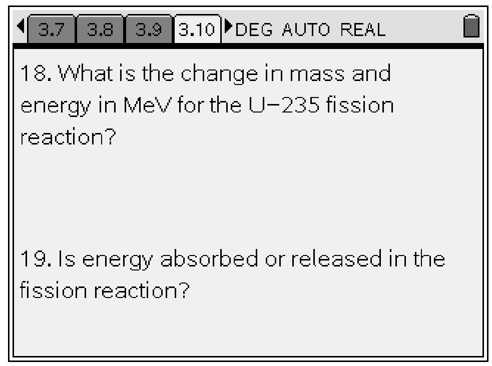


Figure 41

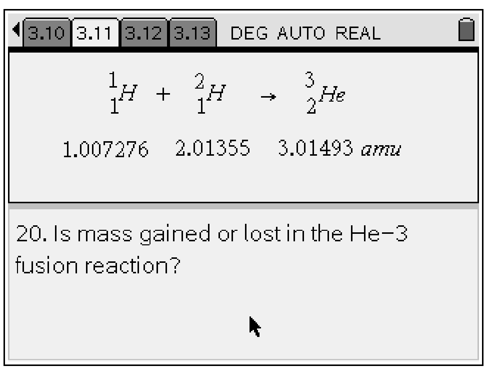


Figure 42

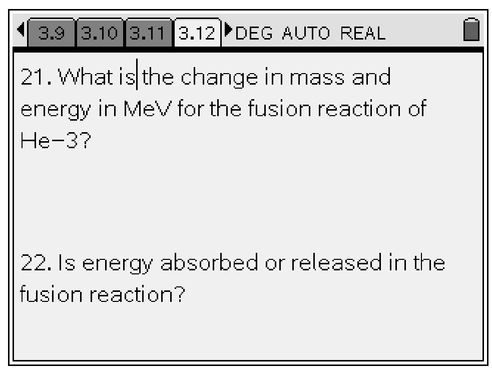


Figure 43

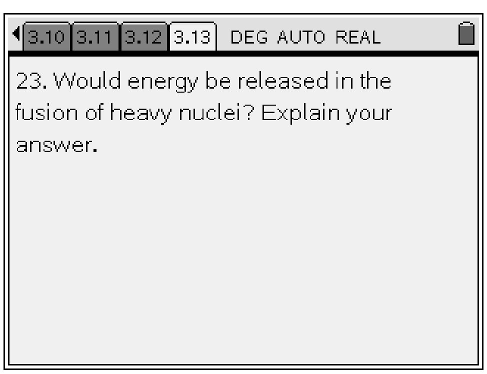


Figure 44

Projectile Trajectory Scenarios

Student Worksheet

Name ________________________

Class ________________________

Note: Sections of this document are numbered to correspond to the pages in the TI-Nspire™ .tns document “ProjectileTrajectory.tns.”

1.1 Trajectories and Velocity Components

What determines the trajectory of a projectile? What happens to its motion during its flight? We can investigate the basics of projectile motion using vectors and parametric equations to define and describe the motion.

A projectile may be traveling both horizontally and vertically at the same time. These two simultaneous motions can be combined with vectors and described separately with a set of parametric equations.

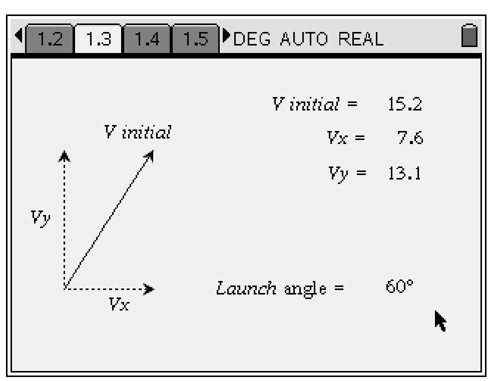
Let's begin by looking a simple projectile launched upwards from "ground level" across a horizontal surface, starting with the initial velocity vectors.

1.2

The path of a projectile depends on the *initial velocity vector*. This initial velocity can have different magnitudes and direction. This causes the *x* (or horizontal) and the *y* (or vertical) components to change.

1. On the next page, drag the tip of the initial velocity vector, and note how the launch angle and the components *V*x and *V*y change.

