Name _____ Date _____ Class _____

LESSON 12-2

Practice A

Series and Summation Notation

Use summation notation to write each series.

1. $2 + 4 + 8 + 16 + 32 + 64$

a. Find a rule for the kth term.

b. Write the notation for the first 6 terms.

2. $-6 - 5 - 4 - 3 - 2$

3. $4 + 7 + 10 + 13 + 16 + 19$

4. $2 + 8 + 18 + 32$

5. $-3 + 9 - 27 + 81 - 243$

6. $16 + 12 + 8 + 4 + 0 - 4 - 8$

7. $-1 - 4 - 9 - 16 - 25$

Expand each series. Then evaluate.

8. $\sum_{k=3}^{7} 2k^2$

a. Expand by replacing k.

b. Simplify.

9. $\sum_{k=1}^{4} \frac{2}{k}$

a. Expand.

b. Simplify.

10. $\sum_{k=5}^{10} (k - 7)$

a. Expand.

b. Simplify.

11. $\sum_{k=2}^{4} 5(2^k)$

a. Expand.

b. Simplify.

12. $\sum_{k=21}^{25} 3(20 - k)$

a. Expand.

b. Simplify.

Solve.

13. Tracy deposits $16 into her savings account each week.

a. Write a series to represent how much she will have deposited in n weeks.

b. Write a series to represent how much she will have deposited in one year.

c. How much will she have deposited in one year?

LESSON 12-2

Practice A

Series and Summation Notation

Use summation notation to write each series.

1. $2 + 4 + 8 + 16 + 32 + 64$
   a. Find a rule for the kth term. $a_k = 2^k$
   b. Write the notation for the first 6 terms. $\sum_{k=1}^{6} 2^k$
2. $-6 - 5 - 4 - 3 - 2$ $\sum_{k=1}^{5} (k - 7)$
3. $4 + 7 + 10 + 13 + 16 + 19$ $\sum_{k=1}^{6} (3k + 1)$
4. $2 + 8 + 18 + 32$ $\sum_{k=1}^{4} (2k^2)$
5. $-3 + 9 - 27 + 81 - 243$ $\sum_{k=1}^{5} (-3)^k$
6. $16 + 12 + 8 + 4 + 0 - 4 - 8$ $\sum_{k=1}^{7} (20 - 4k)$
7. $-1 - 4 - 9 - 16 - 25$ $\sum_{k=1}^{5} -(k^2)$

Expand each series. Then evaluate.

8. $\sum_{k=3}^{7} 2k^2$
   a. Expand by replacing k. $18 + 32 + 50 + 72 + 98$
   b. Simplify. 270
9. $\sum_{k=1}^{4} \frac{2}{k}$
   a. Expand. $2 + 1 + \frac{2}{3} + \frac{1}{2}$
   b. Simplify. $4\frac{1}{6}$
10. $\sum_{k=5}^{10} (k - 7)$
    a. Expand. $-2 - 1 + 0 + 1 + 2 + 3$
    b. Simplify. 3
11. $\sum_{k=2}^{4} 5(2^k)$
    a. Expand. $20 + 40 + 80$
    b. Simplify. 140
12. $\sum_{k=21}^{25} 3(20 - k)$
    a. Expand. $-3 - 6 - 9 - 12 - 15$
    b. Simplify. -45

Solve.

13. Tracy deposits \$16 into her savings account each week.
    a. Write a series to represent how much she will have deposited in n weeks. $\sum_{k=1}^{n} 16$
    b. Write a series to represent how much she will have deposited in one year. $\sum_{k=1}^{52} 16$
    c. How much will she have deposited in one year? \$832

LESSON 12-2

Practice B

Series and Summation Notation

Write each series in summation notation.

1. $-2 + 4 - 8 + 16 - 32$ $\sum_{k=1}^{5} (-2)^k$
2. $\frac{1}{10} + \frac{1}{100} + \frac{1}{1{,}000} + \frac{1}{10{,}000}$ $\sum_{k=1}^{4} \left(\frac{1}{10}\right)^k$
3. $-6 - 1 + 4 + 9 + 14 + 19$ $\sum_{k=1}^{6} (5k - 11)$
4. $\frac{1}{3} + \frac{1}{6} + \frac{1}{9} + \frac{1}{12} + \frac{1}{15} + \frac{1}{18}$ $\sum_{k=1}^{6} \frac{1}{3k}$
5. $7 + 13 + 19 + 25 + 31$ $\sum_{k=1}^{5} (6k + 1)$
6. $-1 + 1 - 1 + 1 - 1 + 1 - 1$ $\sum_{k=1}^{7} (-1)^k$

Expand each series and evaluate.

7. $\sum_{k=4}^{8} \frac{k}{4}$
   a. Expand. $1 + \frac{5}{4} + \frac{6}{4} + \frac{7}{4} + 2$
   b. Simplify. $7\frac{1}{2}$
8. $\sum_{k=1}^{4} 5^{k-2}$
   a. Expand. $\frac{1}{5} + 1 + 5 + 25$
   b. Simplify. $31\frac{1}{5}$
9. $\sum_{k=2}^{6} (-2^k)$
   a. Expand. $-4 - 8 + 16 - 32 + 64$
   b. Simplify. 44
10. $\sum_{k=30}^{39} (70 - 2k)$
    a. Expand. $10 + 8 + 6 + 4 + 2 + 0 - 2 - 4 - 6 - 8$
    b. Simplify. 10

Evaluate each series.

