Name _____ Date _____ Class _____

LESSON 4-5

Practice B

Matrix Inverses and Solving Systems

Determine whether the given matrices are inverses.

1. $\begin{bmatrix} -5 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} -0.2 & 0 \\ 0.8 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & -4 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -0.6 & -0.8 \\ -0.4 & -0.2 \end{bmatrix}$

3. $\begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ -1 & -2 \end{bmatrix}$

_____ _____ _____

Find the inverse of the matrix, if it is defined.

4. $\begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}$

5. $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$

6. $\begin{bmatrix} 8 & 4 \\ -5 & -3 \end{bmatrix}$

_____ _____ _____

7. $\begin{bmatrix} 3 & -3 \\ -2 & 1 \end{bmatrix}$

8. $\begin{bmatrix} -4 & 4 \\ 5 & -4 \end{bmatrix}$

9. $\begin{bmatrix} 6 & -6 \\ 1 & -1 \end{bmatrix}$

_____ _____ _____

Write the matrix equation for the system, and solve.

10. $\begin{cases} 3x + 2y = -5 \\ 4x + 3y = -9 \end{cases}$

11. $\begin{cases} -6x + 4y = 8 \\ 5x - 3y = -5 \end{cases}$

_____ _____

12. $\begin{cases} 4x + 5y = 0 \\ 5x + 3y = 13 \end{cases}$

13. $\begin{cases} 5x - 3y = 8 \\ 6x - 5y = 4 \end{cases}$

_____ _____

Solve.

14. Keith paid \$39 for 3 pounds of pistachios and 2 pounds of cashews. Tracey paid \$23 for 2 pounds of pistachios and 1 pound of cashews.

a. Write a system of equations. Let x = the cost of a pound of pistachios, and y = the cost of a pound of cashews. _____

b. Write the matrix equation and solve. _____

LESSON 4-5

Practice A

Matrix Inverses and Solving Systems

Multiply the matrices two ways to determine if they are inverses.

1. $\begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$ and $\begin{bmatrix} -2 & -1 \\ -3 & -1 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 & -1 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} -2 & -1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$

Yes

2. $\begin{bmatrix} 6 & 5 \\ -2 & -2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2.5 \\ -1 & -3 \end{bmatrix}$

Yes

Find the inverse of the matrix, if it is defined. First find the determinant, and then find the inverse.

3. $\begin{bmatrix} 2 & 5 \\ 2 & 4 \end{bmatrix}$

$D = -2$; $\begin{bmatrix} -2 & \frac{5}{2} \\ 1 & -1 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$

$D = -10$; $\begin{bmatrix} -0.2 & 0.3 \\ 0.4 & -0.1 \end{bmatrix}$

5. $\begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$

$D = -2$; $\begin{bmatrix} 0 & \frac{1}{2} \\ 1 & \frac{3}{2} \end{bmatrix}$

6. $\begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}$

$D = -4$; $\begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{3}{4} \end{bmatrix}$

Write the matrix equation $AX = B$ for each system. Then solve.

7. $\begin{cases} 5x + 3y = -12 \\ 2x + 2y = -4 \end{cases}$

a. Matrix equation $\begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -12 \\ 4 \end{bmatrix}$

b. A^{-1} $\begin{bmatrix} \frac{1}{2} & -\frac{3}{4} \\ -\frac{1}{2} & \frac{5}{4} \end{bmatrix}$

c. Solve for x and y. $(-3, 1)$

8. $\begin{cases} 3x - 2y = 4 \\ 2x - 3y = 11 \end{cases}$ $\begin{bmatrix} 3 & -2 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 11 \end{bmatrix}$; $(-2, -5)$

9. $\begin{cases} 3x + y = 3 \\ 2x + 3y = 9 \end{cases}$ $\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \end{bmatrix}$; $(0, 3)$

10. $\begin{cases} 2x + 3y = -1 \\ x + 4y = -8 \end{cases}$ $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -8 \end{bmatrix}$; $(4, -3)$

LESSON 4-5

Practice B

Matrix Inverses and Solving Systems

Determine whether the given matrices are inverses.