1.3

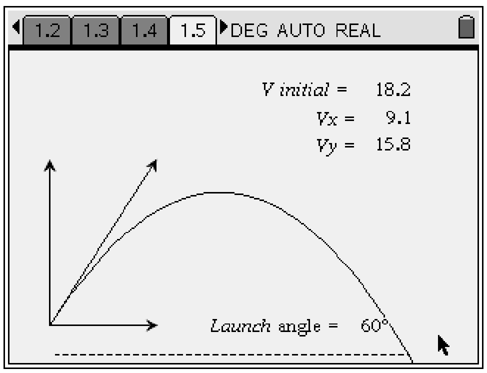


1.4

A projectile trajectory determined by the initial velocity is shown on page 1.4.

2. Drag the tip of the initial velocity, and note how the trajectory changes.
3. Press menu 5 1 for Menu 5: Trace, 1: Graph Trace.
4. Move the trace point along the graph to see the *x* (horizontal distance), the *y* (vertical distance), and the *t* (time of flight) for each point on the trajectory.
   - Clicking will lock a trace point, giving the x- and y-coordinates of that point.

1.5



1.6

5. By experimenting with the *initial velocity vector*, what two things tend to make the projectile travel higher?

1.7

6. What two things can you do to make the projectile travel farther?

1.8

7. Set a suitable trajectory back on page 1.5.
   - What is the maximum height of that trajectory?
8. Lock in a trace point.

1.9

9. When did the projectile reach the maximum height, and what was its horizontal displacement at that time?

1.10

10. When and where did the projectile land on the ground?

2.1 Changing Velocity Components

As a projectile flies through the air, its motion changes. Vertical and horizontal motions are separate and independent. For simple projectile motion consideration (ignoring air resistance, earth movement, curvature of the earth, etc...), horizontal motion is uniform and based on the *V*x (horizontal component of the initial velocity). The vertical motion is controlled by gravity and based on *V*y (vertical component of the initial velocity).

2.2

11. What is the equation to determine distance for *uniform horizontal motion*?

2.3

12. What is the equation for vertical height in freefall motion?

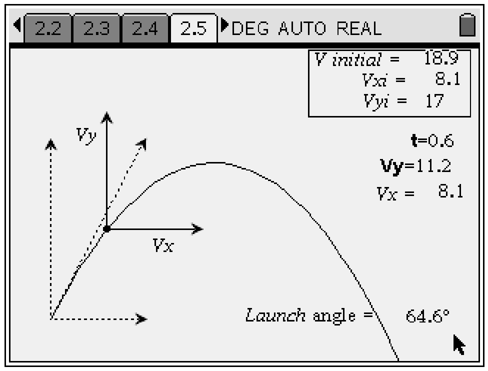
2.4

Parametric equations relate two separate relations to a third variable—usually *time*. This works perfectly for projectiles because we can determine horizontal and vertical displacements from different velocities—both at the same times.

Note: The parametric equations for vertical and horizontal motions are stepped through chosen time intervals to create the trajectories in this document.

On the next page of the TI-Nspire™ document is a trajectory that you can vary by changing the initial velocity. The projectile is attached to the trajectory path, and the velocity of the projectile is shown as it moves along the trajectory.

2.5



2.6

13. What happens to the horizontal component of velocity (*V*x) as the projectile moves along the trajectory?

2.7

14. What happens to the vertical velocity (*V*y) as the projectile moves along the trajectory?

2.8

15. When does the vertical velocity become zero? Why? What is this point of the trajectory called?

2.9

16. What happens to the vertical velocity (*V*y) after the projectile reaches maximum height? Explain.

2.10

On the next page of the TI-Nspire™ document is a spreadsheet with columns for the time and vertical velocity component as the projectile moves along the trajectory.

17. Go back to the last trajectory on page 2.5.

18. Set a suitable trajectory, and move the projectile near to the beginning of the trajectory.

19. Press ctrl ..

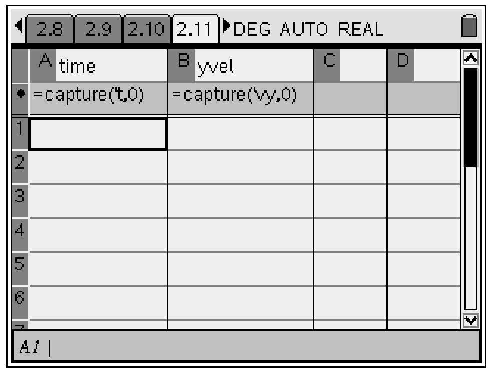
- These keystrokes will capture data from this point for the spreadsheet.

