Math Objectives

- Students will be able to state and apply the properties of logarithms:
  - $\log_a(mn) = \log_a(m) + \log_a(n)$
  - $\log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n)$
  - $\log_a(m^n) = n\log_a(m)$
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).

Vocabulary

- logarithmic expressions
- equivalent expressions
- exponential properties

About the Lesson

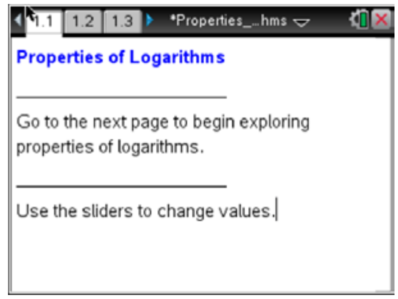
- This lesson is based on observing counter-examples to logarithmic rules.
- This lesson involves comparing the values of several base 2 logarithmic expressions to discover which produce the same result. As a result, students will:
- Generalize these rules for base *a*, where *a* is a real number, $a > 0$ and $a \neq 1$.
- Compare these logarithmic properties to their exponential counterparts.

TI-Nspire™ Navigator™ System

- Use Quick Poll to assess students' understanding throughout the activity.
- Use Live Presenter for student demonstrations.
- Use Screen Capture to examine patterns that emerge.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity

- Properties_of_Logarithms_Student.pdf
- Properties_of_Logarithms_Student.doc

TI-Nspire document

- Properties_of_Logarithms.tns

Discussion Points and Possible Answers

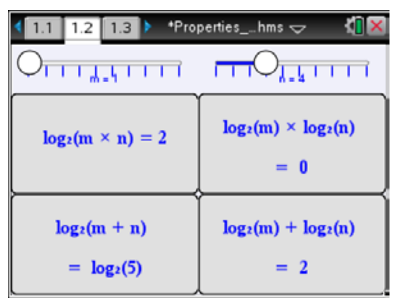
TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 1 at the end of this lesson.

Tech Tip: All the pages after the title screen are designed to easily allow students to drag the slider values. Instruct the students to move the cursor to the point *m* or *n* until they get the open hand (☝) and select the Touchpad or select [enter]. The point should slowly blink. Then the point can be moved by pressing the directional arrows of the Touchpad.

Tech Tip: All the pages after the title screen are designed to easily allow students to drag the slider values. To move the point on each slider, tap on the point and begin sliding it.

Teacher Tip: At some point during the investigation, you may want to discuss some of the historical implications of the properties of logarithms. The usefulness of logarithms in calculations is based on these properties. The first two properties are the basis for the slide rule, a mechanical computation device that preceded the electronic calculator.

Move to page 1.2.



For this activity, the expression used is $\log_2(x)$.

The investigations also work for any base > 0 and any base ≠ 1.

1. As you drag the sliders for *m* and *n,* note what happens as these values are substituted into the four expressions.
    a. Which expressions, if any, appear to be equivalent independent of the values of *m* and *n?*

    Answer: $\log_2(mn)$ and $\log_2(m)+\log_2(n)$

b. Set m = 8 and n = 4. Substitute these values into the logarithmic expressions you found to be equivalent in part 1a, and simplify these expressions to show they are indeed equivalent.

Answer: $\log_2(8\cdot 4)=\log_2(32)=5$
$\log_2(8)+\log_2(4)=3+2=5$

c. Use the expressions you found in parts 1a and 1b to write a general logarithmic property for $\log_a mn$ where a is a real number, $a>0$ and $a\neq 1$.

Answer: $\log_a(mn)=\log_a(m)+\log_a(n)$

d. How do the operations in the logarithmic property in part 1c relate to the operations in the exponential property $a^m a^n=a^{m+n}$?

Answer: For the exponential rule, if the bases are the same and the expressions are being multiplied, the exponents are added. Because a logarithm is also an exponent, the rule applies similarly.

