

LESSON

Practice B

4-6 *Row Operations and Augmented Matrices*

Write the augmented matrix for each system of equations.

1.
$$\begin{cases} 2x + 1 = y \\ x + y + z = 1 \\ 4y + 5z = 3 \end{cases}$$

2.
$$\begin{cases} 3x = 2y + 4 \\ x - y = 3z \\ 2y + 8z = x \end{cases}$$

3.
$$\begin{cases} x + z = 1 \\ 3x - 5y = 12 \\ 2y - 3z = 9 \end{cases}$$

Write the augmented matrix, and use row reduction to solve.

4.
$$\begin{cases} 4x + 3y = -11 \\ 2x - 3y = 17 \end{cases}$$

5.
$$\begin{cases} 3x + 7y = -1 \\ 6x + 11y = 10 \end{cases}$$

6.
$$\begin{cases} 2x = 3y - 1 \\ 5x - 12y = 2 \end{cases}$$

7.
$$\begin{cases} x + 6y = 0 \\ 2x + 9y = -3 \end{cases}$$

Solve.

8. Dimitri has \$4.95 in dimes and quarters. He has 3 fewer dimes than quarters.

a. Write a system of equations.

Let d = the number of dimes and
 q = the number of quarters.

b. Write the augmented matrix for the system.

c. How many of each coin does Dimitri have?

9. Clara has a bag of 60 coins with a value of \$2.00. The coins are all pennies and nickels. How many of each coin are in the bag?

LESSON Practice A
4-6 Row Operations and Augmented Matrices

Write the augmented matrix for each system of equations.

- $$\begin{cases} 3x + 2y = 2 \\ x = 4y + 24 \end{cases}$$

a. Write each equation in $Ax + By = C$ form. $\begin{cases} 3x + 2y = 2 \\ x - 4y = 24 \end{cases}$

b. Use the coefficients and constants to write the augmented matrix. $\left[\begin{array}{cc|c} 3 & 2 & 2 \\ 1 & -4 & 24 \end{array} \right]$
- $$\begin{cases} 2x + 5y = 1 \\ x - y = 4 \end{cases}$$

$$\left[\begin{array}{cc|c} 2 & 5 & 1 \\ 1 & -1 & 4 \end{array} \right]$$
- $$\begin{cases} 5x = 2y \\ 3x - 4y = 14 \end{cases}$$

$$\left[\begin{array}{cc|c} 5 & -2 & 0 \\ 3 & -4 & 14 \end{array} \right]$$
- $$\begin{cases} 4x = 9y + 1 \\ y = 2x + 1 \end{cases}$$

$$\left[\begin{array}{cc|c} 4 & -9 & 1 \\ -2 & 1 & 1 \end{array} \right]$$

Use row reduction on each matrix to find the reduced row-echelon form.

- $$\left[\begin{array}{cc|c} 3 & 2 & 2 \\ 1 & -4 & 24 \end{array} \right]$$

a. Multiply row 1 by 2. $\left[\begin{array}{cc|c} 6 & 4 & 4 \\ 1 & -4 & 24 \end{array} \right]$

b. Add rows 1 and 2. Write the sum in row 1. $\left[\begin{array}{cc|c} 7 & 0 & 28 \\ 1 & -4 & 24 \end{array} \right]$

c. Divide row 1 by 7. Write the quotient in row 1. $\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 1 & -4 & 24 \end{array} \right]$

d. Subtract row 2 from row 1. Write the result in row 2. $\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 4 & -20 \end{array} \right]$

e. Divide row 2 by 4. Write the result in row 2. $\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -5 \end{array} \right]$

f. Solve for x and y . $x = 4, y = -5$
- $$\begin{cases} 2x + 5y = 1 \\ x - 1y = 4 \end{cases}$$

$$(3, -1)$$
- $$\begin{cases} 5x - 2y = 0 \\ 3x - 4y = 14 \end{cases}$$

$$(-2, -5)$$
- $$\begin{cases} 4x - 9y = 1 \\ -2x + 1y = -11 \end{cases}$$

$$(7, 3)$$

LESSON Practice B
4-6 Row Operations and Augmented Matrices

Write the augmented matrix for each system of equations.