20. Move the projectile along the trajectory, and press ctrl . again to gather another set of data at a new point.

21. Do this for at least ten points along the range of the trajectory.

22. Look at the spreadsheet on page 2.11, and then move on to page 2.12.

2.11



2.12

In the spreadsheet on the previous page, you captured data of time and *y*-velocity along the trajectory. As you can see, the *y*-velocity changes. By plotting this data, we can find the how the *y*-velocity changes with time.

2.13

On the bottom half of this page is a data plot.

- Press ctrl tab to toggle between the two work areas of this page.

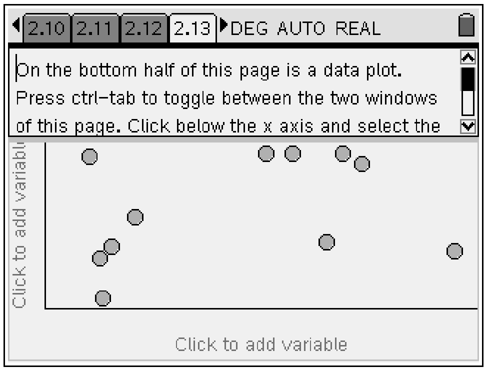
23. Move the cursor, click below the x-axis, and select the variable "`time`".

24. Click beside the y-axis, and select "`yvel`".

- This plots the captured data from the spreadsheet.

25. Click in the data plot, and press menu 3 5 2 for Menu 3: Actions, 5: Regression, 2: Show Linear (ax+b).

- Note the equation that appears.
- Click on the line if the equation disappears.



2.14

26. What does the slope of a velocity vs. time graph represent? What is the slope of this equation? What units should the slope have? Explain this value.

2.15

27. What is the vertical axis intercept (*y*-intercept)? What does this mean?

2.16

28. How does the *y*-intercept relate to the original trajectory on page 2.5?

2.17

29. What does the point where the line crosses the x-axis represent? Where is this seen on the trajectory?

2.18

30. What happens to the vertical velocity at the point where the graph crosses the x-axis? Explain what is going on here.

2.19

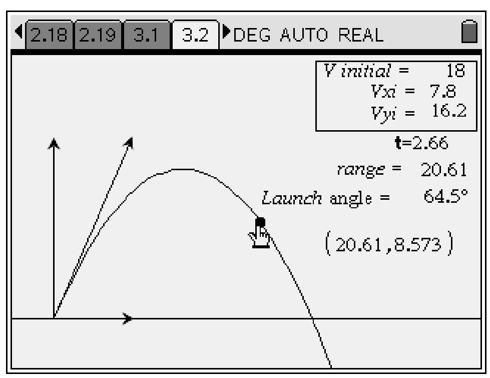
31. Why is the vertical acceleration negative throughout the entire trajectory?

3.1 Maximum Range

How can you maximize the range of a projectile? Shooting it faster is a first, obvious solution; what else can you do?

On the next page is a trajectory for a projectile launched at a fixed, maximum speed. Explore how to make the projectile go farther before landing on the ground. You can drag the initial velocity vector, and you can drag the trace point on the trajectory.

3.2



3.3

32. In this example, what launch angle gives the maximum range?

3.4

33. What is the maximum range?

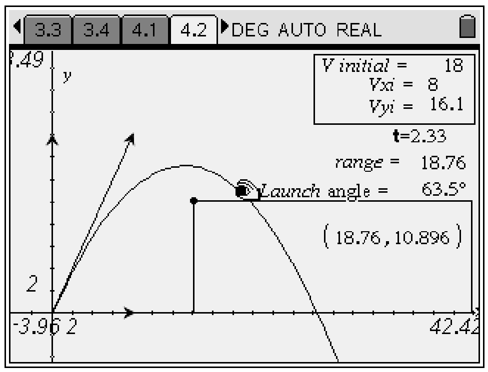
4.1 Optimum Angles?

You now launch the projectile towards an elevated landing area.

34. Does this affect the optimum angle for maximum range?

35. Explore this by dragging the corner of the elevated surface, the tip of the initial velocity vector, and the trace point.

