Name _____ Date _____ Class _____

LESSON 4-4

Practice B

Determinants and Cramer's Rule

Find the determinant of each matrix.

1. $\begin{bmatrix} 8 & 2 \\ 4 & -1 \end{bmatrix}$

2. $\begin{bmatrix} -6 & 3 \\ 9 & -5 \end{bmatrix}$

3. $\begin{bmatrix} -2 & 8 \\ -3 & 7 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 0 & -1 \\ 5 & -2 & 0 \\ 1 & 6 & 2 \end{bmatrix}$

5. $\begin{bmatrix} 0 & -4 & 5 \\ 2 & 4 & 3 \\ 1 & 1 & -1 \end{bmatrix}$

6. $\begin{bmatrix} -4 & 3 & 1 \\ 7 & -2 & 0 \\ 1 & -1 & 2 \end{bmatrix}$

Use Cramer's rule to solve each system of equations.

7. $\begin{cases} 2x + 3y = -1 \\ 3x + 2y = 16 \end{cases}$

8. $\begin{cases} 4x - 3y = 9 \\ 3x + 2y = 28 \end{cases}$

9. $\begin{cases} 8x - 3y = 20 \\ 3x - 2y = 11 \end{cases}$

10. $\begin{cases} 4y = -5x + 33 \\ 2y = 3x - 11 \end{cases}$

11. $\begin{cases} 27 + 4y = 3x \\ y = \frac{1}{3}x - 8 \end{cases}$

12. $\begin{cases} 7 - 5y + 4x = 0 \\ 16 - 2y - 5x = 0 \end{cases}$

Solve.

13. On Monday, Marla babysat for 4 hours, did yard work for 2 hours, and earned a total of \$41. On Friday, she babysat for 5 hours, did yard work for 3 hours, and earned a total of \$55.

a. Write a system of equations. Let x = Marla's hourly rate for babysitting, and y = her hourly rate for yard work.

b. Write the coefficient matrix. Evaluate its determinant.

c. Use Cramer's rule to find x and y.

d. What is Marla's hourly rate for each activity?

LESSON 4-4

Practice A

Determinants and Cramer's Rule

Find the determinant of each matrix.

1. $\begin{bmatrix} 6 & -2 \\ 1 & 10 \end{bmatrix}$

$= 6(\underline{10}) - (\underline{1})(\underline{-2})$

$= 60 - (-2) = 62$

2. $\begin{bmatrix} 3 & -1 \\ -7 & 2 \end{bmatrix}$

-1

3. $\begin{bmatrix} 2 & 9 \\ 1 & -3 \end{bmatrix}$

-15

4. $\begin{bmatrix} 5 & 6 & -1 \\ -3 & 2 & 0 \\ 2 & -3 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 6 & -1 & 5 & 6 \\ -3 & 2 & 0 & -3 & 2 \\ 2 & -3 & 4 & 2 & -3 \end{bmatrix} \rightarrow$ (5·2·4) + (6·0·2) + (−1·−3·−3) − (2·2·−1) − (−3·0·5) − (4·−3·6)

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Use Cramer's rule to solve each system of equations.

5. $\begin{cases} x - 2y = -9 \\ 3x + y = 1 \end{cases}$

a. Write the coefficient matrix. $\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$

b. Find D, the determinant of the coefficient matrix. 7

c. Use Cramer's rule to write the solutions for x and y.

$x = \dfrac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{D} = \dfrac{\begin{vmatrix} \underline{-9} & \underline{-2} \\ \underline{1} & \underline{1} \end{vmatrix}}{\underline{7}}$ $\qquad y = \dfrac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{D} = \dfrac{\begin{vmatrix} \underline{1} & \underline{-9} \\ \underline{3} & \underline{1} \end{vmatrix}}{\underline{7}}$

d. Evaluate the determinants in the numerators and solve for x and y. $x = -1;\ y = 4$

6. $\begin{cases} 2x + 3y = 4 \\ x - 2y = 9 \end{cases}$ $x = 5;\ y = -2$

7. $\begin{cases} 3x + y = 5 \\ 2x - 3y = 18 \end{cases}$ $x = 3;\ y = -4$

8. $\begin{cases} x + 5y = 11 \\ 2x - 3y = 9 \end{cases}$ $x = 6;\ y = 1$

LESSON 4-4

Practice B

Determinants and Cramer's Rule

Find the determinant of each matrix.

