Name ______________________________ Date __________ Class __________

LESSON **4-6**

Problem Solving

Row Operations and Augmented Matrices

At the annual craft show, the Ceramics Club members sell mugs for \$6.00, bowls for \$5.50, and plates for \$9.50. They have for sale one more bowl than the number of plates and 3 times as many mugs as plates. They sold everything for a total of \$236.50. How many of each item did they sell?

1. Write a system of equations to represent the problem, using *m*, *b*, and *p* for the variables. ______________________

2. Write the augmented matrix for the system of equations. ______________________

3. Use your calculator to find the reduced row-echelon form of the augmented matrix. ______________________

4. How many of each item did the Ceramics Club sell? ______________________

Students earned points for finishing first, second, and third in the field day games. Jake earned a total of 38 points, Wanda earned 33 points, and Jill earned 29 points. How many points were earned for each first-, second-, and third-place finish? Choose the letter for the best answer.

Field Day Tally

	Jake	Wanda	Jill
First	IIII	I	I
Second		卌	III
Third	II		II

5. Which augmented matrix models the problem?

A $\left[\begin{array}{ccc:c} 4 & 1 & 1 & 38 \\ 0 & 5 & 3 & 33 \\ 1 & 0 & 2 & 29 \end{array}\right]$ **B** $\left[\begin{array}{ccc:c} 4 & 2 & 0 & 38 \\ 1 & 5 & 0 & 33 \\ 0 & 3 & 2 & 29 \end{array}\right]$

C $\left[\begin{array}{ccc:c} 4 & 0 & 2 & 38 \\ 1 & 5 & 0 & 33 \\ 1 & 3 & 2 & 29 \end{array}\right]$ **D** $\left[\begin{array}{ccc:c} 4 & 1 & 2 & 38 \\ 0 & 3 & 5 & 33 \\ 1 & 2 & 3 & 29 \end{array}\right]$

6. Which matrix in reduced row-echelon form is the solution to the problem?

A $\left[\begin{array}{ccc:c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right]$ **B** $\left[\begin{array}{ccc:c} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 3 \end{array}\right]$

C $\left[\begin{array}{ccc:c} 0 & 0 & 1 & 8 \\ 0 & 1 & 0 & 5 \\ 1 & 0 & 0 & 3 \end{array}\right]$ **D** $\left[\begin{array}{ccc:c} 0 & 0 & 1 & 10 \\ 0 & 1 & 0 & 6 \\ 1 & 0 & 0 & 3 \end{array}\right]$

LESSON **4-6**

Reteach

Row Operations and Augmented Matrices (continued)

You can use row operations to change an augmented matrix into **reduced row-echelon form**. In reduced row-echelon form, the solution to the system appears in the constant column of the augmented matrix.

Reduced row-echelon form shows the identity matrix on the left. $\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \end{array}\right]$

$1x + 0y = 2$ or $x = 2$

$0x + 1y = -3$ or $y = -3$

To solve $\begin{cases} x + y = 4 \\ 2x - y = -7 \end{cases}$, use the augmented matrix $\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 2 & -1 & -7 \end{array}\right]$.

$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 2 & -1 & -7 \end{array}\right]$

Add row 1 to row 2. $\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 3 & 0 & -3 \end{array}\right]$ Row 1 stays the same. The sum becomes row 2.

Switch rows 1 and 2. $\left[\begin{array}{cc|c} 3 & 0 & -3 \\ 1 & 1 & 4 \end{array}\right]$

Divide row 1 by 3. $\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 1 & 1 & 4 \end{array}\right]$

Subtract row 1 from row 2. $\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 5 \end{array}\right]$ $x = -1$, $y = 5$

Solve.