4.2



4.3

36. As the elevated landing area becomes higher, what happens to the angle needed for maximum range?

4.4

37. As an extension to this activity, try to create another trajectory where the launch point is elevated above the landing area.

38. What does this do to the range and to the optimum angle for maximum range?

Projectile Trajectory Scenarios

Concepts

- Projectile Motion
- Velocity vectors
- Component vectors

Materials

- TI-Nspire™ Math and Science Learning Handheld

Overview

This activity document contains four problems to explore different aspects of projectile motion.

Introduction

This activity contains four problems as seen in the TI-Nspire™ .tns document "ProjectileTrajectory.tns." The pages and problems contained in the .tns file serve as the sections in this activity document.

Problem 1 illustrates how the *initial velocity vector* has both vertical and horizontal components that act independently of each other and how changes in the initial velocity can change the components and the resulting trajectory.

Problem 2 looks at changes in the velocity components as a projectile moves along its trajectory. By varying trajectories, students can see that horizontal velocity is always constant (ignoring air, etc.) and that the vertical component is always the same as simple freefall motion.

Problem 3 allows students to explore graphically (simulate) how to determine the range of a projectile, and find the optimum angle to maximize range.

Problem 4 extends this exploration to include having the projectile land on an elevated surface.

Notes & Hints

1. Diagrams have much background information that is hidden.
   - For example, a vector may be constructed on a line segment to limit the parameters of the vector. The line segment is then hidden so that the vector is essentially "locked" and cannot be moved without showing the line segment first.
   - Students could mess up diagrams by "Showing" and then changing some points or information. Students should immediately save the document under a slightly changed name so that they will always have a clean, original copy to work with.
   - You, of course, should have a back-up master copy on your handheld and on your computer.
2. These activities involve the dragging of points to change the initial velocity vector of a projectile, the trace points on trajectories, and the parameters of a landing area.
   - Data will be captured by students during some of this dragging.
   - After some initial random, fun exploration, students should be encouraged to drag (explore) systematically.
3. The trajectories are created through parametric equations.
   - For simplicity, the time increment used is 0.1 s.
   - This relatively large time increments means some loss of precision for maximum height or landing point calculations, but it makes the dragging simpler and easier to follow.

Further notes and suggested answers are included with some of the page descriptions and screen shots below (in bold text).

1.1 Trajectories and Velocity Components

What determines the trajectory of a projectile? What happens to its motion during its flight? We can investigate the basics of projectile motion using vectors and parametric equations to define and describe the motion.

A projectile may be traveling both horizontally and vertically at the same time. These two simultaneous motions can be combined with vectors and described separately with a set of parametric equations.

Let's begin by looking a simple projectile launched upwards from "ground level" across a horizontal surface, starting with the initial velocity vectors.

The rhetorical question may be a good jumping off point for discussion of terminology and perceptions.

1.2

The path of a projectile depends on the *initial velocity vector*. This initial velocity can have different magnitudes and direction. This causes the x (or horizontal) and y (or vertical) components to change.

1. On the next page, drag the tip of the initial velocity vector, and note how the launch angle and the components *V*x and *V*y change.

The only "hot spot" is the tip of *V* initial. Show the students that to drag something they "grab" it by moving the cursor over the point until the open hand appears. Then either hold the click button for one second, or press ctrl to grab.

1.3

See Figure 1 for page 1.3 with the hand cursor ‘grabbing’ the vector.

1.4

A projectile trajectory determined by the initial velocity is shown on page 1.4.

2. Drag the tip of the initial velocity, and note how the trajectory changes.
3. Press menu 5 1 for Menu 5: Trace, 1: Graph Trace.
4. Move the trace point along the graph to see the *x* (horizontal distance), the *y* (vertical distance), and the *t* (time of flight) for each point on the trajectory.
   - Clicking will lock a trace point, giving the x- and y-coordinates of that point.

Every click will drop another trace point. Extra trace points and coordinates can be removed. Press esc, grab the unwanted point, and delete with clear.

Use right and left arrows only to trace. Up or down arrows change the trace mode. If the wrong mode appears, just restart the trace menu.

1.5

See Figure 2 for page 1.5 with a locked and a dynamic trace point.

