Math Objectives

- Students will explore the nature and needs of a word problem and look at the expression and equations implied by the situation.
- Students will explore the graph of the function that best models the scenario in the word problem and examine the relationship between the values available for the Domain and Range as they relate to the context of the problem.
- Students will focus on the real number types needed in the situation and examine these data types graphically, numerically, and analytically.
- Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically (CCSS).
- Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies (CCSS).
- Make sense of problems and persevere in solving them (CCSS Mathematics Practice).

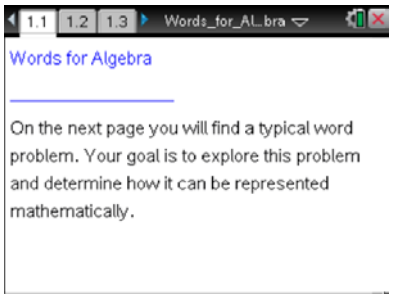
Vocabulary

- expression
- equation
- solution
- whole numbers
- rational numbers
- domain
- range

About the Lesson

- This lesson involves starting with the context of a word problem and then examining it from several different perspectives in an effort to build expressions and equations that model the problem
- As a result, students will:
  - Consider elements of the situation that determine the data types that must be used.
- Operate under the following constraints: 1) fruit will be sold to the nearest hundredth, or thousandth of a Newton, but pencils are only sold in whole number quantities; and 2) a set amount of money.

TI-Nspire™ Navigator™ System



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Files:

Student Activity

Words_for_Algebra_Student.pdf
Words_for_Algebra_Student.doc

TI-Nspire document

Words_for_Algebra.tns

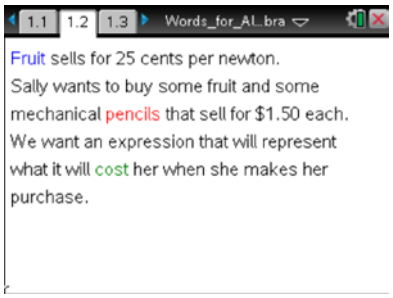
Visit www.mathnspired.com for lesson updates and tech tip videos.

- Transfer a File.
- Use Screen Capture to examine student work and responses.
- Use Live Presenter to allow the students to demonstrate their technique.
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll for Formative Assessment.

Discussion Points and Possible Answers

Teacher Tip: The goal of this activity is to teach the students how to parse the word problem in such a way that they come to know the expression and then the equation that best models the story, rather than just asking the students what the answer is. Attention to Domain and Range issues from the context of the problem and therefore the data types is a point of focus. That is, in this case, you can only have Whole numbers of pencils but the amount of fruit could be any non-negative rational number, limited only by size and accuracy of the weighing scale at the store. Some parts of the US apparently do not use SI, so you might want to explain the relationship between Newtons and Pounds (1N is about 0.23 Lbs)

Move to page 1.2.



1. Consider this problem. Discuss with your table and the class how you could represent the relationship between the variables, **fruit**, **pencil**, and **cost**. Talk about how one purchases fruit and how this is different from how pencils are purchased.

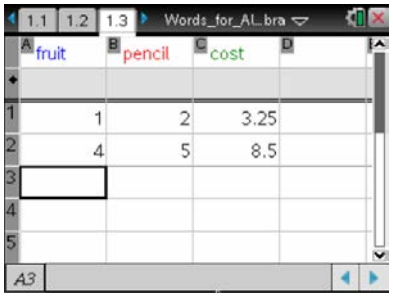
   Give one set of values for the situation.
   Fruit = _____ Pencils = _____ Cost = _____

   Sample Answers:
   Fruit = __3 N____ Pencils = __11____ Cost = _$17.25_
   [3*0.25 + 11*1.50 = 17.25]

   Fruit = __2.8 N____ Pencils = __11____ Cost = _$17.20_
   [2.8*0.25 + 11*1.50 = 17.20]

Teacher Tip: There are currently three variables in the problem (amount of fruit, number of pencils, and the total cost of the items). Once the students fully understand the relationship between the amount of fruit they buy, the number of pencils they get and the cost, we will set a total amount for the cost so that we will only be working with the number of pencils and the weight of the fruit. Note that the fruit is 25 cents a Newton, so you could get 0.77 newtons of fruit for about 19 cents. The pencils are purchased, or not, in theses amounts {0,1,2,3,…}, so the price of pencils will be some multiple of 150 cents. In addition, the students will need to round values for the total cost if they get a fraction of a cent in their calculations.