1. $\begin{bmatrix} -5 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} -0.2 & 0 \\ 0.8 & 1 \end{bmatrix}$ **Yes**

2. $\begin{bmatrix} 1 & -4 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -0.6 & -0.8 \\ -0.4 & -0.2 \end{bmatrix}$ **Yes**

3. $\begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ -1 & -2 \end{bmatrix}$ **Yes**

Find the inverse of the matrix, if it is defined.

4. $\begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}$

5. $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$ $\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$

6. $\begin{bmatrix} 8 & 4 \\ -5 & -3 \end{bmatrix}$ $\begin{bmatrix} \frac{3}{4} & 1 \\ -\frac{5}{4} & -2 \end{bmatrix}$

7. $\begin{bmatrix} 3 & -3 \\ -2 & 1 \end{bmatrix}$ $\begin{bmatrix} -\frac{1}{3} & -1 \\ -\frac{2}{3} & -1 \end{bmatrix}$

8. $\begin{bmatrix} -4 & 4 \\ 5 & -4 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ \frac{5}{4} & 1 \end{bmatrix}$

9. $\begin{bmatrix} 6 & -6 \\ 1 & -1 \end{bmatrix}$ **The inverse does not exist.**

Write the matrix equation for the system, and solve.

10. $\begin{cases} 3x + 2y = -5 \\ 4x + 3y = -9 \end{cases}$ $\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ -9 \end{bmatrix}$; $(3, -7)$

11. $\begin{cases} -6x + 4y = 8 \\ 5x - 3y = -5 \end{cases}$ $\begin{bmatrix} -6 & 4 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$; $(2, 5)$

12. $\begin{cases} 4x + 5y = 0 \\ 5x + 3y = 13 \end{cases}$ $\begin{bmatrix} 4 & 5 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 13 \end{bmatrix}$; $(5, -4)$

13. $\begin{cases} 5x - 3y = 8 \\ 6x - 5y = 4 \end{cases}$ $\begin{bmatrix} 5 & -3 \\ 6 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$; $(4, 4)$

Solve.

14. Keith paid \$39 for 3 pounds of pistachios and 2 pounds of cashews. Tracey paid \$23 for 2 pounds of pistachios and 1 pound of cashews.

a. Write a system of equations. Let x = the cost of a pound of pistachios, and y = the cost of a pound of cashews. $\begin{cases} 3x + 2y = 39 \\ 2x + y = 23 \end{cases}$

b. Write the matrix equation and solve. $\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 29 \\ 23 \end{bmatrix}$; pistachios: \$7 per pound, cashews: \$9 per pound

LESSON 4-5

Practice C

Matrix Inverses and Solving Systems

Find the inverse of the coefficient matrix for the system and solve.

1. $\begin{cases} 4x + 7y = 11 \\ 2x + 5y = 1 \end{cases}$ $\begin{bmatrix} \frac{5}{6} & -\frac{7}{6} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$; $(8, -3)$

2. $\begin{cases} 21x - 5y = 5 \\ 13x - 3y = 3 \end{cases}$ $\begin{bmatrix} -\frac{3}{2} & \frac{5}{2} \\ -\frac{13}{2} & \frac{21}{2} \end{bmatrix}$; $(0, -1)$

3. $\begin{cases} 2x - 7y = -4 \\ -3x + 11y = 6 \end{cases}$ $\begin{bmatrix} 11 & 7 \\ 3 & 2 \end{bmatrix}$; $(-2, 0)$

4. $\begin{cases} 3x + 8y = 10 \\ 2x + 7y = 5 \end{cases}$ $\begin{bmatrix} \frac{7}{5} & -\frac{8}{5} \\ -\frac{2}{5} & \frac{3}{5} \end{bmatrix}$; $(6, -1)$