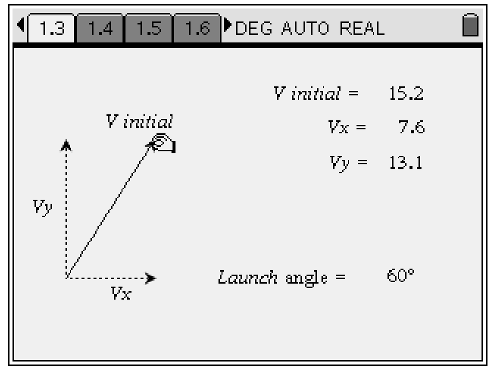


Figure 1

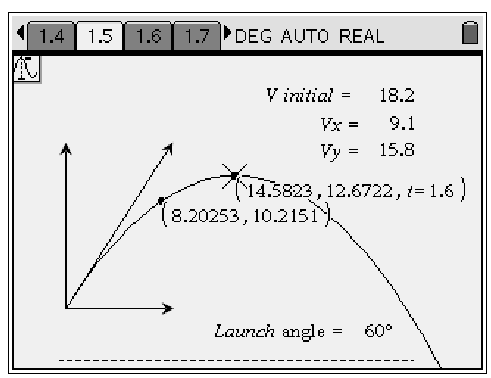


Figure 2

1.6

5. By experimenting with the initial velocity vector, what two things tend to make the projectile travel higher?

Longer *initial velocity vector* (greater speed) and larger angle (more vertical) make the projectile go higher.

1.7

6. What two things can you do to make the projectile travel farther?

Longer *initial velocity vector* (greater speed) and an optimum angle make the projectile go farther. Optimum angle is explored later. Here it is 45°.

1.8

7. Set a suitable trajectory on page 1.5.
   - What is the maximum height of that trajectory?
8. Lock in a trace point.

Any trajectory that fits on the screen is suitable. Values will vary with students. Note that exact maximum heights are not available due to the time increment limit of 0.1 s.

1.9

9. When did the projectile reach the maximum height, and what was its horizontal displacement at that time?

Read values from the trace point coordinates.

Note: The live trace coordinates are (*x*,*y*,*t*); the locked trace point coordinates give only (*x*,*y*).

1.10

10. When and where did the projectile land on the ground?

Trace as close to the ground (y = 0) as possible, given the time increments of 0.1 s.

2.1 Changing Velocity Components

As a projectile flies through the air, its motion changes. Vertical and horizontal motions are separate and independent. For simple projectile motion consideration (ignoring air resistance, earth movement, curvature of the earth, etc...), horizontal motion is uniform and based on the *V*x (horizontal component of the initial velocity). The vertical motion is controlled by gravity and based on *V*y (vertical component of the initial velocity).

2.2

11. What is the equation to determine distance for *uniform horizontal motion*?

$$\mathbf{dx} = \boldsymbol{Vx \cdot t} \quad \textbf{(where } \boldsymbol{V}\mathbf{x} = \boldsymbol{V}_{\textbf{initial}}\ \mathbf{cos}\ \boldsymbol{\theta}\textbf{)}$$

2.3

12. What is the equation for vertical height in freefall motion?

$$\mathbf{dy} = V\mathrm{y_i} \cdot t + \tfrac{1}{2}\, g t^2$$

$$(\text{where } V\mathrm{y_i} = V_{\mathrm{initial}} \sin\theta \text{ and } g = -9.8\,\mathrm{m/s^2})$$

2.4

Parametric equations relate two separate relations to a third variable—usually *time*. This works perfectly for projectiles; we can determine horizontal and vertical displacements from different velocities—both at the same times.

Note: The parametric equations for vertical and horizontal motions are stepped through chosen time intervals to create the trajectories in this document.

On the next page is a trajectory that you can vary by changing the initial velocity. The projectile is attached to the trajectory path, and the velocity of the projectile is shown as it moves along the trajectory.

In this example, steps have been taken to make the diagram less likely to be disturbed:

- **Graph type is parametric.**
- **Parametric graph 1 was left empty, and equations are in parametric graph 2.**
- **Entry Line was changed back to function and then hidden.**

The parametric equations are based on measured components (that are hidden) stored as variables:

- $x_2(t) = Vx \cdot t$
- $y_2(t) = V\mathrm{y_i} \cdot t - 4.9t^2$
- $t_{\mathrm{step}} = 0.1$ **with limits** $0<t<5$

2.5

See Figure 3 for page 2.5.