1. $\begin{bmatrix} 8 & 2 \\ 4 & -1 \end{bmatrix}$ -16

2. $\begin{bmatrix} -6 & 3 \\ 9 & -5 \end{bmatrix}$ 3

3. $\begin{bmatrix} -2 & 8 \\ -3 & 7 \end{bmatrix}$ 10

4. $\begin{bmatrix} 1 & 0 & -1 \\ 5 & -2 & 0 \\ 1 & 6 & 2 \end{bmatrix}$ -36

5. $\begin{bmatrix} 0 & -4 & 5 \\ 2 & 4 & 3 \\ 1 & 1 & -1 \end{bmatrix}$ -30

6. $\begin{bmatrix} -4 & 3 & 1 \\ 7 & -2 & 0 \\ 1 & -1 & 2 \end{bmatrix}$ -31

Use Cramer's rule to solve each system of equations.

7. $\begin{cases} 2x + 3y = -1 \\ 3x + 2y = 16 \end{cases}$ $(10, -7)$

8. $\begin{cases} 4x - 3y = 9 \\ 3x + 2y = 28 \end{cases}$ $(6, 5)$

9. $\begin{cases} 8x - 3y = 20 \\ 3x - 2y = 11 \end{cases}$ $(1, -4)$

10. $\begin{cases} 4y = -5x + 33 \\ 2y = 3x - 11 \end{cases}$ $(5, 2)$

11. $\begin{cases} 27 + 4y = 3x \\ y = \frac{1}{3}x - 8 \end{cases}$ $(-3, -9)$

12. $\begin{cases} 7 - 5y + 4x = 0 \\ 16 - 2y - 5x = 0 \end{cases}$ $(2, 3)$

Solve.

13. On Monday, Marla babysat for 4 hours, did yard work for 2 hours, and earned a total of $41. On Friday, she babysat for 5 hours, did yard work for 3 hours, and earned a total of $55.

a. Write a system of equations. Let x = Marla's hourly rate for babysitting, and y = her hourly rate for yard work. $\begin{cases} 4x + 2y = 41 \\ 5x + 3y = 55 \end{cases}$

b. Write the coefficient matrix. Evaluate its determinant. $\begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix}$; $\det = \begin{vmatrix} 4 & 2 \\ 5 & 3 \end{vmatrix} = 2$

c. Use Cramer's rule to find x and y. $x = 6.5;\ y = 7.5$

d. What is Marla's hourly rate for each activity? Babysitting: $6.50, yard work: $7.50

LESSON 4-4

Practice C

Determinants and Cramer's Rule

Find the determinant of each matrix.

1. $\begin{bmatrix} 12 & 5 \\ -14 & -3 \end{bmatrix}$ 34

2. $\begin{bmatrix} -6 & -1 & -2 \\ 2 & 5 & 0 \\ 4 & 3 & 1 \end{bmatrix}$ 0

3. $\begin{bmatrix} 2 & 4 & -1 \\ 0 & 3 & -3 \\ 1 & 0 & 6 \end{bmatrix}$ 27

Use Cramer's rule to solve each system of equations.