Teacher Tip: If desired, you may want students to prove question 1d instead. Here is a sample answer:

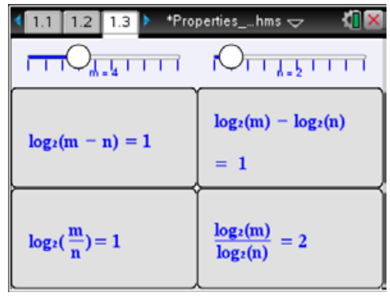
Using the definition of logarithms that states $\log_a x=y \Leftrightarrow a^y=x$ and the stated exponential property, you can verify the logarithm property from question 1c.

$\log_a(m)=y_1 \Leftrightarrow a^{y_1}=m$
$\log_a(n)=y_2 \Leftrightarrow a^{y_2}=n$
$\log_a(m)+\log_a(n)=y_1+y_2$
$mn=a^{y_1}a^{y_2}=a^{y_1+y_2} \Leftrightarrow \log_a(mn)=y_1+y_2$
Thus, $\log_a(m)+\log_a(n)=\log_a(mn)$

Tech Tip: To reset, students can select >**Controls**> **Reset**.

TI-Nspire Navigator Opportunity: ***Screen Capture*** **and** ***Quick Poll***
See Note 2 at the end of this lesson.

Move to page 1.3.



2. As you drag the sliders for *m* and *n*, note what happens as these values are substituted into the four expressions.

   a. Which expressions, if any, appear to be equivalent independent of the values of *m* and *n*?

   <u>Answer:</u> $\log_2\left(\frac{m}{n}\right)$ and $\log_2(m)-\log_2(n)$

   b. Set *m* = 8 and *n* = 4. Substitute these values into the logarithmic expressions you found to be equivalent in question 2a, and simplify these expressions to show they are indeed equivalent.

   <u>Answer:</u> $\log_2\left(\frac{8}{4}\right)=\log_2(2)=1$

   $\log_2(8)-\log_2(4)=3-2=1$

   c. Use the expressions you found in parts 2a and 2b to write a general logarithmic property for $\log_a\left(\frac{m}{n}\right)$ where *a* is a real number and $a>0$ and $a\neq 1$.

   <u>Answer:</u> $\log_a\left(\frac{m}{n}\right)=\log_a(m)-\log_a(n)$

   d. How do the operations in the logarithmic property in part 2c relate to the operations in the exponential property $\frac{a^m}{a^n}=a^{m-n}$?

   <u>Answer:</u> The log of a quotient is the difference of the logs. Since logarithms are exponents, the exponent of a quotient is the difference of the exponents.

Teacher Tip: If desired, you may want students to prove question 2d instead. Here is a sample answer:

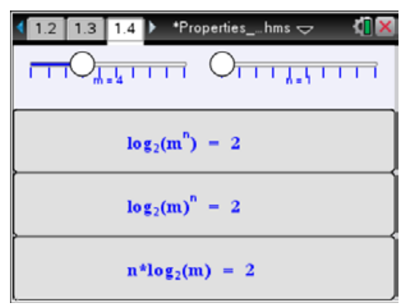
Using the definition of logarithms that states $\log_a x = y \Leftrightarrow a^y = x$ and the stated exponential property, you can verify the logarithm property from question 2c.

$$\log_a(m) = y_1 \Leftrightarrow a^{y_1} = m$$
$$\log_a(n) = y_2 \Leftrightarrow a^{y_2} = n$$
$$\log_a(m) - \log_a(n) = y_1 - y_2$$
$$\frac{m}{n} = \frac{a^{y_1}}{a^{y_2}} = a^{y_1 - y_2} \Leftrightarrow \log_a\left(\frac{m}{n}\right) = y_1 - y_2$$

Thus, $\log_a(m) - \log_a(n) = \log_a\left(\frac{m}{n}\right)$

TI-Nspire Navigator Opportunity: *Screen Capture and Quick Poll*
See Note 3 at the end of this lesson.

Move to page 2.4.



3. As you drag the sliders for *m* and *n*, note what happens as these values are substituted into the three expressions.
   a. Which expressions, if any, appear to be equivalent independent of the values of *m* and *n*?

