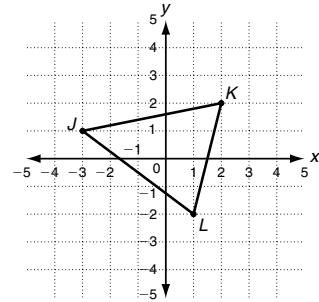


LESSON
4-3

Practice B

Using Matrices to Transform Geometric Figures

Triangle *JKL* has vertices *J*(−3, 1), *K*(2, 2), and *L*(1, −2).

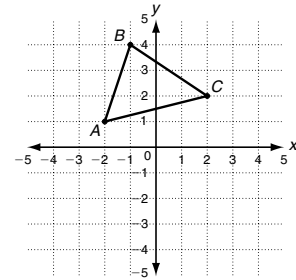


Use a matrix to transform triangle *JKL*. Find the coordinates of the vertices of the image.

1. Translate 5 units right, 6 units down.
2. Translate 2 units left, 4 units up.

3. Enlarge by a factor of 7.
4. Reduce by a factor of 0.25.

Reflect or rotate triangle *ABC* with vertices *A*(−2, 1), *B*(−1, 4), and *C*(2, 2). Find the coordinates of the vertices of the image. Describe the transformation.



5. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

6. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

7. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

8. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Solve.

9. a. Natalie drew a figure with vertices *H*(−3, −2), *O*(−3, 3), *U*(0, 5), *S*(3, 3), *E*(3, −2) to use as a pattern on a sweatshirt. Write a matrix that defines the figure.

- b. Natalie wants to enlarge the figure by a factor of 5. Describe a method she can use.

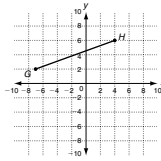
- c. What are the coordinates of Natalie's enlarged figure?

H' _____ *O'* _____ *U'* _____ *S'* _____ *E'* _____

LESSON Practice A

4-3 Using Matrices to Transform Geometric Figures

Line segment GH has endpoints $G(-7, 2)$ and $H(4, 6)$. Use line segment GH for Exercises 1–6.



Use a matrix to transform line segment GH . Find the coordinates of the image endpoints $G'H'$.

- Translate 2 units right and 8 units down.
- Translate 5 units right and 1 unit up.
- Translate 6 units left and 3 units down.

$$\begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ -8 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} -7+2 & 4+2 \\ 2+(-8) & 6+(-8) \end{bmatrix}$$

$$\begin{matrix} G'(-5, -6), & H'(6, -2) \\ G'(-2, 3), & H'(9, 7) \end{matrix}$$

$$\begin{matrix} G'(-13, -1), \\ H'(-2, 3) \end{matrix}$$

- Enlarge by a factor of 8.
- Enlarge by a factor of 5.
- Reduce by a factor of 0.5.

$$8 \begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 8(-7) & 8(4) \\ 8(2) & 8(6) \end{bmatrix}$$

$$\begin{matrix} G'(-56, 16), & H'(32, 48) \\ G'(-35, 10), & H'(20, 30) \\ G'(-3.5, 1), & H'(2, 3) \end{matrix}$$

Use each matrix to reflect the given point. Write the coordinates of the image. Tell which axis the point is reflected across.

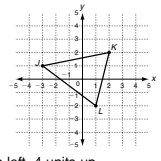
- $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$; $(2, -3)$
 - $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$; $(-10, 1)$
 - $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$; $(5, 4)$
- $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -1(2) + 0(-3) \\ 0(2) + 1(-3) \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$; y -axis
 $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -10 \\ 1 \end{bmatrix} = \begin{bmatrix} 1(-10) + 0(1) \\ 0(-10) + (-1)(1) \end{bmatrix} = \begin{bmatrix} -10 \\ -1 \end{bmatrix}$; x -axis
 $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix} = \begin{bmatrix} -1(5) + 0(4) \\ 0(5) + 1(4) \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$; y -axis

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LESSON Practice B

4-3 Using Matrices to Transform Geometric Figures

Triangle JKL has vertices $J(-3, 1)$, $K(2, 2)$, and $L(1, -2)$.



Use a matrix to transform triangle JKL . Find the coordinates of the vertices of the image.

- Translate 5 units right, 6 units down.
- Translate 2 units left, 4 units up.

$$J'(2, -5), K'(7, -4), L'(6, -8)$$

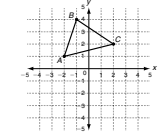
$$J'(-5, 5), K'(0, 6), L'(-1, 2)$$

- Enlarge by a factor of 7.
- Reduce by a factor of 0.25.

$$J'(-21, 7), K'(14, 14), L'(7, -14)$$

$$J'(-0.75, 0.25), K'(0.5, 0.5), L'(0.25, -0.5)$$

Reflect or rotate triangle ABC with vertices $A(-2, 1)$, $B(-1, 4)$, and $C(2, 2)$. Find the coordinates of the vertices of the image. Describe the transformation.



$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\begin{matrix} A'(2, 1), B'(1, 4), C'(-2, 2); \\ \text{reflection across the } y\text{-axis} \end{matrix}$$

$$\begin{matrix} A'(1, 2), B'(4, 1), C'(2, -2); \\ 90^\circ \text{ clockwise rotation} \end{matrix}$$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{matrix} A'(-1, -2), B'(-4, -1), \\ C'(-2, 2); 90^\circ \text{ counterclockwise} \\ \text{rotation} \end{matrix}$$

$$\begin{matrix} A'(-2, -1), B'(-1, -4), \\ C'(2, -2); \text{reflection across the } \\ x\text{-axis} \end{matrix}$$

Solve.