4. $\begin{cases} 4x - 3y = 3 \\ -3x + 2y = -1 \end{cases}$ $(-3, -5)$

5. $\begin{cases} 5x - 4y = 22 \\ 4x + 3y = -1 \end{cases}$ $(2, -3)$

6. $\begin{cases} 6x - 7y = -11 \\ 5x + 4y = 40 \end{cases}$ $(4, 5)$

7. $\begin{cases} 8x - 5y = 61 \\ 3x + 4y = 17 \end{cases}$ $(7, -1)$

8. $\begin{cases} x - 6y = 21 \\ 3x + 5y = 17 \end{cases}$ $(9, -2)$

9. $\begin{cases} 5x - 6y = -2 \\ 4x - 5y = -3 \end{cases}$ $(8, 7)$

10. $\begin{cases} 3x - 2y + 4z = 0 \\ 6x + 5y - 3z = 7 \\ 5x + 3y + 5z = 11 \end{cases}$ $(0, 2, 1)$

11. $\begin{cases} 4x - 2y + z = -6 \\ 3x + 3y + 5z = -8 \\ 2x - 4y - 3z = 2 \end{cases}$ $(1, 3, -4)$

12. $\begin{cases} -2x + 6y + 3z = -10 \\ 5x - 5y - 4z = 9 \\ 3x + 2y = 0 \end{cases}$ $(2, -3, 4)$

Solve.

13. Travis invested $20,000 in two simple interest accounts. He invested part at 4.5% interest and the rest at 3.5% interest. He earned $785 in total interest per year.

a. Write the problem as a system of equations. $\begin{cases} x + y = 20{,}000 \\ 0.045x + 0.035y = 785 \end{cases}$

b. Find the value of the determinant of the coefficient matrix. -0.01

c. Use Cramer's rule to write the solution for the amount Travis invested at 4.5%. $\dfrac{\begin{vmatrix} 20{,}000 & 1 \\ 785 & .035 \end{vmatrix}}{-.01}$

d. How much did Travis invest at 4.5% interest? $8500

LESSON 4-4

Reteach

Determinants and Cramer's Rule

A **square matrix** has the same number of rows as columns. The **determinant** of a square matrix is shown by $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$.

To find the determinant of a 2×2 matrix, find the product of each diagonal, beginning at the upper left corner. Then subtract.

$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$

$\det \begin{bmatrix} 2 & 3 \\ 5 & 9 \end{bmatrix} = \begin{vmatrix} 2 & 3 \\ 5 & 9 \end{vmatrix} = 2(9) - 5(3) = 18 - 15 = 3$

Vertical brackets indicate a determinant.

Find the determinant of each matrix.

1. $\det \begin{bmatrix} -1 & 2 \\ -5 & 4 \end{bmatrix} = \begin{vmatrix} -1 & 2 \\ -5 & 4 \end{vmatrix} = -1(4) - (-5)(2) = 6$

2. $\det \begin{bmatrix} \frac{3}{2} & -\frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix} = \begin{vmatrix} \frac{3}{2} & -\frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} \end{vmatrix} = \frac{3}{2}\left(\frac{1}{4}\right) - \left(\frac{1}{2}\right)\left(-\frac{1}{4}\right) = \frac{1}{2}$

3. $\det \begin{bmatrix} -3 & -4 \\ -1 & -6 \end{bmatrix} = \begin{vmatrix} -3 & -4 \\ -1 & -6 \end{vmatrix} = -3(\underline{-6}) - (-1)(\underline{-4}) = 14$

4. $\det \begin{bmatrix} -2.4 & 0.5 \\ 1.2 & 2 \end{bmatrix} = \begin{vmatrix} -2.4 & 0.5 \\ 1.2 & 2 \end{vmatrix} = -5.4$

5. $\det \begin{bmatrix} \frac{1}{6} & 9 \\ \frac{2}{3} & -12 \end{bmatrix} = \begin{vmatrix} \frac{1}{6} & 9 \\ \frac{2}{3} & -12 \end{vmatrix} = -8$

6. $\det \begin{bmatrix} 8 & \frac{2}{5} \\ -15 & \frac{3}{4} \end{bmatrix} = \begin{vmatrix} 8 & \frac{2}{5} \\ -15 & \frac{3}{4} \end{vmatrix} = 12$