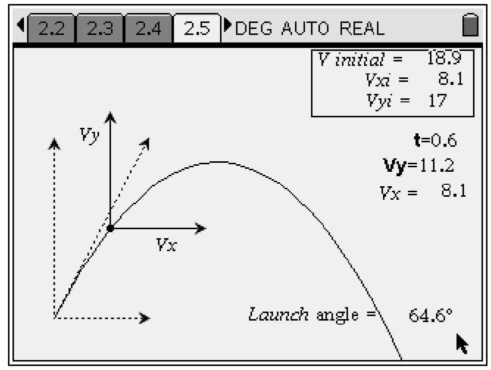


Figure 3

2.6

13. What happens to the horizontal component of velocity (*V*x) as the projectile moves along the trajectory?

***V*x remains constant.**

2.7

14. What happens to the vertical velocity (*V*y) as the projectile moves along the trajectory?

***V*y decreases on the way up, changes direction to downwards at the top, and increases on the way down.**

2.8

15. When does the vertical velocity become zero? Why? What is this point of the trajectory called?

***V*y becomes zero at the apex of the trajectory. The explanation by students is often backwards. *V*y becomes zero because gravity is continually pulling the projectile down. This causes the vertical motion to stop. Because the upward vertical motion stops, this means the projectile will go no higher, thus the apex of the trajectory. Immediately, the projectile changes directions and starts to fall faster and faster. This will be explored more shortly.**

2.9

16. What happens to the vertical velocity (Vy) after the projectile reaches maximum height? Explain.

2.10

On the next page of the TI-Nspire™ document is a spreadsheet with columns for the time and vertical velocity component as the projectile moves along the trajectory (Figure 4).

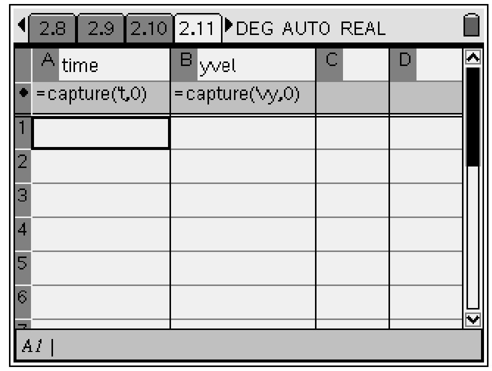


Figure 4

17. Go back to the last trajectory on page 2.5 (Figure 5).

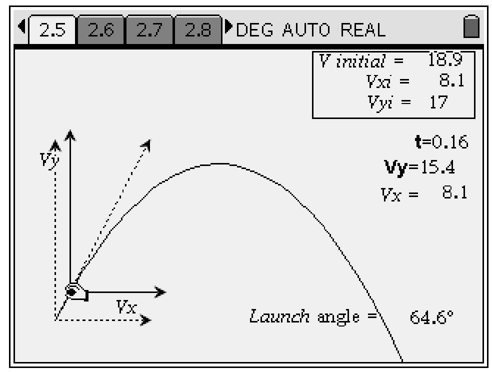


Figure 5

18. Set a suitable trajectory, and move the projectile near the beginning of the trajectory.

19. Press ctrl .

- These keystrokes will capture data from this point for the spreadsheet (Figure 6).

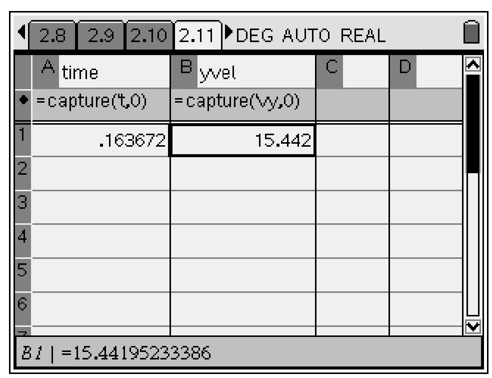


Figure 6

20. Move the projectile along the trajectory, and again press ctrl . to gather another set of data at a new point.

21. Do this for at least ten points along the range of the trajectory.

22. Look at the data captured in the spreadsheet on page 2.11, and then move on to page 2.12.

Encourage students to capture date over a wide range of the trajectory including on the way up, near the apex (but not necessarily right at the peak), and on the way down. Spreading out the capture points will make the next graph easier to interpret.

2.12

In the spreadsheet on the page 2.11, you captured data of time and *y*-velocity along the trajectory. As you can see, the *y*-velocity changes. By plotting this data, we can find the how the *y*-velocity changes with time.