- $$\begin{cases} 2x + 1 = y \\ x + y + z = 1 \\ 4y + 5z = 3 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 0 & -1 \\ 1 & 1 & 1 & 1 \\ 0 & 4 & 5 & 3 \end{array} \right]$$
- $$\begin{cases} 3x = 2y + 4 \\ x - y = 3z \\ 2y + 8z = x \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & -2 & 0 & 4 \\ 1 & -1 & -3 & 0 \\ -1 & 2 & 8 & 0 \end{array} \right]$$
- $$\begin{cases} x + z = 1 \\ 3x - 5y = 12 \\ 2y - 3z = 9 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 3 & -5 & 0 & 12 \\ 0 & 2 & -3 & 9 \end{array} \right]$$

Write the augmented matrix, and use row reduction to solve.

- $$\begin{cases} 4x + 3y = -11 \\ 2x - 3y = 17 \end{cases}$$

$$\left[\begin{array}{cc|c} 4 & 3 & -11 \\ 2 & -3 & 17 \end{array} \right]; (1, -5)$$
 - $$\begin{cases} 2x = 3y - 1 \\ 5x - 12y = 2 \end{cases}$$

$$\left[\begin{array}{cc|c} 2 & -3 & -1 \\ 5 & -12 & 2 \end{array} \right]; (-2, -1)$$
 - $$\begin{cases} 3x + 7y = -1 \\ 6x + 11y = 10 \end{cases}$$

$$\left[\begin{array}{cc|c} 3 & 7 & -1 \\ 6 & 11 & 10 \end{array} \right]; (9, -4)$$
 - $$\begin{cases} x + 6y = 0 \\ 2x + 9y = -3 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & 6 & 0 \\ 2 & 9 & -3 \end{array} \right]; (-6, 1)$$
- Solve.**
- Dimitri has \$4.95 in dimes and quarters. He has 3 fewer dimes than quarters.

a. Write a system of equations. Let d = the number of dimes and q = the number of quarters.

$$\begin{cases} 10d + 25q = 495 \\ d = q - 3 \end{cases}$$

$$\left[\begin{array}{cc|c} 10 & 25 & 495 \\ 1 & -1 & -3 \end{array} \right]$$

12 dimes and 15 quarters
 - Clara has a bag of 60 coins with a value of \$2.00. The coins are all pennies and nickels. How many of each coin are in the bag?

35 nickels and 25 pennies

LESSON Practice C
4-6 Row Operations and Augmented Matrices

Write the augmented matrix, and use row reduction to solve.

- $$\begin{cases} 5x - 3y = 14 \\ 3x + 4y = -9 \end{cases}$$

$$\left[\begin{array}{cc|c} 5 & -3 & 14 \\ 3 & 4 & -9 \end{array} \right]; (1, -3)$$
- $$\begin{cases} 9x + 11y + 19 = 0 \\ 15x + 22y + 17 = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 9 & 11 & 19 & 0 \\ 15 & 22 & 17 & 0 \end{array} \right]; (-7, 4)$$
- $$\begin{cases} 7x + 2y + 4 = 0 \\ 5x - 3y = 37 \end{cases}$$

$$\left[\begin{array}{cc|c} 7 & 2 & -4 \\ 5 & -3 & 37 \end{array} \right]; (2, -9)$$
- $$\begin{cases} 6x + 5 = 5y \\ 3y + 19 = 8x \end{cases}$$

$$\left[\begin{array}{cc|c} 6 & -5 & -5 \\ -8 & 3 & -19 \end{array} \right]; (5, 7)$$
- $$\begin{cases} x + z + 1 = y \\ 4z = y \\ 2x = z + 8 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -1 \\ 0 & -1 & 4 & 0 \\ 2 & 0 & -1 & 8 \end{array} \right]; (5, 8, 2)$$
- $$\begin{cases} 3y + z = -1 \\ 5y + 2z = 3x \\ x + y = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 0 & 3 & 1 & -1 \\ -3 & 5 & 2 & 0 \\ 1 & 1 & 0 & 0 \end{array} \right]; (-1, 1, -4)$$

Solve.