5. $\begin{cases} 4x + 3y = 3 \\ 9x + 5y = -2 \end{cases}$ $\begin{bmatrix} -\frac{5}{7} & \frac{3}{7} \\ \frac{9}{7} & -\frac{4}{7} \end{bmatrix}$; $(-3, 5)$

6. $\begin{cases} 2x + 7y = -5 \\ 3x + 5y = 9 \end{cases}$ $\begin{bmatrix} -\frac{5}{11} & \frac{7}{11} \\ \frac{3}{11} & -\frac{2}{11} \end{bmatrix}$; $(8, -3)$

Use the integers 3, −4, −6, and 8.

7. Create a matrix whose inverse is undefined. Possible answer: $\begin{bmatrix} 3 & -4 \\ -6 & 8 \end{bmatrix}$

8. Create a matrix whose inverse is defined. Possible answer: $\begin{bmatrix} 3 & 8 \\ -6 & -4 \end{bmatrix}$

Solve.

9. Matrix $A = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$. The determinant of A is -2 and A^{-1} is $\begin{bmatrix} \frac{5}{2} & 4 \\ -\frac{1}{2} & -1 \end{bmatrix}$. Find e, f, g, and h. $e = 2, f = 8, g = -1, h = -5$

10. Frank and Juanita sold tickets for the charity fund-raiser. They sold both single tickets and 5-ticket books. Write the appropriate matrix equation and find the price of a single ticket and a book of tickets.

Fund-Raiser Tickets Sold			
	Single	Book	Total Sales
Frank	12	4	70
Juanita	8	3	50

$\begin{bmatrix} 12 & 4 \\ 8 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 70 \\ 50 \end{bmatrix}$; a single ticket costs \$2.50 and a book of tickets costs \$10.00.

LESSON 4-5

Reteach

Matrix Inverses and Solving Systems

The identity matrix of a 2×2 matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. If a square matrix A has an inverse A^{-1}, then the product of A and A^{-1} is the identity matrix.

Use the following rule to find the **inverse of a 2×2 matrix.**

The inverse of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Think: **"Switch ops."**
Switch a and d and take the **opposites** of b and c.

If the determinant is 0, the matrix has no inverse.

To find the inverse of $A = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$, first find the determinant.

$\det\begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix} = \begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} = 2 - 4 = -2$

The determinant exists, so the matrix has an inverse.

Then **switch ops** and multiply by $-\frac{1}{2}$.

$A^{-1} = -\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}(1) & -\frac{1}{2}(-1) \\ -\frac{1}{2}(-4) & -\frac{1}{2}(2) \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ 2 & -1 \end{bmatrix}$

Find the inverse of each matrix.

1. $A = \begin{bmatrix} 2 & 7 \\ -1 & -2 \end{bmatrix}$

$\det\begin{bmatrix} 2 & 7 \\ -1 & -2 \end{bmatrix} = \begin{vmatrix} 2 & 7 \\ -1 & -2 \end{vmatrix} = \mathbf{3}$

$\frac{1}{\det A} = \frac{\mathbf{1}}{\mathbf{3}}$

$A^{-1} = \frac{1}{3}\begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & -\frac{7}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

2. $A = \begin{bmatrix} 6 & 1 \\ 8 & 2 \end{bmatrix}$

$\det\begin{bmatrix} 6 & 1 \\ 8 & 2 \end{bmatrix} = \mathbf{4}$

$\frac{1}{\det A} = \frac{\mathbf{1}}{\mathbf{4}}$

$A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -2 & \frac{3}{2} \end{bmatrix}$

3. $A = \begin{bmatrix} -4 & 6 \\ 1 & -2 \end{bmatrix}$ $\det A = \mathbf{2}$ $A^{-1} = \begin{bmatrix} -1 & -3 \\ -\frac{1}{2} & -2 \end{bmatrix}$