- Natalie drew a figure with vertices $H(-3, -2)$, $O(-3, 3)$, $U(0, 5)$, $S(3, 3)$, $E(3, -2)$ to use as a pattern on a sweatshirt. Write a matrix that defines the figure.

-3	-3	0	3	3
-2	3	5	3	-2
- Natalie wants to enlarge the figure by a factor of 5. Describe a method she can use.

Multiply each entry in the matrix by 5.
- What are the coordinates of Natalie's enlarged figure?

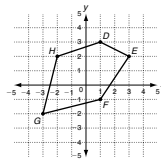
$$H'(-15, -10), O'(-15, 15), U'(0, 25), S'(15, 15), E'(15, -10)$$

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LESSON Practice C

4-3 Using Matrices to Transform Geometric Figures

Use a matrix to transform figure $DEFGH$ with coordinates $D(1, 3)$, $E(3, 2)$, $F(1, -1)$, $G(-3, -2)$, and $H(-2, 2)$. Give the transformation matrix or scalar and the coordinates of the image.



- Translate 9 units left and 4 units up.

$$\begin{bmatrix} -9 & -9 & -9 & -9 & -9 \\ 4 & 4 & 4 & 4 & 4 \end{bmatrix}$$

$$D'(-8, 7), E'(-6, 6), F'(-8, 3), G'(-12, 2), H'(-11, 6)$$

- Reduce by a factor of 0.1.

$$\text{scalar } 0.1; D'(0.1, 0.3), E'(0.3, 0.2), F'(0.1, -0.1), G'(-0.3, -0.2), H'(-0.2, 0.2)$$

- Rotation 90° clockwise

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}; D'(3, -1), E'(2, -3), F'(-1, -1), G'(-2, 3), H'(2, 2)$$

- Rotation 90° counterclockwise

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}; D'(-3, 1), E'(-2, 3), F'(1, 1), G'(2, -3), H'(-2, -2)$$

- Reflection across the x -axis

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}; D'(1, -3), E'(3, -2), F'(1, 1), G'(-3, 2), H'(-2, -2)$$

- Reflection across the y -axis

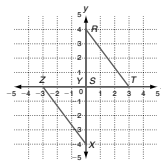
$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}; D'(-1, 3), E'(-3, 2), F'(-1, -1), G'(3, -2), H'(2, 2)$$

- Reflection across $y = x$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; D'(3, 1), E'(2, 3), F'(-1, 1), G'(-2, -3), H'(2, -2)$$

Solve.

- Yung Li drew triangle RST with coordinates $R(0, 4)$, $S(0, 0)$, and $T(3, 0)$. Then she drew triangle XYZ with coordinates $X(0, -4)$, $Y(0, 0)$, and $Z(-3, 0)$.



- Graph triangles RST and XYZ .
- Write a coordinate matrix to represent each triangle.

$$\begin{bmatrix} 0 & 0 & 3 \\ 4 & 0 & 0 \end{bmatrix}; \begin{bmatrix} 0 & 0 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

- Use the matrices to show the transformation of triangle RST into triangle XYZ .

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 3 \\ 4 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

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LESSON Reteach

4-3 Using Matrices to Transform Geometric Figures

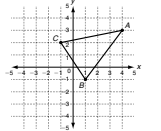
A matrix can define a polygon in the coordinate plane.

Vertices of $\triangle ABC$:

$$A(4, 3), B(1, -1), C(-1, 2)$$

Write each pair of coordinates in a column.

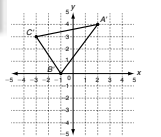
$$\text{Matrix for } \triangle ABC: \begin{bmatrix} 4 & 1 & -1 \\ 3 & -1 & 2 \end{bmatrix} \begin{matrix} x\text{-coordinates} \\ y\text{-coordinates} \end{matrix}$$



To translate $\triangle ABC$ 2 units left and 1 unit up, add a translation matrix to the matrix for $\triangle ABC$.

$$\text{Translation matrix: } \begin{bmatrix} -2 & -2 & -2 \\ 1 & 1 & 1 \end{bmatrix} \begin{matrix} \text{The } x\text{-coordinates are} \\ \text{translated 2 units left.} \end{matrix}$$

$$\begin{matrix} \text{The } y\text{-coordinates are} \\ \text{translated 1 unit up.} \end{matrix}$$



Add the matrices to find the vertices of the translated image.

$$\begin{bmatrix} 4 & 1 & -1 \\ 3 & -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -3 \\ 4 & 0 & 3 \end{bmatrix}$$

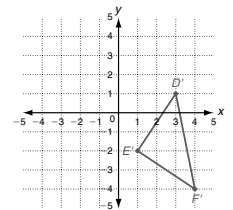
Translated image, $A'(2, 4)$, $B'(-1, 0)$, $C'(-3, 3)$.

Solve.

- $\triangle DEF$ has vertices $D(0, 3)$, $E(-2, 0)$, and $F(1, -2)$. Write the matrix for $\triangle DEF$.
- Write the translation matrix to translate $\triangle DEF$ 3 units right and 2 units down.
- Add the matrices to find the coordinates of the vertices of the image $\triangle D'E'F'$. Then graph $\triangle D'E'F'$.

$$\begin{bmatrix} 3 & 1 & 4 \\ 1 & -2 & -4 \end{bmatrix}; D'(3, 1), E'(1, -2), F'(4, -4)$$

$$\begin{bmatrix} 0 & -2 & 1 \\ 3 & 0 & -2 \end{bmatrix}; \begin{bmatrix} 3 & 3 & 3 \\ -2 & -2 & -2 \end{bmatrix}$$



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