2.13

On the bottom half of this page is a data plot.

- Press ctrl tab to toggle between the two work areas of this page.

23. Click below the x-axis, and select the variable "`time`".

24. Now click beside the y-axis, and select "`yvel`" (Figure 7).

- This plots the captured data from the spreadsheet.

25. Click in the data plot, and press menu 3 5 2 for Menu 3: Actions, 5: Regression, 2: Show Linear (ax+b).

- Note the equation that appears (Figure 8).
- Click on the line if the equation disappears.

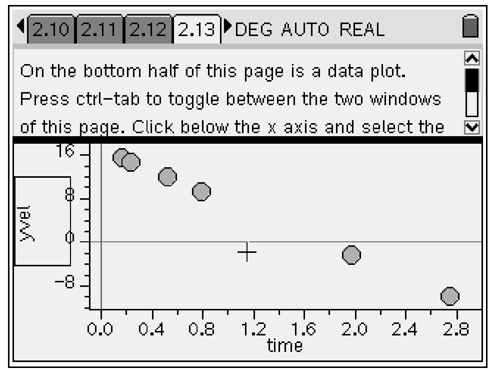


Figure 7

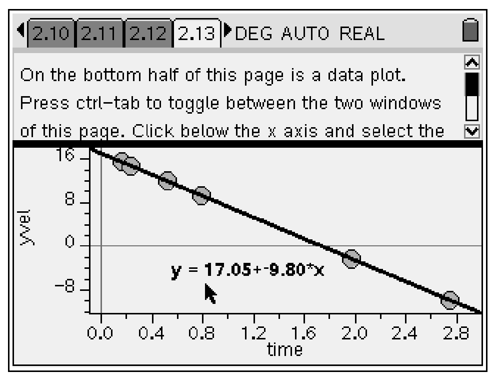


Figure 8

2.14

26. What does the slope of a velocity vs. time graph represent? What is the slope of this equation? What units should the slope have? Explain this value.

The general equation that applies here is:

$$\mathbf{V_2 = V_1 + at}$$

The slope of the graph is the acceleration:

$$\mathbf{-\ 9.8\ m/s^2}$$

2.15

27. What is the vertical axis intercept (*y*-intercept)? What does this mean?

The vertical axis intercept represents the initial speed: Vy_1 = 17.05 m/s in this example.

2.16

28. How does the *y*-intercept relate to the original trajectory on page 2.5?

This is the original vertical component of the *initial projectile velocity*, *V*y_i.

2.17

29. What does the point where the line crosses the x-axis represent? Where is this seen on the trajectory?

The point where the graph crosses the trajectory represents a vertical speed of zero. This occurs at the apex of the trajectory—at about 1.7s in this example.

2.18

30. What happens to the vertical velocity at the point where the graph crosses the x-axis? Explain what is going on here.

The graph is a straight line, indicating constant (negative) acceleration. Immediately above the horizontal axis, the graph indicates positive (upwards) motion; immediately below the axis, it indicates negative (downwards) motion. As the graph crosses the line, the projectile stops going up and starts going down—with no change in the acceleration (slope of the graph). How long is the projectile stopped at the top?

2.19

31. Why is the vertical acceleration negative throughout the entire trajectory?

Gravity causes the acceleration and is acting downwards continually throughout the entire motion.

3.1 Maximum Range

How can you maximize the range of a projectile? Shooting it faster is a first, obvious solution; what else can you do?

On the next page is a trajectory for a projectile launched at a fixed, maximum speed. Explore how to make the projectile go farther before landing on the ground. You can drag the initial velocity vector, and you can drag the trace point on the trajectory.

Changing the angle of launch is the other factor affecting the range.

In this problem, the *initial velocity vector* was built to a circle (hidden). This fixes the length (magnitude) of the vector but allows it to be dragged to different launch angles.

3.2

See Figure 9 for page 3.2

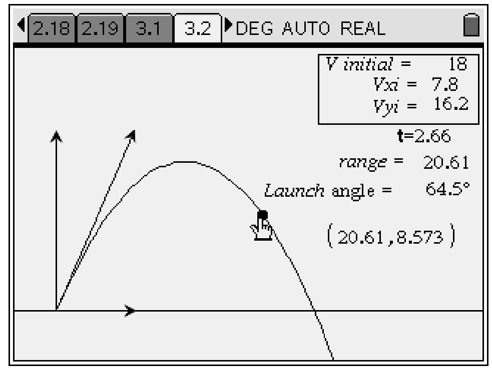


Figure 9

3.3

32. In this example, what launch angle gives the maximum range?

Students can drag the initial velocity vector to different angles to see where the trajectory crosses the horizontal.