Move to page 1.3.



2. a. This spreadsheet shows some possible scenarios related to the problem. In the first row, we see that Sally buys exactly one newton of fruit and two pencils for a total cost of $3.25. What is the meaning of the second row of values?

Sample Answers: Sally buys 4 newtons of fruit and 5 pencils for a total cost of $8.50

b. Examine the rest of the rows, and then determine several other possible situations. Fill these in on the spreadsheet in the TI-Nspire Document.

Teacher Tip: Make sure the students express the quantities correctly. That is, she bought 4 newtons of fruit, not 4 pieces of fruit. You can use the formula $0.25*fruit + $1.50*pencil = cost to fill in the spreadsheet. Students will probably need to repeat the pattern many times to come up with the formula. This will vary by class and student, but the redundancy of the calculation should lead to the correct expression.

Tech Tip: Starting with an = sign will help facilitate the distinction between a variable and a number in the spreadsheet. Students might want to use the Scratchpad for their calculations. The Cost column could be calculated with the expression "0.25***fruit** + 1.50***pencil**" on the diamond line. If the students save this document and you collect it later through the TI-Nspire Navigator or C2C, the file will be tagged with their name since they will need to login to transfer the file. If they just send the file using the TI-Nspire Link, they might want to use the Save As… option and save the document with their name.

TI-Nspire Navigator Opportunity: *Screen Capture, Live Presenter, and Quick Poll*
See Note 1 at the end of this lesson.

3. Could the value in the fruit column ever be a decimal (rational number)? Could the number of pencils ever be a decimal? Explain.

Sample Answers: Yes, you could buy a half of a newton or 2.7N of fruit, according to the scale and the amount of fruit. For example 3 apples might weigh 3N or 4.5N depending on their size. No; at a store, you can't buy a fraction of a pencil.

Teacher Tip: You might need to refresh the students' memory on rational numbers and their decimal and fraction representations. Note that you will need to round up (ala Superman III) to get the amount you pay (Cost).

Tech Tip: Have the students add a Notes page and answer this question and others there.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 2 at the end of this lesson.

4. Could Sally ever spend the same amount of money for different quantities of fruit and pencils? Give a specific example of this.

Sample Answers: Yes, if you have no fruit and 1 pencil, it will cost $1.50. If you get 6N of fruit and no pencils, it will cost the same. The only trick is to watch for multiples of 25. In the equation 25*fruit + 150*pencil = cost, you could see this factor of 25 leading to: fruit + 6*pencil = cost/25.

5. What would happen if Sally bought the same number of pencils as she had newtons of fruit? Can this happen more than once. What pattern do you notice?

Answer: Beside zero items, you get the following pattern:

Fruit	Pencil	Cost
0	0	0.00
1	1	1.75
2	2	3.50
3	3	5.25
4	4	7.00
n	n	1.75n

6. If Sally buys a set number of pencils, say five, and just increases the number of newtons of fruit she buys, what pattern do you see in the cost?

Sample Answers: As the number of number of newtons of fruit increases, the cost will go up 25 cents for each additional newton, with a base cost of $7.50. cost = $0.25*fruit + $7.50

Fruit	Pencil	Cost
0	5	7.50
1	5	7.75
2	5	8.00
3	5	8.25
4	5	8.50

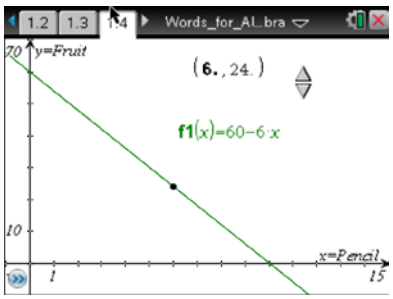
7. What if she buys 4.7 newtons of fruit and then increases the number of pencils that she gets, what pattern do you see in the cost? Write an equation to represent this situation.

Sample Answers: This is the same scenario as before, but now the base cost is $1.18 and the cost will increase by $1.50 with each pencil purchased. cost = $1.18 + $1.50*pencil.