5. $\begin{cases} x + y = -2 \\ x - 2y = 7 \end{cases}$

a. Write the augmented matrix. $\left[\begin{array}{cc|c} 1 & 1 & -2 \\ \mathbf{1} & \mathbf{-2} & \mathbf{7} \end{array}\right]$

b. Subtract row 1 from row 2. $\left[\begin{array}{cc|c} 1 & 1 & -2 \\ \mathbf{0} & \mathbf{-3} & \mathbf{9} \end{array}\right]$

c. Divide row 2 by **−3**. $\left[\begin{array}{cc|c} 1 & 1 & -2 \\ \mathbf{0} & \mathbf{1} & \mathbf{-3} \end{array}\right]$

d. Subtract row 2 from row 1. $\left[\begin{array}{cc|c} \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{-3} \end{array}\right]$

e. $x =$ **1** $y =$ **−3**

LESSON **4-6**

Challenge

Using an Augmented Matrix to Determine an Inverse

To *augment* means to make greater. Augment a square matrix on its right side by an identity matrix of the same dimensions and then use row operations until the identity matrix appears on the left half of the matrix. This is the inverse of the original matrix.

- **Example**

Use an augmented matrix to find the inverse of the 2 × 2 matrix, A, that is shown at right. $A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$

- **Solution**

Augment matrix A by the 2 × 2 identity matrix on the right. Use row operations until the identity matrix appears on the left. $\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right]$

$(3R_1 - R_2) \rightarrow R_2 \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & -1 & 3 & -1 \end{array}\right] (R_1 + R_2) \rightarrow R_1 \left[\begin{array}{cc|cc} 1 & 0 & 4 & -1 \\ 0 & -1 & 3 & -1 \end{array}\right] -R_2 \rightarrow R_2 \left[\begin{array}{cc|cc} 1 & 0 & 4 & -1 \\ 0 & 1 & -3 & 1 \end{array}\right]$

So, $A^{-1} = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$

Some square matrices do not have inverses. If the reduced augmented matrix you create has one or more rows that contain all zero elements to the left of the vertical line, then the given matrix has no inverse.

Use an augmented matrix to find the inverse of the given matrix, if it is defined. Verify your result.

1. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ **Answer:** $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

2. $\begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}$ **Answer:** $\begin{bmatrix} \frac{2}{11} & \frac{3}{11} \\ -\frac{1}{11} & \frac{4}{11} \end{bmatrix}$

3. $\begin{bmatrix} -2 & 6 \\ -1 & 3 \end{bmatrix}$ **The inverse is not defined.**

4. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 7 \\ 0 & 1 & 2 \end{bmatrix}$ **Answer:** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -7 \\ 0 & -1 & 4 \end{bmatrix}$

5. $\begin{bmatrix} -2 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{bmatrix}$ **Answer:** $\begin{bmatrix} -\frac{4}{3} & -\frac{5}{3} & 1 \\ -\frac{4}{3} & -\frac{8}{3} & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix}$

6. $\begin{bmatrix} -2 & 5 & 3 \\ 4 & -1 & 3 \\ 4 & -10 & -6 \end{bmatrix}$ **The inverse is not defined.**

LESSON **4-6**

Problem Solving

Row Operations and Augmented Matrices

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1. Write a system of equations to represent the problem, using *m*, *b*, and *p* for the variables.

$\begin{cases} b = p + 1 \\ m = 3p \\ 6m + 5.5b + 9.5p = 236.5 \end{cases}$

2. Write the augmented matrix for the system of equations.

$\left[\begin{array}{ccc|c} 0 & 1 & -1 & 1 \\ 1 & 0 & -3 & 0 \\ 6 & 5.5 & 9.5 & 236.5 \end{array}\right]$

3. Use your calculator to find the reduced row-echelon form of the augmented matrix.

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 21 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 7 \end{array}\right]$

4. How many of each item did the Ceramics Club sell? **21 mugs, 8 bowls, 7 plates**

Students earned points for finishing first, second, and third in the field day games. Jake earned a total of 38 points, Wanda earned 33 points, and Jill earned 29 points. How many points were earned for each first-, second-, and third-place finish? Choose the letter for the best answer.