- Jonah is buying a cell phone. He has a choice of 2 plans. The monthly base price of each plan includes 500 minutes. There is a charge for each additional minute over 500 minutes.

	Base Price (500 min)	Additional (per min)
Plan A	\$40	\$0.50
Plan B	\$75	\$0.25

a. For what number of minutes is the total monthly cost the same for each plan?
640 minutes

b. Jonah expects to use between 11 and 12 hours of cell phone time each month. Which plan is the better buy for him?
Plan B

8. Andrea has \$2.10 in nickels, dimes, and quarters. She has a total of 18 coins with as many nickels as she has dimes and quarters combined. How many dimes does Andrea have?
4 dimes

LESSON Reteach
4-6 Row Operations and Augmented Matrices

To write the augmented matrix of a system of linear equations, use the coefficients and the constant terms of the system.

Write linear systems in two variables in the form $Ax + By = C$ to write the augmented matrix.

System of Linear Equations	Augmented Matrix
$\begin{cases} x + 6 = 4y \\ y - 3 = 2x \end{cases} \rightarrow \begin{cases} x - 4y = -6 \\ 2x - y = -3 \end{cases}$	$\left[\begin{array}{cc c} 1 & -4 & -6 \\ 2 & -1 & -3 \end{array} \right]$

The line separates the coefficients from the constants.

Write linear systems in three variables in the form $Ax + By + Cz = D$ to write the augmented matrix.

System of Linear Equations	Augmented Matrix
$\begin{cases} x + y = z + 5 \\ 2z - x = 3 \\ y = 4z - 1 \end{cases} \rightarrow \begin{cases} x + y - z = 5 \\ -x + 0y + 2z = 3 \\ 0x + y - 4z = -1 \end{cases}$	$\left[\begin{array}{ccc c} 1 & 1 & -1 & 5 \\ -1 & 0 & 2 & 3 \\ 0 & 1 & -4 & -1 \end{array} \right]$

Write the augmented matrix for the system of equations.

- $$\begin{cases} 5x - 1 = 7y \\ y - 3 = 2x \end{cases} \rightarrow \begin{cases} 5x - 7y = 1 \\ 2x - y = -3 \end{cases}$$

$$\left[\begin{array}{cc|c} 5 & -7 & 1 \\ 2 & -1 & -3 \end{array} \right]$$
- $$\begin{cases} 8x = y - 9 \\ -x - 7 = 4y \end{cases}$$

$$\left[\begin{array}{cc|c} 8 & -1 & -9 \\ -1 & -4 & 7 \end{array} \right]$$
- $$\begin{cases} x + y = z + 5 \\ 2z - x = 3 \\ y = 4z - 1 \end{cases} \rightarrow \begin{cases} x + y - z = 5 \\ -x + 0y + 2z = 3 \\ 0x + y - 4z = -1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ -1 & 0 & 2 & 3 \\ 0 & 1 & -4 & -1 \end{array} \right]$$
- $$\begin{cases} x - y = 1 - z \\ 3x = 5z + 2 \\ z = 6y - 8 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 3 & 0 & -5 & 2 \\ 0 & -6 & 1 & -8 \end{array} \right]$$
- $$\begin{cases} z + 3y = x \\ 2x = y - 8z \\ y + 4 = z + z \end{cases}$$

$$\left[\begin{array}{ccc|c} -1 & 3 & 1 & 0 \\ 2 & -1 & 8 & 0 \\ -1 & 1 & -1 & -4 \end{array} \right]$$