11. $\sum_{k=12}^{20} 3$ 27
12. $\sum_{k=1}^{40} k$ 820
13. $\sum_{k=1}^{10} k^2$ 385

Solve.

14. One day, Hannah starts a new online Internet club by convincing two of her friends to join. The next day, each member convinces two more people to join. The third day of the club, each member convinces two more people to join, and so on for a full week.
    a. Write a series that represents the number of club members at the end of n days. $\sum_{k=1}^{n} 3^k$
    b. Write a series that represents the number of club members at the end of one week. $\sum_{k=1}^{7} 3^k$
    c. How many members will the club have at the end of a week? 3279

LESSON 12-2

Practice C

Series and Summation Notation

Write each series in summation notation.

1. $6 + 12 + 24 + 48 + 96$ $\sum_{k=1}^{5} 3(2^k)$
2. $\frac{3}{2} + \frac{3}{4} + \frac{1}{2} + \frac{3}{8} + \frac{3}{10} + \frac{1}{4} + \frac{3}{14}$ $\sum_{k=1}^{7} \frac{3}{2k}$
3. $1 - 1 + 1 - 1 + 1 - 1 + 1$ $\sum_{k=1}^{7} (-1)^{k-1}$
4. $10 + 1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000}$ $\sum_{k=1}^{5} \left(\frac{1}{10}\right)^{k-2}$
5. $-\frac{1}{4} + 2 + \frac{23}{4} + 11 + \frac{71}{4} + 26$ $\sum_{k=1}^{6} \left(\frac{3}{4}k^2 - 1\right)$
6. $0 + 2 + 6 + 12 + 20 + 30$ $\sum_{k=1}^{6} (k^2 - k)$

Evaluate each series.

7. $\sum_{k=2}^{5} \frac{k+1}{k-2}$ $8\frac{1}{6}$
8. $\sum_{k=4}^{8} (0.5k^2 - 3)$ 80
9. $\sum_{k=2}^{6} (-3)^{k-4}$ $6\frac{5}{9}$
10. $\sum_{k=7}^{22} 8.1$ 129.6
11. $\sum_{k=1}^{45} k$ 1035
12. $\sum_{k=1}^{30} k^2$ 9455

Solve.

13. Kate is unpacking boxes of books and arranging them on her new bookcases. She places 5 books on the top shelf. Each shelf contains 4 more books than the shelf above it. There are 6 shelves.
    a. How many books are on the bottom shelf? 25 books
    b. How many books are in the bookcase? 90 books
14. After a 20-minute jog on his treadmill, Trent cools down by gradually slowing until he reaches 0.5 mile per hour, the slowest speed on his treadmill. At minute 21, he moves his speed down 0.1 mile per hour. At minute 22, he moves his speed down 0.2 mile per hour. At minute 23, he moves his speed down 0.3 mile per hour. Each minute of cooldown his speed is reduced by 0.1 mile more than the previous minute.
    a. Write a series representing the total reduction in speed at minute 30. $\sum_{k=21}^{30} \frac{(k - 20)}{10}$
    b. By how much has he reduced his speed after 10 minutes? 5.5 miles per hour

LESSON 12-2

Reteach

Series and Summation Notation

A series is the sum of the terms of a sequence.
Summation notation is used to represent a series.

The Greek letter sigma, or Σ, denotes the sum.

$\sum_{k=1}^{n} f(k)$

Last value for k

Sequence rule

First value for k

To evaluate a series, expand the series by writing the terms in the sequence up to the nth term. Then add to find the sum.

$k = 1$ $k = 2$ $k = 3$ $k = 4$ $k = 5$

$$\sum_{k=1}^{5} (k^2 + 1) = (1^2 + 1) + (2^2 + 1) + (3^2 + 1) + (4^2 + 1) + (5^2 + 1)$$
$$= 2 + 5 + 10 + 17 + 26$$
$$= 60$$

To write the terms, substitute the whole numbers from 1 through 5 for k.

Simplify. Then add to evaluate.

$$\sum_{k=2}^{4} -2(3)^k = -2(3)^2 + [-2(3)^3] + [-2(3)^4]$$
$$= -2(9) - 2(27) - 2(81)$$
$$= -18 - 54 - 162 = -234$$

This series starts with $k = 2$ and ends with $k = 4$.

Expand each series and evaluate.

1. $\sum_{k=2}^{5} (4k - 1) = (4(2) - 1) + (4(3) - 1) + (4(4) - 1) + (4(5) - 1)$
   $= 7 + 11 + 15 + 19$
   $= 52$
2. $\sum_{k=1}^{4} \frac{1}{k} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$
   $= \frac{12}{12} + \frac{6}{12} + \frac{4}{12} + \frac{3}{12}$
   $= \frac{25}{12} = 2\frac{1}{12}$
3. $\sum_{k=1}^{4} (5)^{k-1} = (5)^{1-1} + (5)^{2-1} + (5)^{3-1} + (5)^{4-1}$
   $= 1 + 5 + 25 + 125$
   $= 156$
4. $\sum_{k=1}^{3} (2^k + 1)$
   $(2^1 + 1) + (2^2 + 1) + (2^3 + 1)$
   $= 3 + 5 + 9 = 17$
5. $\sum_{k=3}^{5} (2k^2 - 1)$
   $[2(3^2) - 1] + [2(4^2) - 1] + [2(5^2) - 1]$
   $= 17 + 31 + 49 = 97$