Students can drag the projectile along the trajectory to see its x- and y-coordinates at points on the trajectory.

The maximum range shown in the screenshot is the best we can do with this time-step setting.

The algebraic approach could be another lesson. For a landing on the same level as the launch, the optimum angle is 45 ° (Figure 10).

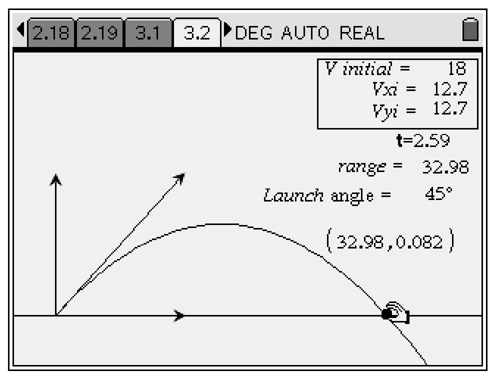


Figure 10

3.4

33. What is the maximum range?

The maximum range for this launch velocity is about 33 m.

4.1 Optimum Angles?

You now launch the projectile towards an elevated landing area.

34. Does this affect the optimum angle for maximum range?

35. Explore this by dragging the corner of the elevated surface, the tip of the initial velocity vector, and the trace point.

The results here are interesting and unexpected for many people. The relative height of the landing area compared to the launch does make a difference to the maximum range and to the optimum launch angle.

Students can explore a variety of different landing heights by dragging the corner of the "cliff" or "building" to get a height they want.

An interesting extreme: can the building be too high?

Yes, with the fixed launch speed, there will be an absolute maximum height for the trajectory. If the landing is higher than that, the projectile will not reach.

The algebraic solution (another lesson) results in complex number values.

The range is found by dragging the projectile to the landing height (Figure 11 showing page 4.2).

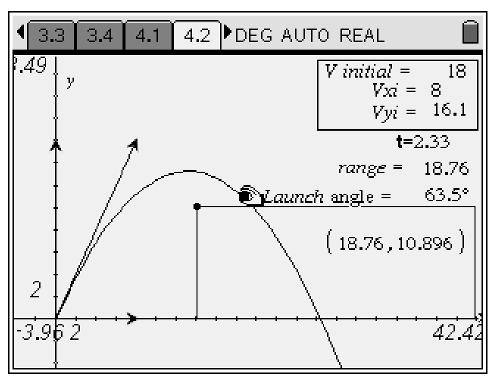


Figure 11

4.3

36. As the elevated landing area becomes higher, what happens to the angle needed for maximum range?

The higher the landing is compared to the launch location, the larger (more vertical) the launch angle needs to be to get maximum range. This can be described conceptually by saying that, because of the higher landing, the projectile will have less time in the air. To add more air time, we need to shoot it higher.

4.4

37. As an extension to this activity, try to create another trajectory where the launch point is elevated above the landing area.
38. What does this do to the range and to the optimum angle for maximum range?

The lower the landing area, the closer the optimum angle is to horizontal. An "extreme" of this is a circular orbit. A satellite has a horizontal speed and "never" lands because it is so high (and, of course, because of the curvature of the earth—the analogy is not perfect).

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Final Notes

Online Evaluation

Please take the time to provide your feedback on this institute by completing the online evaluation at:
http://deploy.ztelligence.com/start/index.jsp?PIN=139ZK2ED2EKU7

An email will also be sent to each participant with a link to the evaluation.

Continuing Education and Graduate Credit

As a part of Texas Instruments' on going commitment to help teachers succeed, T^3 is working with several institutions to provide CEU and/or Graduate credit for our courses for teachers.

General information can be found at

http://education.ti.com/educationportal/sites/US/sectionHome/pd_credit.html

Adams State College
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Under the "What's New Section", click on Texas Instruments

That action will then direct you to an ASC/Texas Instruments Welcome page

You will be prompted to click on a “REGISTER BUTTON” for the graduate credit

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http://www.actprogram.info/ti-nspire.html

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