   Answer: $\log_2(m^n)$ and $n\log_2(m)$

b. Set m = 4 and n = 3. Substitute these values into the logarithmic expressions you found in part 3a, and simplify these expressions to show they are equivalent.

Answer:
$$\log_2\left(4^3\right) = \log_2(64) = 6$$
$$3\log_2(4) = 3\cdot 2 = 6$$

c. Use the expressions you found in parts 3a and 3b to write a general logarithmic property for $\log_a(m)^n$ where a is a real number, $a > 0$ and $a \neq 1$.

Answer: $\log_a\left(m^n\right) = n\log_a(m)$

d. How do the operations in the logarithmic property in part 3c relate to the operations in the exponential property $\left(a^m\right)^n = a^{mn}$?

Answer: When an expression with an exponent is raised to an exponent, the exponents are multiplied. Roughly speaking, this is a restatement in logarithm form of the law of exponents.

Teacher Tip: If desired, you may want students to prove question 3d instead. Here is a sample answer:

$$\log_a\left(m^n\right) = y \Leftrightarrow a^y = m^n$$
$$\left(a^y\right)^{1/n} = \left(m^n\right)^{1/n}$$
$$a^{y/n} = m \Leftrightarrow \log_a m = \frac{y}{n}$$
$$n\log_a m = y$$
$$\text{Or } \log_a\left(m^n\right) = \underbrace{\log_a\left(m\cdot m\cdot m\cdot\cdots\cdot m\right)}_{n\text{ times}} =$$
$$\underbrace{\log_a(m) + \log_a(m) + \cdots + \log_a(m)}_{n\text{ times}} = n\log_a(m)$$

e. Use the logarithmic property you proved in part 3c to show that $\log_a a = 1$ for all values of a where $a > 0$ and $a \neq 1$.

Answer: Since $a = a^1$ then $\log_a a^1 = 1$.

f. Use the logarithmic property you proved in part 3c to show that $\log_a 1 = 0$ for all values of a where $a > 0$ and $a \neq 1$.

Answer: Since $1 = a^0$ then $\log_a a^0 = 0$.

Teacher Tip: The last page does not allow for the *n* value to be negative, so this is a perfect opportunity to discuss with students how this can be applied as well. In other words, discuss why $\log_2\left(\frac{1}{x}\right) = -\log_2 x$. This discussion can provide review of other laws of exponents as well.

TI-Nspire Navigator Opportunity: *Screen Capture* and *Quick Poll*
See Note 4 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students know and can apply the following logarithm properties:

- $\log_a(mn) = \log_a(m) + \log_a(n)$
- $\log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n)$
- $\log_a(m)^n = n\log_a(m)$

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Note 1

Question 1, *Live Presenter*: You may want to demonstrate how to drag the point on the slider using Live Presenter.

Note 2

Question 1c, *Screen Capture*: Take a Screen Capture of page 1.2 where students are on different *m*-values and *n*-values. As a class, discuss the various cases that occur and the fact that the two same expressions are always equivalent.

Question 1.c, *Quick Poll (Open Response):* Send an Open Response Quick Poll asking students to submit their answers to question 1c. If students struggle to write the general logarithmic property, revisit the Screen Capture and use equivalent expressions to write the general property.

Note 3

Question 2c, *Screen Capture:* Take a Screen Capture of page 1.3 when students are on different *m*-values and *n*-values. As a class, discuss the various cases that occur and the fact that the two same expressions are always equivalent.

Question 2c, *Quick Poll (Open Response)*: Send an Open Response Quick Poll asking students to submit their answer to question 2c. If students struggle to write the general logarithmic property, revisit the Screen Capture and use equivalent expressions to write the general property.

Note 4

Question 3c, *Screen Capture:* Take a Screen Capture of page 1.4 when students are on different *m*-values and *n*-values. As a class, discuss the various cases that occur and the fact that the two same expressions are always equivalent.

Question 3c, 3e, and 3f, *Quick Polls (Open Response):* Send Open Response Quick Polls asking students to submit their answers to questions 3c, 3e, and 3f. If students struggle to write the general logarithmic property, revisit the Screen Capture and use equivalent expressions to write the general property.