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	Jake	Wanda	Jill
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5. Which augmented matrix models the problem?

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(C) $\left[\begin{array}{ccc|c} 4 & 0 & 2 & 38 \\ 1 & 5 & 0 & 33 \\ 1 & 3 & 2 & 29 \end{array}\right]$ D $\left[\begin{array}{ccc|c} 4 & 1 & 2 & 38 \\ 0 & 3 & 5 & 33 \\ 1 & 2 & 3 & 29 \end{array}\right]$

6. Which matrix in reduced row-echelon form is the solution to the problem?

(A) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right]$ B $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 3 \end{array}\right]$

C $\left[\begin{array}{ccc|c} 0 & 0 & 1 & 8 \\ 0 & 1 & 0 & 5 \\ 1 & 0 & 0 & 3 \end{array}\right]$ D $\left[\begin{array}{ccc|c} 0 & 0 & 1 & 10 \\ 0 & 1 & 0 & 6 \\ 1 & 0 & 0 & 3 \end{array}\right]$

LESSON **4-6**

Reading Strategies

Understand Vocabulary

<table>
<tr><td>Augmented Matrix
$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases} \longrightarrow \left[\begin{array}{cc|c} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{array}\right]$
$\begin{cases} 4x + 3y = 10 \\ x - 2y = -3 \end{cases} \longrightarrow \left[\begin{array}{cc|c} 4 & 3 & 10 \\ 1 & -2 & -3 \end{array}\right]$</td><td>A system of linear equations can be represented as an augmented matrix. In this form the coefficient terms are to the left of the vertical line, and the constant terms are to the right.</td></tr>
<tr><td>Row Operations
For example subtract row 2 from row 1 to create a new row 2.
$\left[\begin{array}{cc|c} a & b & c \\ d & e & f \end{array}\right] = \left[\begin{array}{cc|c} a & b & c \\ a-d & b-e & c-f \end{array}\right]$
$\left[\begin{array}{cc|c} 4 & 3 & 10 \\ 1 & -2 & -3 \end{array}\right] = \left[\begin{array}{cc|c} 4 & 3 & 10 \\ 3 & 5 & 13 \end{array}\right]$</td><td>Row operations change the form of an augmented matrix in the process of solving a system of equations. The new matrix formed is equivalent to the original matrix.</td></tr>
<tr><td>Row Reduction
Use the row reduction feature on your graphing calculator, rref, to find the reduced row-echelon form of an augmented matrix.
$\left[\begin{array}{cc|c} 4 & 3 & 10 \\ 1 & -2 & -3 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right]$
$x = 1$ and $y = 2$</td><td>A series of row operations is referred to as row reduction. The object is to find an equivalent form of the augmented matrix that solves the system of equations. This form is called the reduced row-echelon form $\left[\begin{array}{cc|c} 1 & 0 & m \\ 0 & 1 & n \end{array}\right]$ where m and n are constants. So $x = m$ and $y = n$.</td></tr>
</table>

Use the augmented matrix $\left[\begin{array}{cc|c} -5 & 10 & 3 \\ 2 & -4 & 1 \end{array}\right]$ **for Exercises 1–3.**

1. Explain how an augmented matrix represents a system of equations.

Possible answer: An augmented matrix shows the coefficients and the constants of the linear equations in the order they appear in the equations.

2. Write the system of equations represented by the augmented matrix.

$\begin{cases} -5x + 10y = 3 \\ 2x - 4y = 1 \end{cases}$

3. Compare multiplying the first equation in the system of equations by 3 and the row operation of multiplying the first row of the augmented matrix by 3. What is the effect of each operation?

Possible answer: Multiplying the equation by 3 gives an equation equivalent to the original equation. In the same way, multiplying one row of the augmented matrix by 3 gives a matrix equivalent to the original matrix.