Fruit	Pencil	Cost
4.7	0	1.18
4.7	1	2.68
4.7	2	4.18
4.7	3	5.68
4.7	4	7.18

This is an expression that you could have used to determine the cost of your purchases: $0.25*fruit + $1.50*pencil. So if Sally has $15 to spend, the equation becomes $0.25*fruit + $1.50*pencil = $15.00.

Move to page 1.4.



8. Since Sally can only buy whole pencils, we could ask how much fruit she could get if she buys "x" number of pencils, with her $15. Highlight the slider; and using the ▲ and ▼ keys, explore the patterns in the numbers where the ordered pair (6, 24) indicates 6 pencils and 24 newtons of fruit will cost $15.

Note: The rule that is being graphed. **f1**(x) = 60 – 6*x. How is this the same as the equation:
$0.25*fruit + $1.50*pencil = $15.00? What is x? What is **f1**(x)? Where is the $15.00?

Sample Answers: The pattern is the combination of whole number newtons of fruit purchased and the number of pencils that would cost a total of $15. When you solve the equation for fruit, you will subtract the cost of the pencils and then divide the amounts by the cost of 1N of fruit. The 60 in the equations occurs because there are 60 quarters in $15; in a similar vein, you have 6 quarters in the cost of one pencil. X is the number of pencils you buy. **f1**(x) is the number of newtons of fruit you buy. The $15 is represented by 60 quarters in the equation.

9. What is the meaning of the point where the line appears to cross the x-axis (Pencil)? What about the point that the line crosses the y-axis (Fruit)?

Answer: You are buying only pencils to spend your $15. You are only buying fruit with your $15.

10. What do all of the points on the graph represent, as they form the line?

Answer: They are the combinations of newtons of fruit and pencils purchased that would cost $15.00. Take note of the fact that all of the x-values (Pencils) are Whole Numbers since you cannot buy a part of a pencil.

11. Considering the limitations of the problem, give the Domain and Range of the function used to model the word problem.

Sample Answers: The Domain is the number of pencils you can buy so that, along with the fruit, the total cost is $15.00, [0.1.2….,10] Є Whole Number. The Range is the number of newtons of fruit you could buy so that, along with the pencils, the total cost is $15.00, [0,60] Є Rational Numbers.

12. Are there points on the line that are not solutions to the equation **f1**(x) = 60 – 6*x in the context of the problem [$0.25*fruit + $1.50*pencil = $15.00]? Explain.

Sample Answers: Yes, but these do not appear when using the slider. In this situation, you could buy fractional parts of newtons of fruit, but this would not work with the combinations of pencils to make the $15.00. That is, the plot should really be just the points that work for the context of the problem.

13. Develop an equation that would relate the number of newtons of fruit and the number of pencils Sally could buy for $21.75.

Answer: $0.25*fruit + $1.50*pencil = $21.75
fruit = 87 – 6*pencil

Teacher Tip: Have the students test this out with List & Spreadsheet, Graph, and Calculator pages.

TI-Nspire Navigator Opportunity: ***Screen Capture, Live Presenter, and Quick Poll***
See Note 3 at the end of this lesson.

Extension

Look at this problem from the perspective of the hot dog and bun scenario (8 hot dogs per package and 12 buns per package) and then relate this to the Limiting Reactants aspect of Stoichiometry.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- Different strategies to move from a word problem to an equation and/or expression.
- How to use a Table, Graph, and/or Calculator to develop and test their solution(s).
- When situations limit or determine the variable type of real number to be used in the solution.

TI-Nspire Navigator

Note 1

Question 2, Name of Feature: Screen Capture, Live Presenter, and Quick Poll

Take a Screen Capture of the student's List & Spreadsheet, let a student be the Student Presenter to show the class how they are filling in the rows, and/or take a Quick Poll asking what they get if you tell them two of the three variables (If you buy 5N of fruit and it costs $71.25 how many pencils did you get?).

Note 2

Question 3, Name of Feature: Screen Capture

Take a Screen Capture of the student handhelds to see their reply to this question, if placed in a Notes page in the document.

Note 3

Question 13, Name of Feature: Screen Capture, Live Presenter, and Quick Poll

Take a Screen Capture of the student handhelds to see their answers. Use the Student Presenter to have a student flesh out their solution and verification. Use a Quick Poll to collect the equations either as text or as a line graph.